CHAPTER -5

# The Energy Equation and its Application

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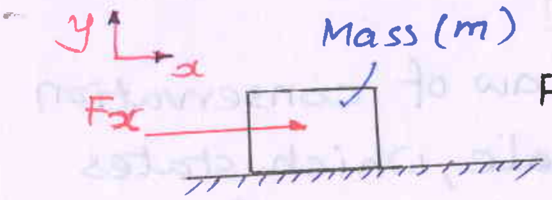
**Introduction:-** This chapter includes the study of forces causing fluid flow. Thus dynamics of fluid is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the Newton's second law of motion, which relates the acceleration with the forces.

## Equations of motion:-

Newton's second law of motion




$$F_x = m \times a_x$$

In the fluid flow, the following forces are present

i) $F_g$, gravity force.

ii) $F_p$, the pressure force

iii) $F_v$, Force due to viscosity.

iv) $F_t$, force due to turbulence

v) $F_c$, force due to compressibility.

the net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$$

i) If the force due to compressibility $F_c$ is negligible

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

Reynold's equation of motion

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ii) For Flow, where $(F_t)$, is negligible

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x$$

Navier-Stokes equation

iii) If the flow is assumed to be ideal viscous force $(F_v)$ is zero

$$F_x = (F_g)_x + (F_p)_x$$

Euler's equation of motion

## The momentum Equation

It is based on the law of conservation of momentum principale, which states that "the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction."

$$F = m \times a$$

a is acceleration acting in the same direction

$$a = \frac{dv}{dt}$$

$$F = m \cdot \frac{dv}{dt}$$

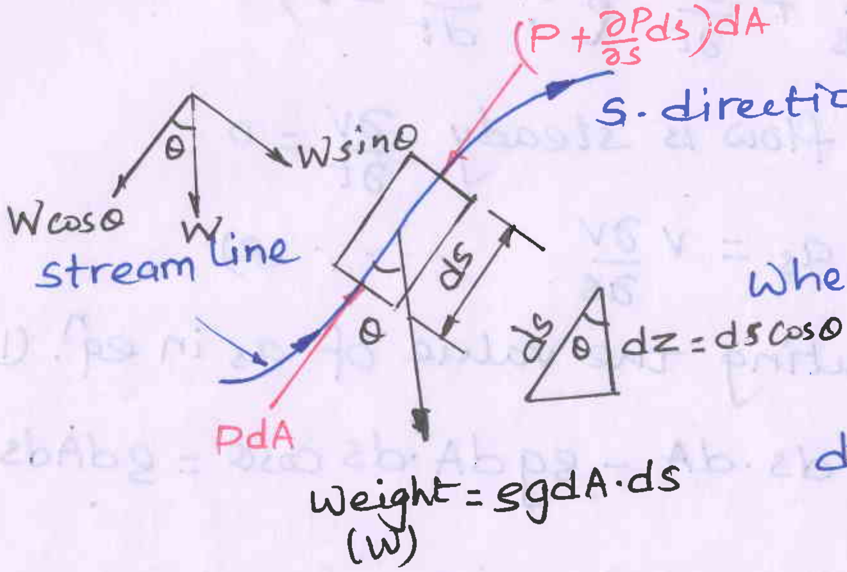
$$= \frac{d(m \cdot v)}{dt} \quad (m \text{ is constant})$$

Equation is known as the momentum principle

Equation can be written as $\boxed{F\,dt = d(mv)}$

which is known as the impuls-momentum eq$^n$.

# Euler's Equation of motion along a streamline




Where dA → Cross-section area of cylinderical elemen

ds → Length of cylinder

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This is equation of motion in which the forces due to gravity and pressure are taken into consideration.

1.) Pressure fore $p.dA$ in the direction of flow.

