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CHAPTER - 3

3 - Hydrostatic Forces on Surface and Buoyancy

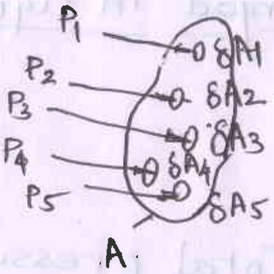
3.1 Introduction:-

This chapter deals with the fluids at rest. This means that there will be no relative motion between adjacent or neighbouring fluid layers.
The velocity gradient, which is equal to the change of velocity between two adjacent fluid layers divided by the distance between the layers, will be zero

$$\frac{du}{dy} = 0 \quad , \quad \tau = 0$$

3.2 Total pressure and Centre of pressure

- Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

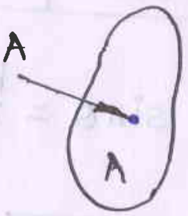


$A = \delta A_1 + \delta A_2 + \delta A_3 \cdots \delta A_n$

$P = P_1 + P_2 + P_3 + \cdots$

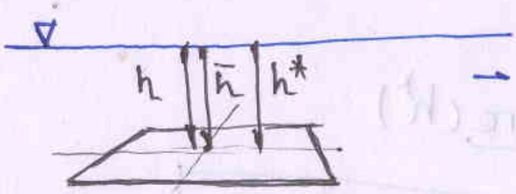
$F_{total} = P.A = P_1 \delta A_1 + P_2 \delta A_2 + P_3 \delta A_3 + \cdots$

$F = P \cdot A$



Centre of pressure is define as the point of application of the total pressure ~~force~~ on the surface.

3.3 Total pressure force and centre of pressure on horizontal plane surface.



→ Total pressure force:-

$$\boxed{F = \rho g A \bar{h}}$$

$h = \bar{h} = h^*$

- centre of pressure $= h^* = \bar{h}$

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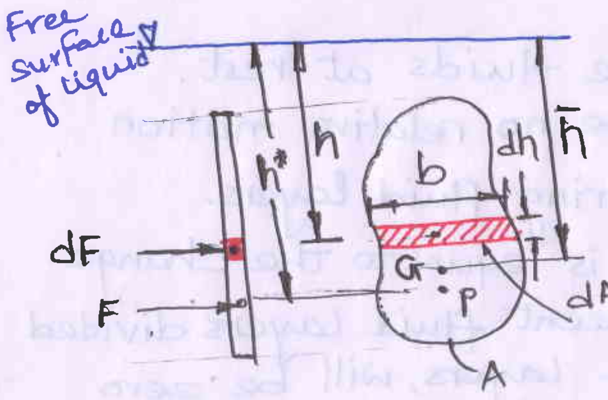
3.4 Vertical plane surface submerged in Liquid



→ Total pressure force

$$\boxed{F = \rho g A\bar{h}}$$

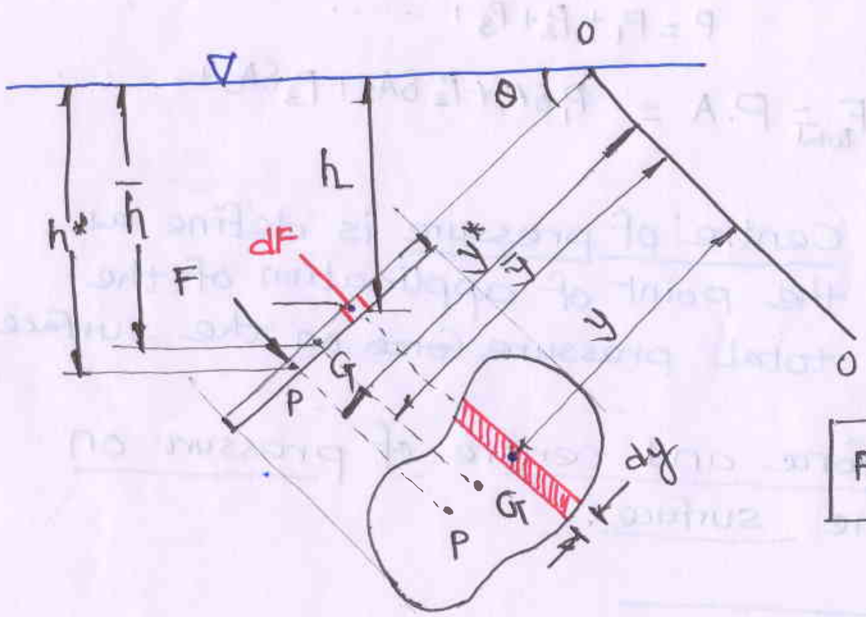
→ Centre of pressure (h^*)

