Basics of Stress & strain

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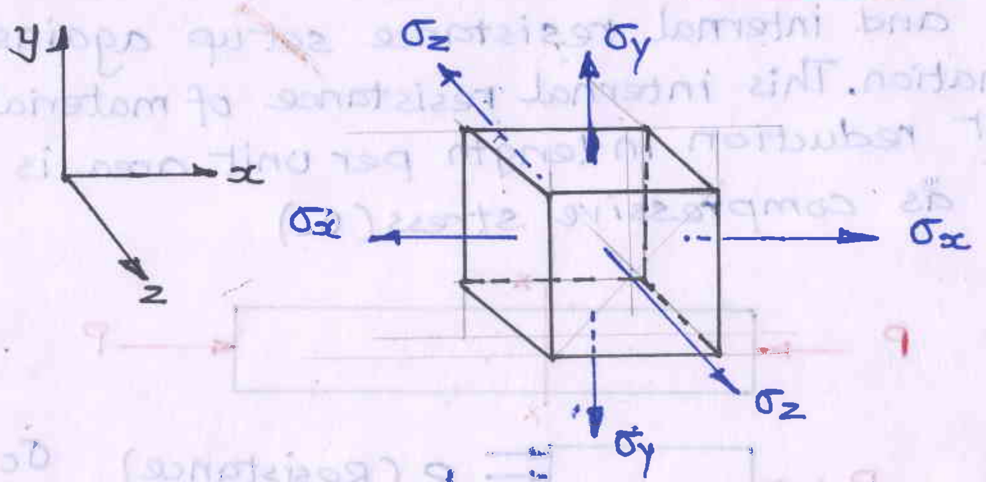
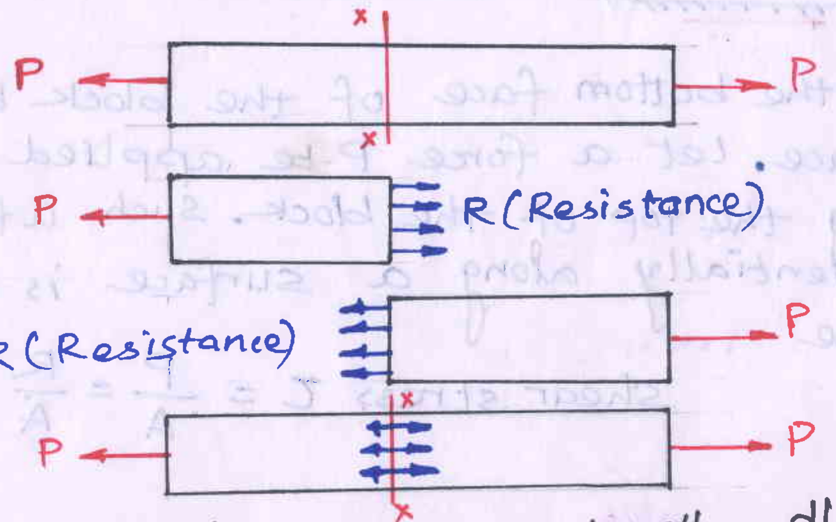
3-D state of stress Normal/axial stresses:-



Stress:- When a material is subjected to external force or load, it deforms and material offers resistance to deformation. The intensity of internal resistance against deformation per unit area is known as stress

$$\text{Stress} = \frac{\text{Internal resistance}}{\text{Cross-Sectional Area}} \quad N/m^2$$

Tensile Stress:- When an element is subjected to an axial pull, the length of member increases and internal resistance set-up against deformation. This internal resistance of material against increase in length per unit area is known as tensile stress (σ_t)



$$\sigma_t = \frac{P}{A} = \frac{R}{A}$$

$$\text{Tensile strain } (\varepsilon) = \frac{\text{Increase in length}}{\text{Original length}} = \frac{dL}{L}$$