2.) Pressure force $(P+\frac{\partial P}{\partial s}.ds)dA$ opposite to the direction of flow

3) Weight of element $\rho g dA.ds$

Let θ is the angle between the direction of flow and the line of action of the weight of element

The resultant force on the fluid element in direction of s must be equal to the mass of fluid element × acceleration in the directions

$$p.dA - (P - \frac{\partial P}{\partial s}ds)dA - \rho g dA ds \cos\theta$$
$$= \rho dA ds \times a_s \quad \cdots ①$$

Where $a_s$ is the acceleration in the direction of s

$$a_s = \frac{dv}{dt}, \text{ where } v \text{ is a function of } s \,\&\, t$$

$$a_s = \frac{dv}{dt} = \frac{\partial v}{\partial s}.\frac{ds}{dt} + \frac{\partial v}{\partial t}$$

$$= v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left(\because \frac{ds}{dt} = v\right)$$

If the flow is steady $\frac{\partial v}{\partial t} = 0$

$$a_s = v\frac{\partial v}{\partial s} \quad \cdots\cdots (2)$$

substituting the value of $a_s$ in $eq^n$. (1)

$$-\frac{\partial P}{\partial s} ds \cdot dA - \rho g\, dA \cdot ds \cos\theta = \rho\, dA\, ds \times \frac{\partial v}{\partial s}$$

Dividing by $\rho\, ds\, dA$,

$$\frac{\partial P}{\rho\, \partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$$

$$\frac{1}{\rho}\frac{\partial P}{\partial s} + g\frac{dz}{ds} + \frac{v\,\partial v}{ds} = 0$$

$$\boxed{\frac{dP}{\rho} + g\,dz + v\,dv = 0}$$

## Bernoulli's theorem:-

"The total mechanical energy of the moving fluid comprising the gravitational potential energy of elevation, the energy associated with the fluid pressure and the kinetic energy of the fluid motion, remain constant"

Let prove above statement

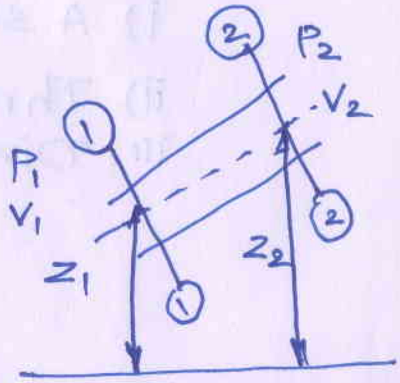
Bernoulli's $eq^n$ is obtained by integrating the Euler's $eq^n$

$$\int \frac{dP}{\rho} + \int g\,dz + \int v\,dv = \text{constant}$$

If flow is incompressible, $\rho$ is constant

$$\frac{P}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\frac{P}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$




$$\boxed{\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

where $\frac{P}{\rho g}$ = Pressure energy per unit weight of fluid or pressure head.

$\frac{V^2}{2g}$ = Kinetic energy per unit weight or kinetic head.

$z$ = Potential energy per unit weight or Potential head.

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Assumtion:-

i) The fluid is ideal i.e. viscosity is zero
ii) The flow is steady
iii) The flow is incompressible
iv) The flow is irrotational