$$\boxed{h^* = \frac{I_G}{A\bar{h}} + \bar{h}}$$

Principle of Moments: moment of Resultant force about axis is equal to the sum of moments of the components about the same axis

Parallel axis theoram
$I_O = I_G + A\bar{h}^2$

3.5 Inclined plane surface submerged in Liquid



Total pressure force

$$\sin\theta = \frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*}$$

$$\boxed{F = \rho g A\bar{h}}$$

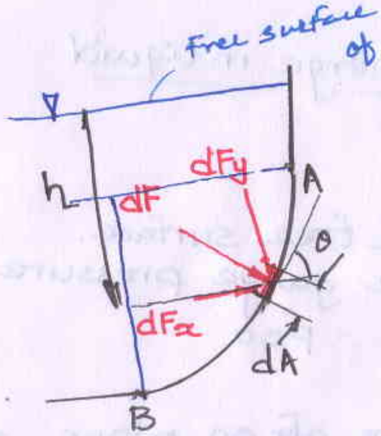
→ Centre of pressure (h^*)

$$\boxed{h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}}$$

General Eq^n

IF $\theta = 90°$ → Vertical
$\theta = 0$ → Horizontal

3.6 Curved surface submerged in Liquid

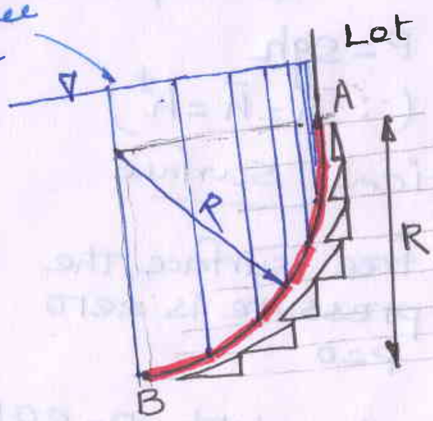


$F_x = \int dF_x$

$F_y = \int dF_y$

$$F = \sqrt{F_x^2 + F_y^2}$$

Let width of curve surface is unity



$F_x = \int dF_x = \rho g \int h \, dA \sin\theta$

$= \rho g \int h \, dA \sin\theta$

$dA \sin\theta$ = Vertical projection area of the curve surface

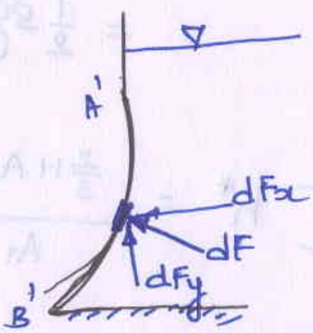
i.e $\boxed{F_x = \rho g \bar{h} A} = \rho g \bar{h} \times R \times 1$

$F_y = \int dF_y = \rho g \int h \, dA \cos\theta$

$dA\cos\theta$ = Horizontal projection area of the curve surface

$\forall = \int h \, dA \cos\theta$ = Total volume supported by the curve surface

$\boxed{F_y = \rho g \forall}$ = Total weight supported by the curve surface.