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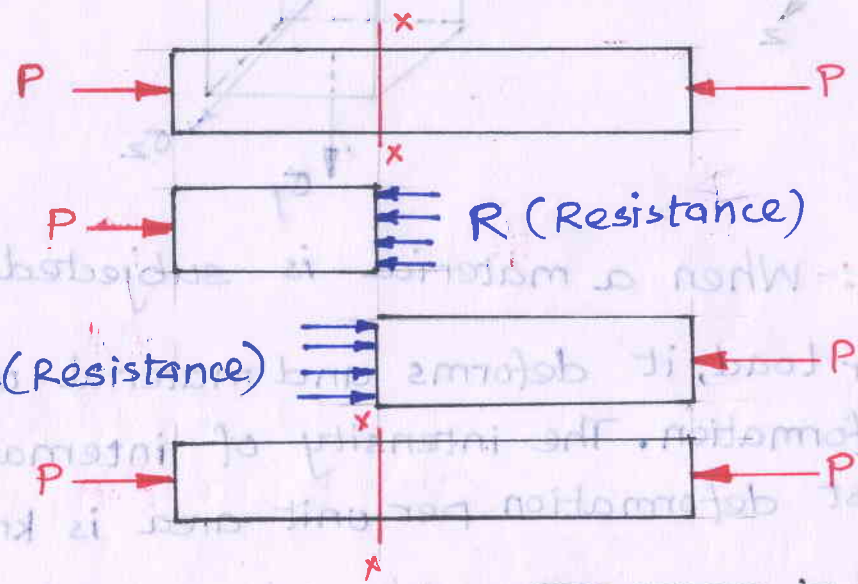
Compressive Stress:– When an element is subjected to an axial push, the length of element reduces and internal resistance setup against deformation. This internal resistance of material against reduction in length per unit area is known as compressive stress (σ_c)

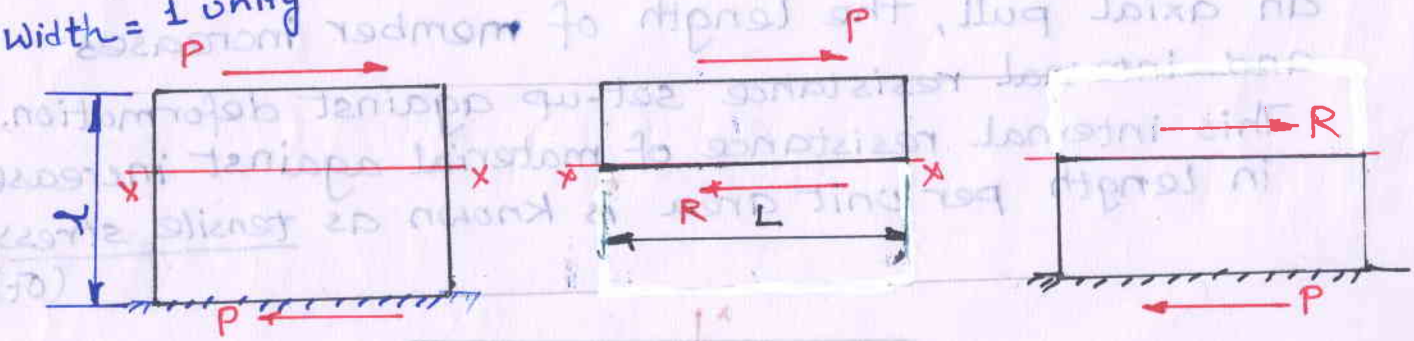


$$\sigma_c = \frac{P}{A} = \frac{R}{A}$$

$$\text{Compressive strain } (\varepsilon) = \frac{\text{Decrease in length}}{\text{Original length}} = \frac{dl}{L}$$

