CHAPTER - 4
TORSION :-

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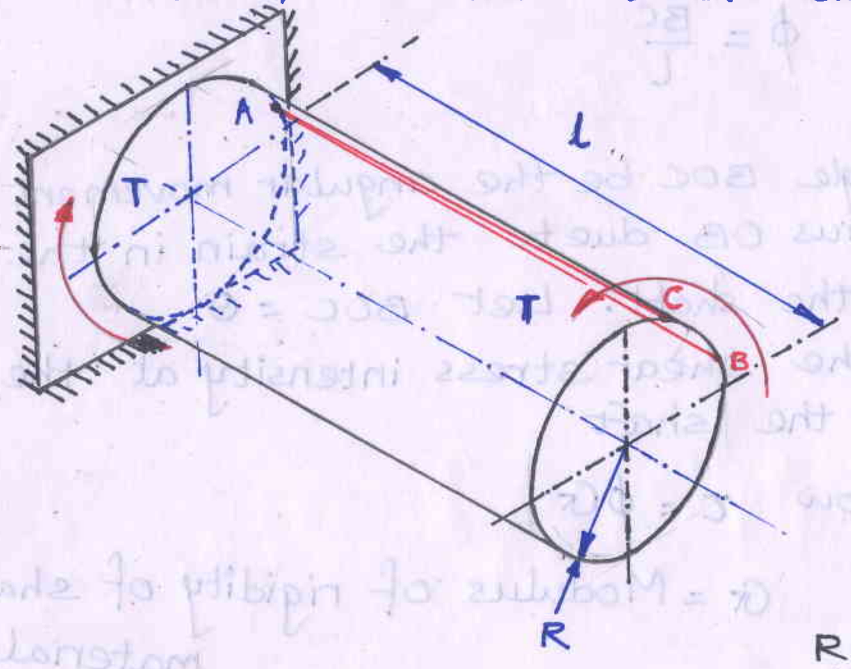
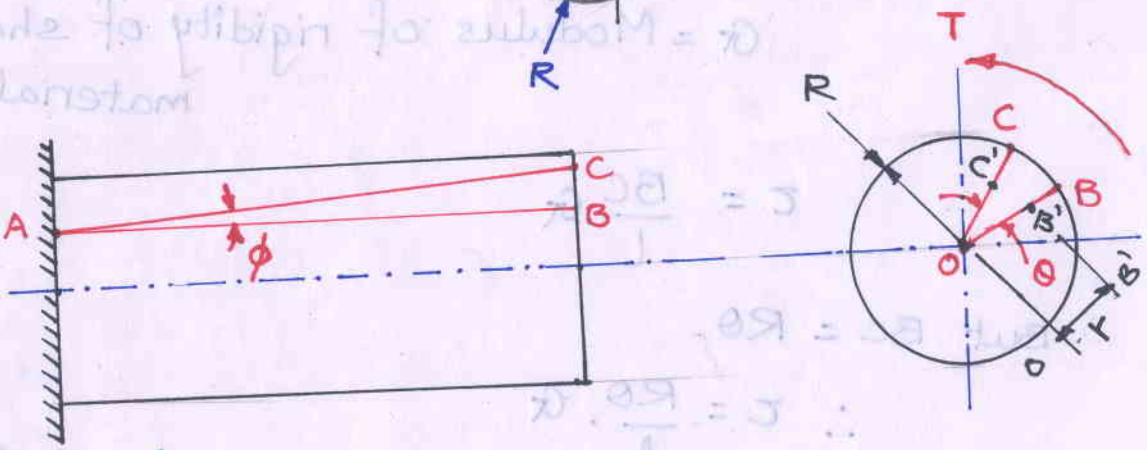
Pure torsion :-

A shaft of circular section is said to be in pure torsion when it is subjected to equal and opposite end couple whose axis coincide with the axis of shaft.

While a beam bends as an effect of bending moment, a shaft twists as an effect of torsion





Theory of pure torsion:-

In above fig. shows a solid cylindrical shaft of radius R and length l subjected to a couple or twisting moment T at one end, while its other end is held or fixed by the application of a balancing couple of the same magnitude.