the curve surface A'B' is not supporting any fluid In such cases, F_y is equal to the weight of the imaginary liquid supported by A'B' up to free surface of liquid

The direction of F_y will be take in upward direction

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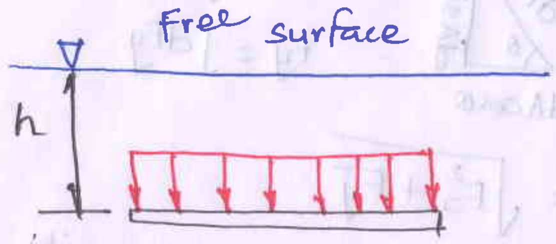
3.7 Pressure Diagram

- Horizontal plane surface submerge in liquid

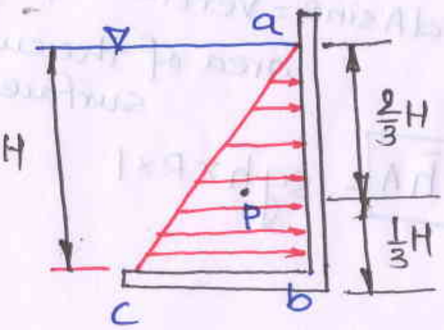


At the free surface. the gauge pressure is zero
$\therefore P = 0$

Pressure ~~of~~ on plane surface
$P = \rho g h$

Pressure force $= \rho g A h = \rho g A \bar{h}$ $\quad (\because h = \bar{h} = h^*)$

- Pressure diagram for vertical surface



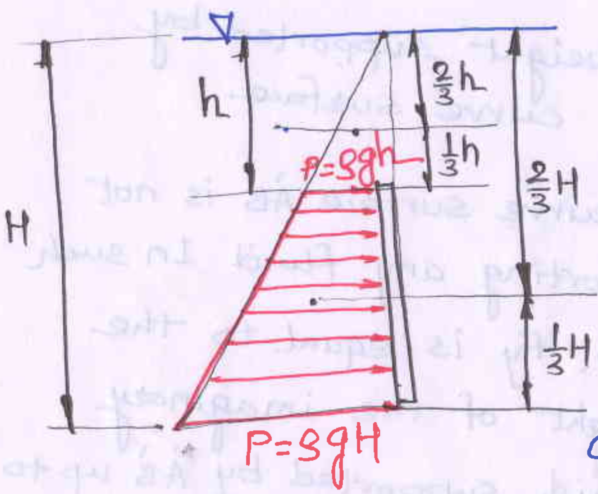
At the free surface, the gauge pressure is zero
at pt. a $\quad P = 0$

Pressure at point b $\quad P = \rho g H$

Area of pressure diagram $= \frac{1}{2}(ab) \times (bc)$
$= \frac{1}{2} H \times \rho g H$