Shear stress:–

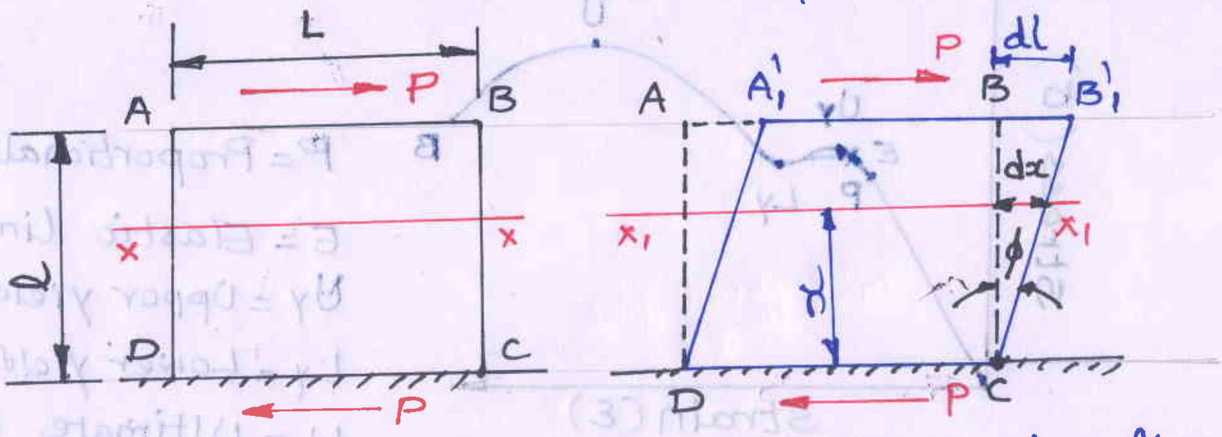
Width = 1 unity



Let the bottom face of the block be fixed to a surface. let a force P be applied tangentially along the top of the block. such a force acting tangentially along a surface is called shear force

$$\text{shear stress } \tau = \frac{P}{A} = \frac{R}{A} = \frac{R}{L \times 1} = \frac{P}{L \times 1}$$

Shear deformation:- When the block does not fail in shear, a shear deformation occurs



If the bottom face of the block be fixed, it can be realized that the block has deformed to position $A_1' B_1' CD$. or we can say, that the face ABCD has been distorted to the position $A_1' B_1' CD$ through the angle $BCB_1' = \phi$

The ratio $\frac{dl}{l} = \frac{\text{Transverse displacement}}{\text{Distance from the lower face}}$ is called the shear train

We could have considered any other horizontal layer say the layer XX which is at distance x from the lower face. Let dx be the horizontal displacement of the layer XX

The shear strain $= \frac{dx}{x}$

ϕ is very small, $\phi = \tan\phi = \frac{dl}{l}$ = shear strain

Elastic Limit:-

A material is said to be elastic when it undergoes a deformation on the application of a loading such that the deformation disappears on the removal of the loading. When a member is subjected to an axial loading, its section will offer a resistance or stress. When the loading is removed, obviously the stress will vanish and the deformation will also vanish.

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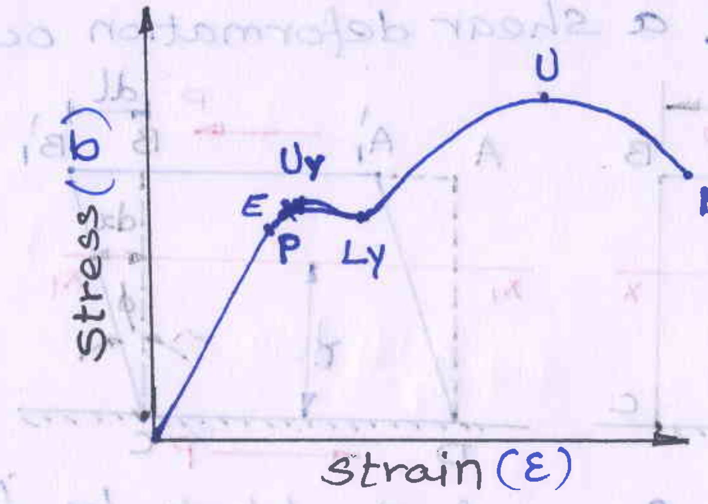
But this is true when the intensity of stress is within a certain limit called the elastic limit



P = Proportionality Limit
E = Elastic limit
Uy = Upper yield point
Ly = Lower yield point
U = Ultimate point
B = Breaking point

Stress - strain behaviour of M.S.

Hooke's law :-

→ The stress is directly proportional to strain within proportionality limit, stress ∝ strain