Bernoulli's eq$^n$. for real fluid

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

where $h_L$ is Loss of energy bet$^n$. 1-1 & 2-2

## Principle of venturimeter:-

A venturimeter is device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts

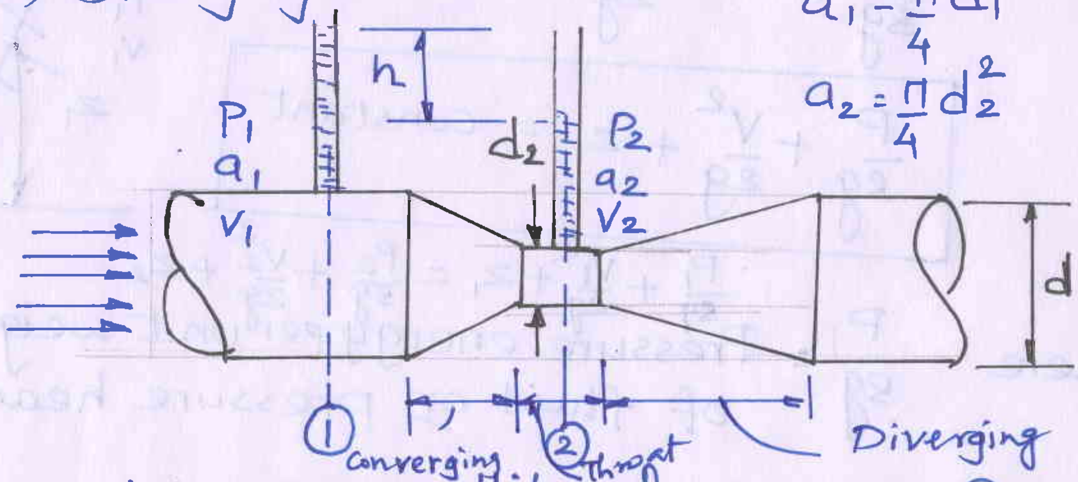
i) A short converging part

ii) Throat

iii) Diverging part

$$a_1 = \frac{\pi}{4} d_1^2$$

$$a_2 = \frac{\pi}{4} d_2^2$$




Applying Bernoulli's eq. at section ① & ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \quad (\because z_1 = z_2 \text{ horizontal pipe})$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad \ldots (i) \quad \left\{ \because \frac{P_1 - P_2}{\rho g} \text{ is the difference of pressure heads at sections ① \& ②} \right\}$$

Applying continuity eq$^n$. at sec$^n$. ① & ②

$$Q = a_1 V_1 = a_2 V_2 \quad \text{or} \quad V_1 = \frac{a_2}{a_1} V_2$$

Substituting this value of $V_1$ in eq$^n$. (i)

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{a_2 V_2}{a_1}\right)^2}{2g}$$

$$h = \frac{V_2^2}{2g}\left[1-\frac{a_2^2}{a_1^2}\right] = \frac{V_2^2}{2g}\left[\frac{a_1^2-a_2^2}{a_1^2}\right]$$

$$V_2^2 = 2gh \times \frac{a_1^2}{a_1^2-a_2^2}$$

$$V_2 = \sqrt{2gh} \times \sqrt{\frac{a_1^2}{a_1^2-a_2^2}} = \frac{a_1}{\sqrt{a_1^2-a_2^2}}\sqrt{2gh}$$

Discharge $Q_{th} = a_2 V_2$

$$Q_{th} = \frac{a_1 a_2}{\sqrt{a_1^2-a_2^2}}\sqrt{2gh}$$

Actual discharge $Q_{act} = C_d Q_{th}$

where ($C_d$ co-efficient of discharge)

$$\boxed{Q_{act} = C_d \frac{a_1 a_2}{\sqrt{a_1^2-a_2^2}} \times \sqrt{2gh}}$$

Value of h given by differential U-tube

case-1 $S_h$ = sp. gravity of the heavier liquid

$S_f$ = sp. gravity of the liquid flow through pipe

$x$ = Difference of the heavier liquid column in U-tube

$$h = x\left[\frac{S_h}{S_f} - 1\right] \text{ or } x\left[\frac{S_h}{S_F} - 1\right]$$

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Case-2 If the differential manometer contains a liquid which is lighter than liquid flowing

$$h = x\left[1 - \frac{S_L}{S_f}\right] \text{ or } h = x\left[1 - \frac{\rho_L}{\rho_f}\right]$$