Pressure Force $= \frac{1}{2} \rho g H^2$ per width

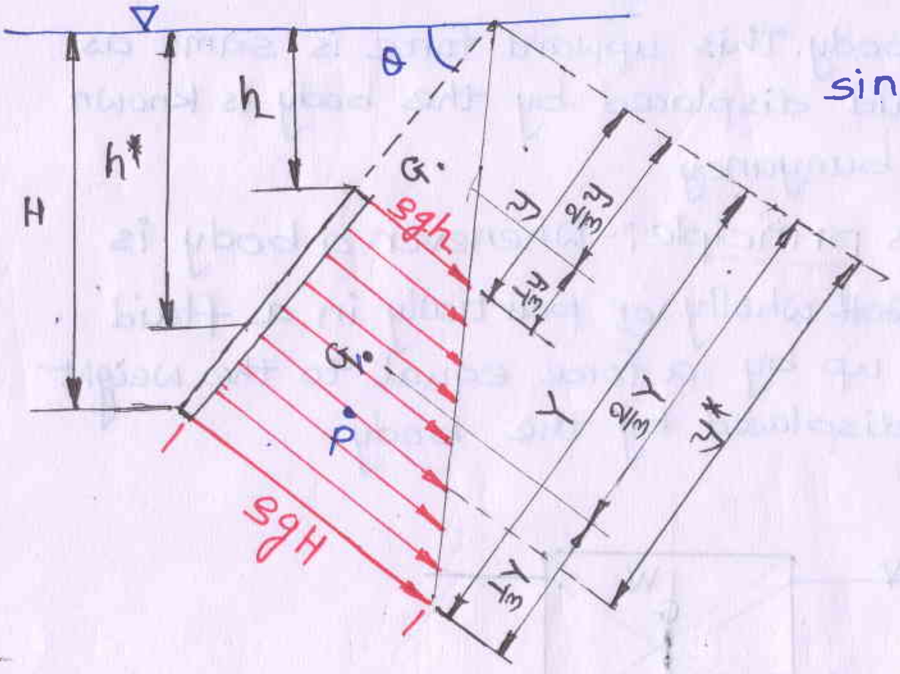
Centre of pressure $h^* = \frac{2}{3} H$



Area of pressure diagram $= A_1 - A_2$
$= \frac{1}{2} H \times \rho g H - \frac{1}{2} h \cdot \rho g h$
$= \frac{1}{2} \rho g (H^2 - h^2)$ per width

Centre of pressure $\quad h^* = \dfrac{\frac{2}{3} H A_1 - \frac{2}{3} h A_2}{A_1 - A_2}$

— pressure diagram for inclined and submerged surface



$$\sin\theta = \frac{h}{y} = \frac{H}{Y} = \frac{h^*}{y^*}$$

F = Area of pressure diagram per unit width

$$= A_1 - A_2$$

$$= \frac{1}{2}\rho g H \cdot Y - \frac{1}{2}\rho g h y$$

$$= \frac{1}{2}\rho g H \frac{H}{\sin\theta} - \frac{1}{2}\rho g h \frac{h}{\sin\theta}$$

$$= \frac{1}{2}\rho g \left(\frac{H^2}{\sin\theta} - \frac{h^2}{\sin\theta}\right)$$

$$= \frac{\rho g}{2 \cdot \sin\theta}(H^2 - h^2)$$

Centre of pressure $h^* = y^* . \sin\theta$

$$y^* = \frac{\frac{2}{3}Y A_1 - \frac{2}{3}y A_2}{A_1 - A_2}$$

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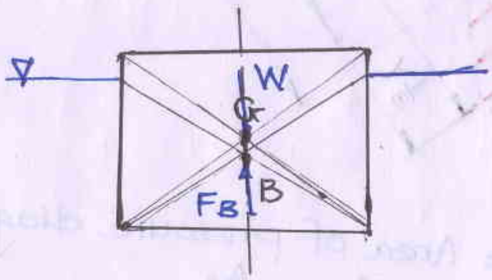
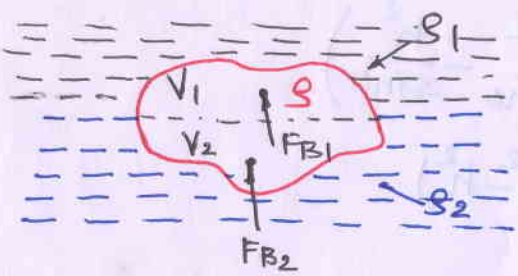
3.8 Buoyancy:-

When a body is immersed in a fluid, there is tendency of a fluid to exert force on the submerged body. This upward force is same as weight of fluid displaced by the body is known as force of buoyancy

- Archimedes principle:- Whenever a body is immersed ~~wall~~ wholly or partially in a fluid it is lifted up by a force equal to the weight of fluid displaced by the body



- Centre of Buoyancy:- It is defined as the point through which the force of buoyancy is supposed to act.