stress = A constant × strain

$$\sigma \propto \varepsilon$$

$$\sigma = E\varepsilon$$ where E is constant of proportionality

$$E = \frac{\sigma}{\varepsilon}$$

→ E is known as Young's modulus or Modulus of elasticity

→ The shear stress is proportional to shear train within elastic limit

$$\tau \propto \phi \qquad G = \frac{\tau}{\phi}$$

$$\tau = G\phi$$ where G is constant

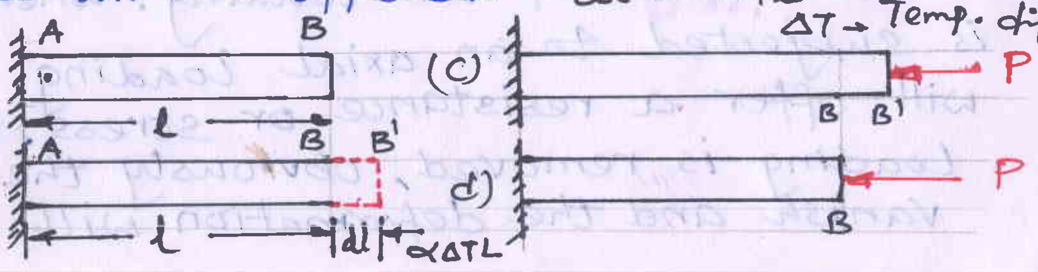
→ G is known as shear modulus or Modulus of rigidity

Thermal stress :-

When the temperature of a material changes there will be corresponding change in dimension. When a member is free to expand or contract due to rise or fall of temperature, no stress will be induced in the member, But, if the natural change in length due to rise or fall of temperature be prevented, stresses will be offered. $dl = \alpha \Delta T l$

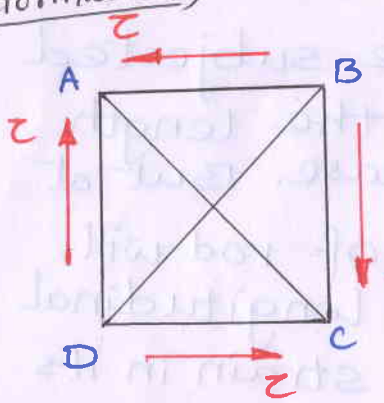
ΔT → Temp. difference

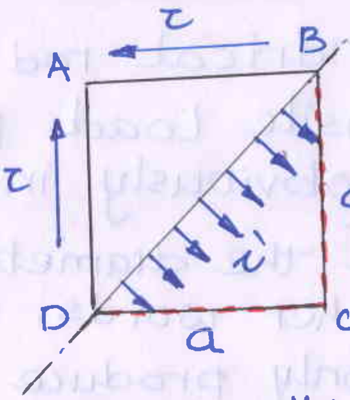
Where α is the coefficient of linear expansion

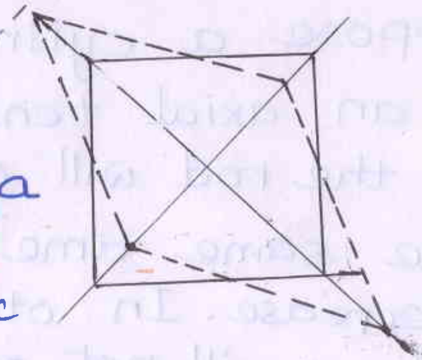


Stresses and strain Along the Diagonals

(Information)







width → unity

Force on $\left.\begin{matrix} AB \\ AD \\ CD \\ BC \end{matrix}\right\} = \tau \times a \times 1$

Length of diagonals BD & AC

$= a\sqrt{2}$

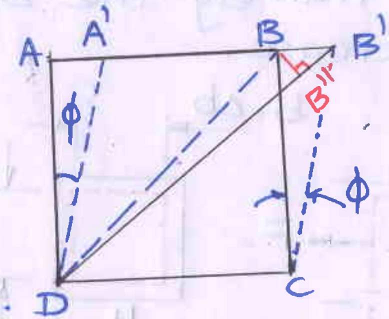
Resolving the forces perpendicular to diagonal BD

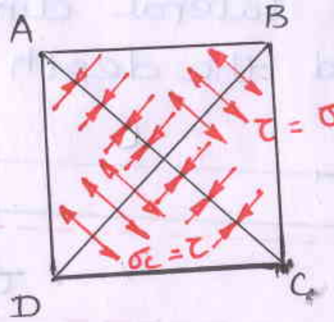
$$\tau' \times a\sqrt{2} \times 1 = 2 \times \tau \times a \times 1 \times \cos 45^\circ$$

$$= 2 \times \tau \times a \times 1 \times \frac{1}{\sqrt{2}} = \tau a \sqrt{2}$$

$$\tau' = \tau$$

The tensile stress (σ_t) along BD and compressive stress (σ_c) along AC are both equal to τ





stain along the diagonals

$$\varepsilon_{BD} = \varepsilon_{AC} = \frac{B'B''}{BD} = \frac{BB' \cos 45^\circ}{CD \sec 45^\circ}$$

$$\varepsilon_{BD} = \varepsilon_{AC} = \frac{1}{2} \frac{BB'}{CD} = \frac{1}{2} \phi$$