Let AB be a line on the surface of the shaft and parallel to axis of the shaft before the deformation of the shaft. As an effect of torsion this line, after the deformation of the shaft, takes the form AC

The angle $CAB = \phi$ represents the shear strain of the shaft material at the surface

$$\therefore BC = l\phi \quad \text{(angle being small)}$$

$$\phi = \frac{BC}{l}$$

Let the angle BOC be the angular movement of the radius OB due to the strain in the length of the shaft. Let $BOC = \theta$

Let τ be the shear stress intensity at the surface of the shaft

We know $\tau = \phi G$

G = Modulus of rigidity of shaft material

$$\tau = \frac{BC}{l} G$$

But $BC = R\theta$

$$\therefore \tau = \frac{R\theta}{l} \cdot G$$

$$\boxed{\frac{\tau}{R} = \frac{G\theta}{l}} \quad \Rightarrow \tau = \left(\frac{G\theta}{L}\right) R$$

Constant

$$\tau \propto R$$

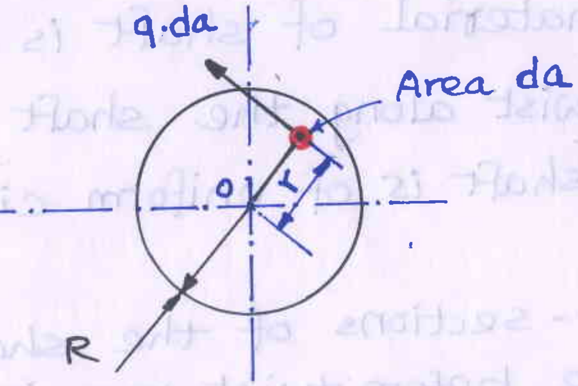
shear stress at any pt. in shaft is proportional to the distance of the point from the axis of the shaft

deformation in interior cylinder at distance r

shear stress intensity q at the radius r is given

$$\frac{q}{r} = \frac{G\theta}{l} = \frac{\tau}{R}$$

Moment of resistance



The section of shaft of radius R subjected to pure torsion. Let τ be the maximum shear stress which occurs at the surface.

Consider an elemental area da at a distance r from the axis of the shaft.

shear stress offered by the elemental area

$$= q = \frac{r}{R}\tau$$

$\therefore$ shear resistance offered by the elemental area

$$q.da = \frac{r}{R}\tau \cdot da$$

$\therefore$ Moment of resistance offered by the elemental

$$= \frac{r}{R}\tau \cdot da \cdot r$$

$$= \frac{\tau}{R} \cdot da \cdot r^2$$

Total moment of resistance offered

$$T = \frac{\tau}{R}\sum da \cdot r^2 \text{ OR } = \frac{\tau}{R}\int da \cdot r^2$$

where $\int da \cdot r^2 = J$ polar moment of inertia

$$\therefore T = \frac{\tau}{R} J \Rightarrow \frac{T}{J} = \frac{\tau}{R}$$

But : $\frac{\tau}{R} = \frac{G\theta}{l} \Rightarrow$ $\boxed{\frac{\tau}{R} = \frac{T}{J} = \frac{G\theta}{l}}$

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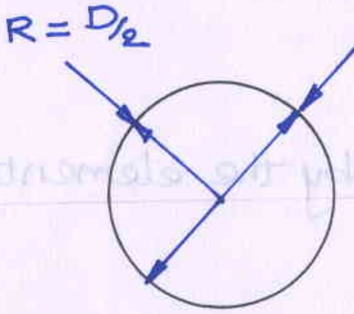
Assumption in the theory of pure torsion :-

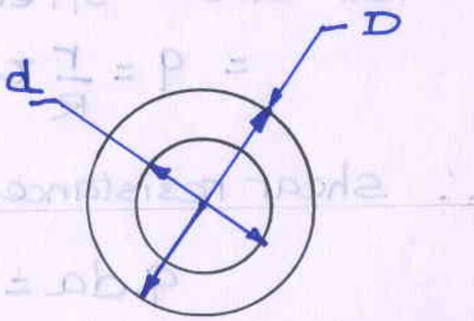
1) The material of shaft is uniform throughout.
2) The twist along the shaft is uniform.
3) The shaft is of uniform circular section throughout.
4) Cross-sections of the shaft, which are plane before twist remain plane after twist
5) All radii which are straight before twist remain straight after twist.

Application of theory of torsion eq^n.

Solid & Hollow circular shaft



Solid shaft



Hollow shaft

Maxi shear stress is on the surface

$$\tau_{max} = \frac{T \times R}{\frac{\pi}{32} D^4} = \frac{16T}{\pi D^3}$$

Max. shear stress is on the surface

$$\tau_{max} = \frac{T \times R}{\frac{\pi}{32}(D^4 - d^4)}$$

$$= \frac{16TD}{\pi(D^4 - d^4)}$$

Torsional Rigidity

$$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{R}$$

$$\frac{T}{(\theta/l)} = GJ$$

GJ is called the torsional Rigidity or stiffness, similar to EI which is called bending stiffness