$S_l$ = Sp. gr. of ligther liquid in U-tube

Case-3 Inclined venturimeter with Diff. U-tube manometer

heavier liquid in U-tube

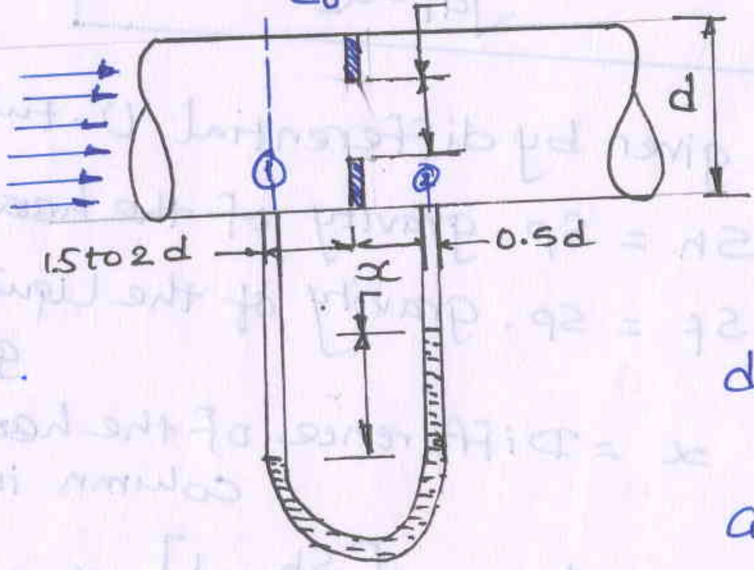
$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_f} - 1\right] = x\left[\frac{\rho_h}{\rho_f} - 1\right]$$

Case-4 Ligther liquid in U-tube

$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_L}{S_f}\right] = x\left[1 - \frac{\rho_L}{\rho_f}\right]$$