The linear strain on the diagonals is equal to half the shear strain

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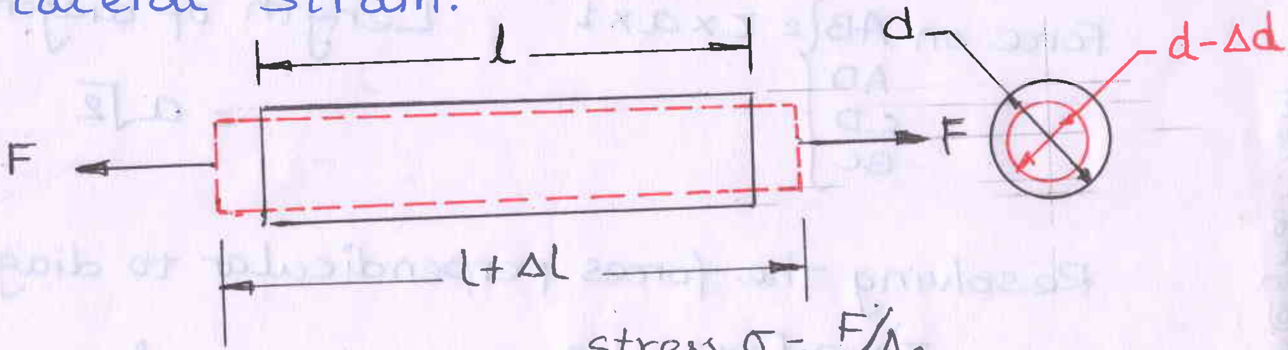
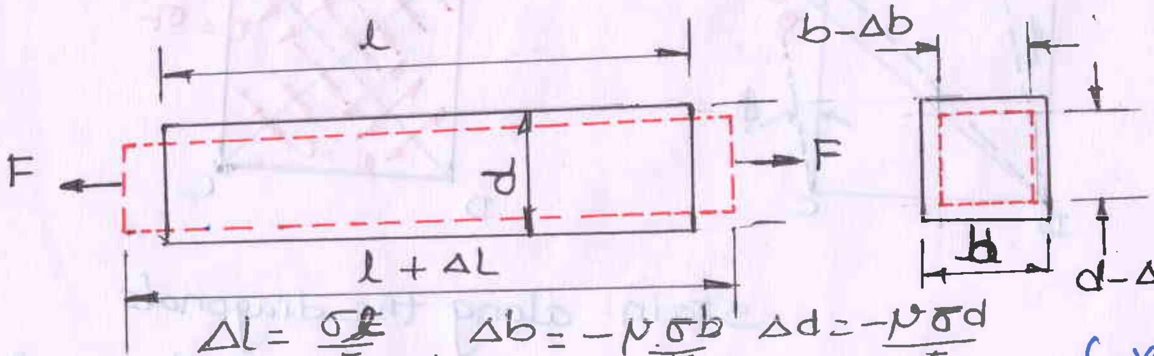
Lateral strain and poisson's Ratio

Suppose a cylindrical rod be subjected to an axial tensile load F, the length of the rod will obviously increase. But at the same time the diameter of rod will decrease. In other words, a longitudinal stress will not only produce a strain in its own direction but will also produce a lateral strain.



stress $\sigma = F/A_c$

Similarly suppose a rectangular bar of width b, depth d and length l be subjected to an axial tensile load. The deformation of the member will take place such that the length of the member will increase while the lateral dimensions namely the breadth and the depth will decrease.



$\varepsilon_l = \frac{\Delta L}{L}$

$\varepsilon_d = -\frac{\Delta d}{d}$

$\varepsilon_b = -\frac{\Delta b}{b}$

(-ve sign for decrease)

$\Delta l = \frac{\sigma l}{E}$, $\Delta b = -\frac{\nu \sigma b}{E}$ $\Delta d = -\frac{\nu \sigma d}{E}$

Let ΔL be the increase in length and Δb and Δd the decrease in width & depth

The ratio $\frac{\Delta L}{L}$ is called the longitudinal strain while $\frac{\Delta b}{b}$ and $\frac{\Delta d}{d}$ is called the Lateral strain.

When the deformation of the member is within the elastic limit it is found that the ratio of the lateral strain to the longitudinal strain is a constant for a given material. This ratio is called Poisson's ratio

$$\boxed{\text{Poisson's ratio}(\nu) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}}$$

$$\nu = \frac{\varepsilon_b}{\varepsilon_L}, \quad \nu = \varepsilon_d/\varepsilon_L$$

Most of the metals we comes across $\frac{1}{\nu}$ lies betn 3 & 4

If $\varepsilon_d = \varepsilon_b$ $\quad \nu = \frac{\varepsilon_b}{\varepsilon_L} = \frac{\varepsilon_d}{\varepsilon_L}$, $\varepsilon_b = \nu\varepsilon_L$, $\varepsilon_d = \nu\varepsilon_L$

Volumetric strain of a Rectangular Bar

Original volume $V = lbd$

Final volume $= (l+\Delta l)(b+\Delta b)(d+\Delta d)$

$= (lb + l\Delta b + \Delta l b + \Delta l\Delta b)(d+\Delta d)$

$= lbd + lb\Delta d + ld\Delta b + l\Delta b\Delta d + \Delta l bd + \Delta l\Delta d b + \Delta l\Delta b d + \Delta l\Delta b\Delta d$

$= lbd + bd\Delta l + lb\Delta d + ld\Delta b$

(Neglecting small quantities)

change in volume $= \Delta V =$ final volume $-$ Original volume

$= bd\Delta l + lb\Delta d + ld\Delta b$

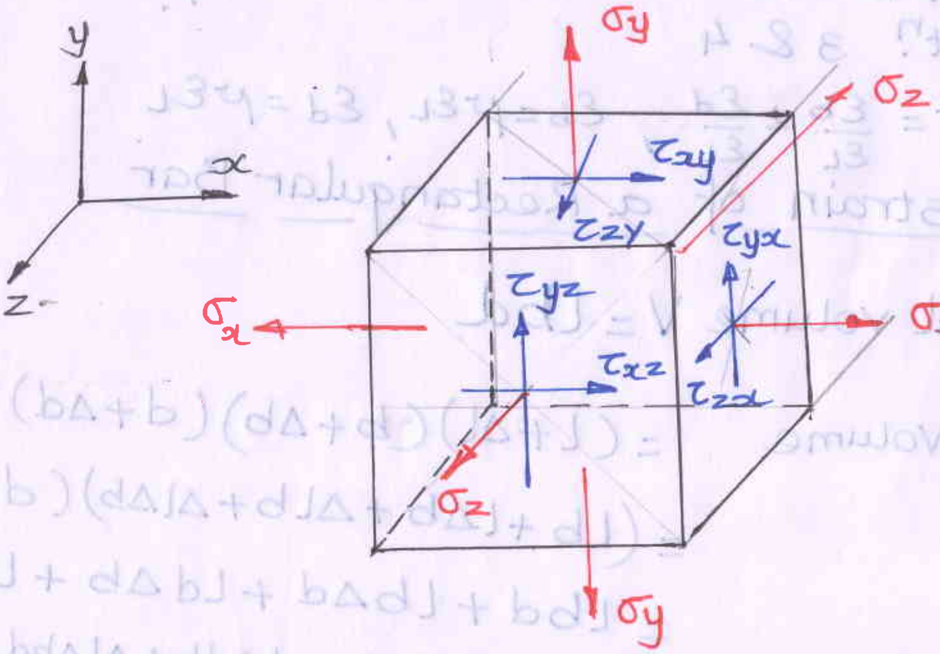
Volumetric strain $\varepsilon_v = \frac{\text{Change in volume}}{\text{Original volume}}$

$$= \frac{bd\Delta l + lb\Delta d + dl\Delta b}{lbd}$$

$$= \frac{\Delta l}{l} + \frac{\Delta d}{d} + \frac{\Delta b}{b}$$

$$\boxed{\varepsilon_v = \varepsilon_l + \varepsilon_d + \varepsilon_b}$$ Volumetric strain