## Principle of orifice

$d_o = 0.5d$

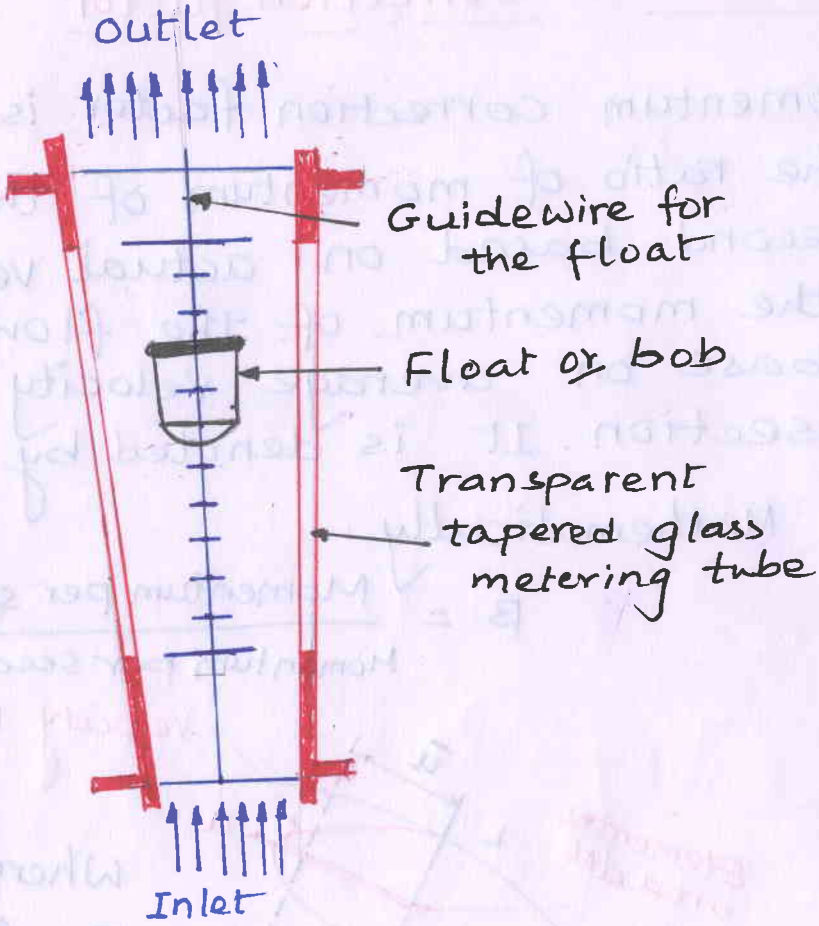



$d_o$ = Diameter of orifice

$$a_o = \frac{\pi}{4} d_o^2$$

$$\cancel{Q_{act}} = \frac{C_d\, a_o a_1}{\sqrt{a_1^2 - a_o^2}} \sqrt{2gh}$$

# Rotameter

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Working :- When the rate of flow increases the float rises in the tube and consequently there is an increase in the annular area between the float and the tube. Thus the float rides higher or Lower depending on the rate of flow

Advantages :-

1) simpler in operation
2) Handling and installation easy
3) Wide variety of corrosive fluid can be handled
4) Low cost, relatively

Limitation :-

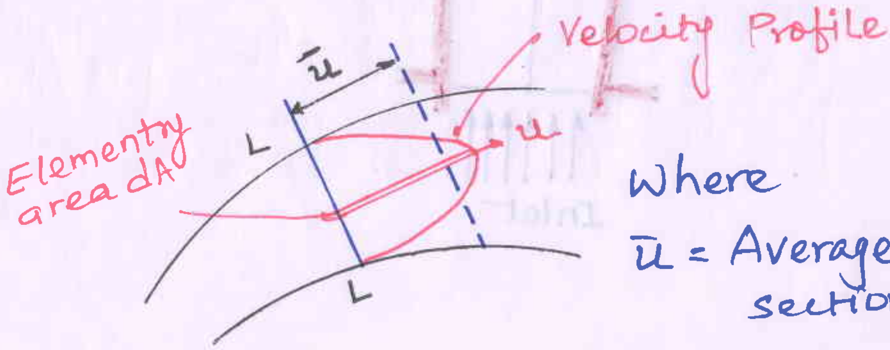
1) Less accurate, compared to venturimeter or orifice
2) Mounted vertically

## Momentum Correction factor

Momentum correction factor is defined as the ratio of momentum of the flow per second based on actual velocity to the momentum of the flow per second base on average velocity across a section. It is denoted by $\beta$

Mathematically

$$\beta = \frac{\text{Momentum per second base on Act. Velocity}}{\text{Momentum per second on avg. Velocity}}$$




Where
$\bar{u}$ = Average velocity at the section L-L
$u$ = Local or point or actual velocity
$dA$ = Elementary area
$A$ = Area of cross-section

The momentum of fluid mass m is $= m\bar{u}$

$$= (\rho A\bar{u})\cdot\bar{u} = \rho A\bar{u}^2$$

The true momentum at the section LL is given

$$\int_{LL} dm\cdot u = \int_{LL} (\rho dA\cdot u)u$$

$$= \int_A \rho u^2 dA$$

$$\beta = \frac{\int_A \rho u^2 dA}{\rho A \bar{u}^2} = \frac{1}{A}\int_A \left(\frac{u}{\bar{u}}\right)^2 dA$$

$\beta = 1$ for uniform flow

$\left.\begin{array}{l}\beta = 1.01 \text{ to } 1.07 \text{ for turbulent flow} \\ \beta = \frac{4}{3} = 1.33 \text{ for Laminar flow}\end{array}\right\}$ in pipe

## Kinetic energy correction factor

Kinetic energy correction factor is defined as the ratio of the kinetic energy of flow per second based on actual velocity across a section to the kinetic energy of flow per second based on average velocity across the same section.