Multi - Axial stresses :-



Normal stresses σ_x, σ_y and σ_z act along the principal direction X, Y, and Z, Let us assume that all the stresses are tensile

strain in X - $\varepsilon_x = \frac{\sigma_x}{E} - \frac{\sigma_y \nu}{E} - \nu\frac{\sigma_z}{E}$

strain in Y - $\varepsilon_y = \frac{\sigma_y}{E} - \frac{\sigma_z \nu}{E} - \frac{\sigma_x \nu}{E}$

strain in Z - $\varepsilon_z = \frac{\sigma_z}{E} - \frac{\sigma_x \nu}{E} - \frac{\sigma_y \nu}{E}$

Shear strain $\phi_{xy} = \frac{\tau_{xy}}{G}$

$\phi_{yz} = \frac{\tau_{yz}}{G}$

$\phi_{zx} = \frac{\tau_{zx}}{G}$

Adding $\varepsilon_x + \varepsilon_y + \varepsilon_z$

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\sigma_x + \sigma_y + \sigma_z}{E} - 2\mu\left[\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right]$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E}\left[1 - 2\mu\right]$$

If $\sigma_x = \sigma_y = \sigma_z = \sigma$ then

$$\varepsilon_v = \frac{3\sigma}{E}(1 - 2\mu)$$

This is a case of hydrostatic tension, where the stresses are equal and tensile in all the three direction

Bulk Modulus

Bulk modulus is defined in terms of stress and volumetric strain

$$K = \frac{\text{stress}}{\text{Volumetric strain}}$$

$$\boxed{K = \frac{\sigma}{\varepsilon_v}}$$

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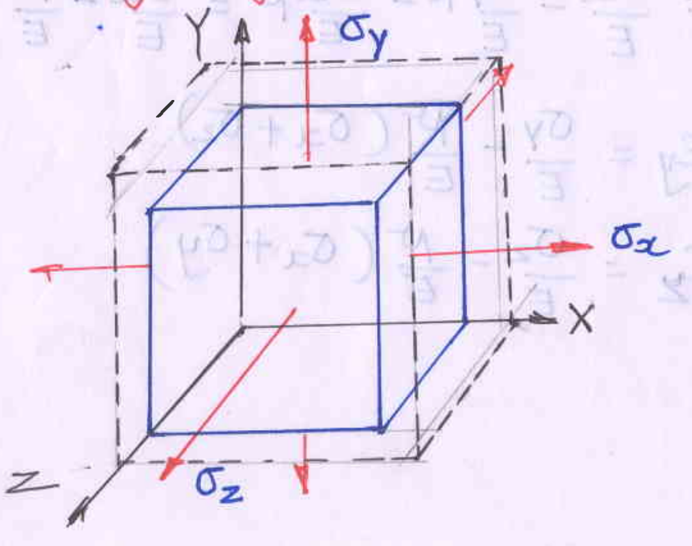
Relationship betn. Youngs Modulus, Modulas of Rigidity & Bulk Modulus



We need the three elastic constants, E, G and K of a material and it Poisson's Ratio ν

Consider a cub of side 1 (unity) subjected to tensile multi-axial stresses σ_x, σ_y and σ_z. Under the action of these stresses, the body deform into a rectangular parallepiped

Volume is given by

$$V' = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)$$

Original volume $V = 1 \times 1 \times 1 = 1$

where ε_x, ε_y and ε_z are corosponding strain in x, y, & z

$$V' = (1+\varepsilon_x+\varepsilon_y+\varepsilon_x\varepsilon_y)(1+\varepsilon_z)$$

$$= (1+\varepsilon_x+\varepsilon_y+\varepsilon_x\varepsilon_y+\varepsilon_z+\varepsilon_x\varepsilon_z+\varepsilon_y\varepsilon_z+\varepsilon_x\varepsilon_y\varepsilon_z)$$

$$= (1+\varepsilon_x+\varepsilon_y+\varepsilon_z) \quad \text{Neglecting small quantities}$$

Change in volume $\Delta V = V' - V$

$$= (1+\varepsilon_x+\varepsilon_y+\varepsilon_z) - 1$$

$$\Delta V = \varepsilon_x+\varepsilon_y+\varepsilon_z = \varepsilon_v = \text{Volumetric strain}$$

Change in Vol.. per unit volume

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\sigma_y}{E}\nu - \frac{\sigma_z}{E}\nu = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y+\sigma_z)$$

Similarly

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x+\sigma_z)$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x+\sigma_y)$$

Adding $\varepsilon_x + \varepsilon_y + \varepsilon_z = \dfrac{\sigma_x + \sigma_y + \sigma_z}{E} - \dfrac{2\mu}{E}(\sigma_x + \sigma_y + \sigma_z) = \varepsilon_v$

$$\varepsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

If $\sigma_x = \sigma_y = \sigma_z = \sigma$ then $\varepsilon_v = \dfrac{3\sigma}{E}(1-2\mu)$

$$\varepsilon_v = 3\frac{\sigma}{E}(1-2\mu)$$

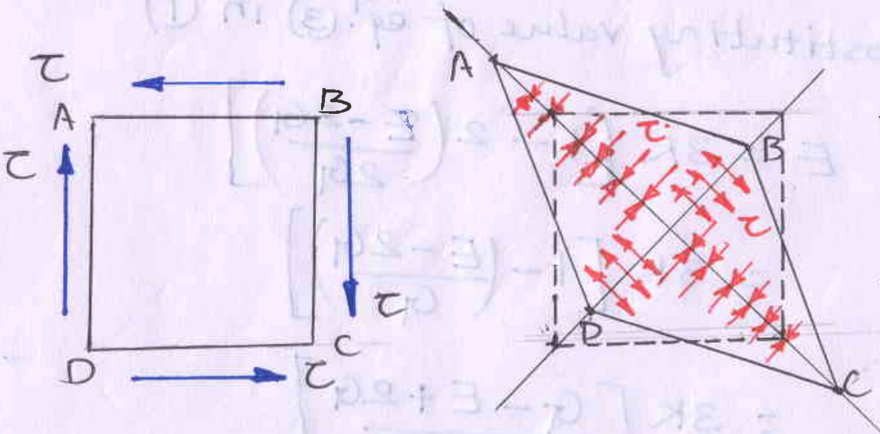
$$E = 3\frac{\sigma}{\varepsilon_v}(1-2\mu)$$

$$\boxed{E = 3K(1-2\mu)}$$

Relation betn. Young's Modulus & Bulk Modulus

To derive a relationship betn. E & G

In this case consider the case of pure shear



→ BD in tension (σ_t)

→ AC in compression (σ_c)

→ Its Magnitude

$\tau = \sigma_t = \sigma_c = \sigma$

The diagonal strains are equal to half the shear strain

$$\varepsilon_{AC} = \varepsilon_{BD} = \frac{\phi}{2} = \frac{\tau}{2G}$$

$$\varepsilon_{BD} = \varepsilon_{AC} = \frac{\sigma}{E} + \mu\frac{\sigma}{E} = \frac{\sigma}{E}(1+\mu) = \frac{\tau}{E}(1+\mu)$$

$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\mu)$$

$$E = 2G(1+\mu)$$

$$\boxed{G = \frac{E}{2(1+\mu)}}$$

This relation betn. E & G

We know that $E = 3K(1-2\nu)$

$\therefore \boxed{G = \frac{3K(1-2\nu)}{2(1+\nu)}}$

Relation bet^n. G & K

We know, Relationship bet E & K and G & E

$\therefore E = 3K(1-2\nu) \quad \ldots (1)$

$\cancel{= 3K/\!\!/}$

$G = \frac{E}{2(1+\nu)} \quad \ldots (2)$

$2G + 2\nu G = E$

$\nu = \frac{E-2G}{2G} \quad \ldots (3)$

Substituting value of eq^n. (3) in (1)

$\therefore E = 3K\left[1 - 2\left(\frac{E-2G}{2G}\right)\right]$

$= 3K\left[1 - \left(\frac{E-2G}{G}\right)\right]$

$= 3K\left[\frac{G - E + 2G}{G}\right]$

$E = \frac{3KG - 3KE + 6KG}{G} = \frac{9KG - 3KE}{G}$

$\therefore EG = 9KG - 3KE$

$EG + 3KE = 9KG$

$E(G + 3K) = 9KG$

$$\boxed{E = \frac{9KG}{(3K+G)}}$$