It is denoted by $\alpha$

$$\alpha = \frac{\text{Kinetic energy per second based on actual velocity}}{\text{Kinetic energy per second based on average velocity}}$$

Total K.E. for the entire section

$$K.E. = \frac{1}{2} m\bar{u}^2 = \frac{1}{2}(\rho A\bar{u})\bar{u}^2 = \frac{1}{2}\rho A\bar{u}^3$$

True K.E. for the entire cross-section

$$= \int \frac{1}{2} dm u^2 = \int \frac{1}{2}(\rho dA \cdot u) u^2$$

$$= \frac{\rho}{2}\int_A u^3 dA$$

$$\alpha = \frac{\frac{\rho}{2}\int_A u^3 dA}{\frac{\rho}{2} A\bar{u}^3} = \frac{1}{A}\int_A \left(\frac{u}{\bar{u}}\right)^3 dA$$

$\alpha = 1$ - uniform

$\alpha = 1.2$ to $1.15$ - turbulent

$\alpha = 2$ - Laminar

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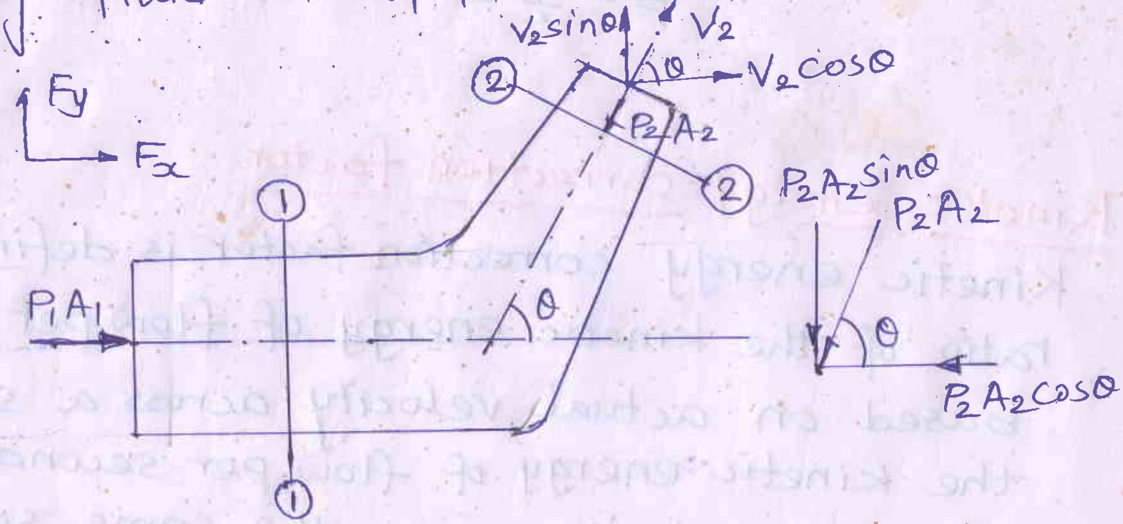
# Momentum equation for 2-D flow along a stream

The impulse-momentum eq$^n$ is used to determine the resultant force exerted by a flowing fluid on a pipe bend




→ Net force acting on fluid in the direction of x = Rate of change of momentum in x-dir$^n$

$$P_1A_1 - P_2A_2\cos\theta - F_x = (\text{mass per sec})(\text{change of velocity})$$

$$= \rho Q (V_{x\,final} - V_{x\,initial})$$

$$= \rho Q (V_2\cos\theta - V_1)$$

$$F_x = \rho Q (V_1 - V_2\cos\theta) + P_1A_1 - P_2A_2\cos\theta$$

→ Similarly in y-direction

$$0 - P_2A_2\sin\theta - F_y = \rho Q (V_2\sin\theta - 0)$$

$$F_y = \rho Q(-V_2\sin\theta) - P_2A_2\sin\theta$$

→ Resultant force $(F_R)$: $= \sqrt{F_x^2 + F_y^2}$

→ Angle made by resultant force: $\tan\phi = \dfrac{F_y}{F_x}$

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