$V = V_1 + V_2$, $W = \rho g V$

$F_{B_1} = \rho_1 g V_1$

$F_{B_2} = \rho_2 g V_2$

Total upward force of buoyancy $F = F_{B_1} + F_{B_2}$

$$= \rho_1 g V_1 + \rho_2 g V_2$$

For equilibrium

Upward force of buoyancy = weight of body

$$\rho_1 g V_1 + \rho_2 g V_2 = \rho g V$$

$$\rho_1 V_1 + \rho_2 V_2 = \rho V$$

3.9 Meta-Centre

Whenever a body floating in liquid, is a given a small angular displacement, it starts oscillating about a point. This point about which starts oscillating is called Metacentre.

The metacentre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement

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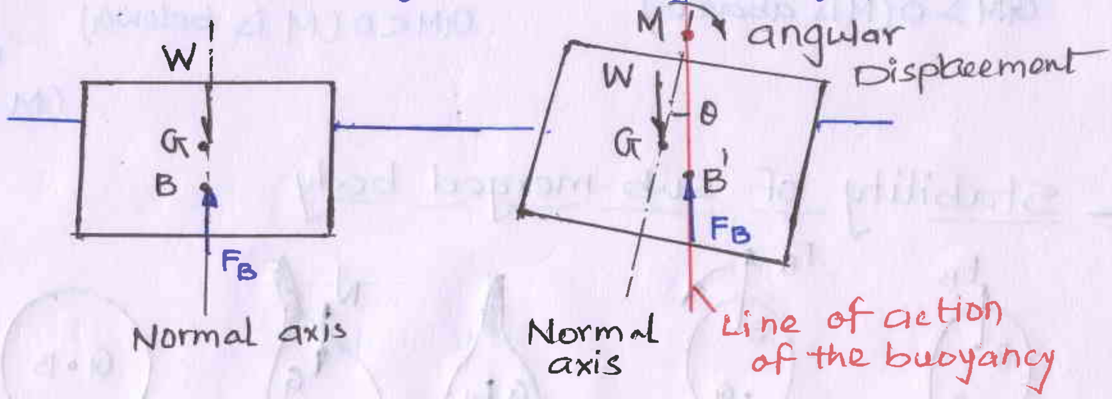
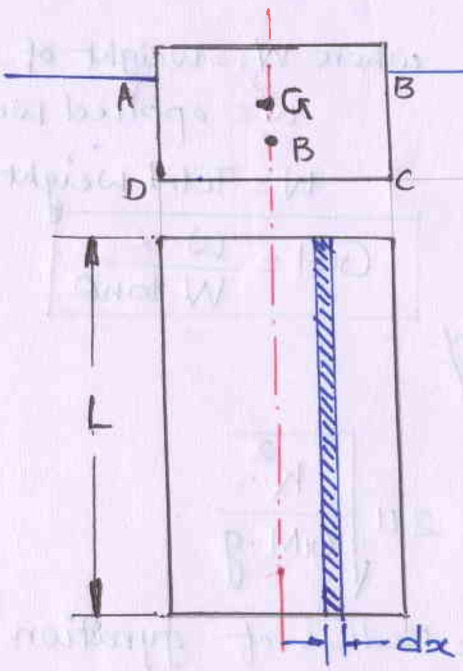
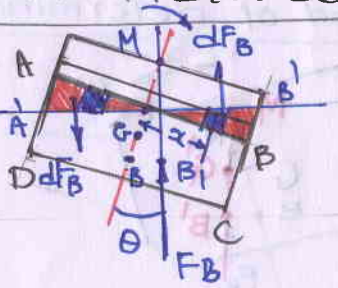


3.10 Meta-centric height (MG)

The distance MG, the distance between the meta centre of a floating body and the centre of gravity of the body is called meta-centric height

3.11 Analytical Method for Meta centre height





$W = F_B$

$$BM = \frac{I}{\forall}$$

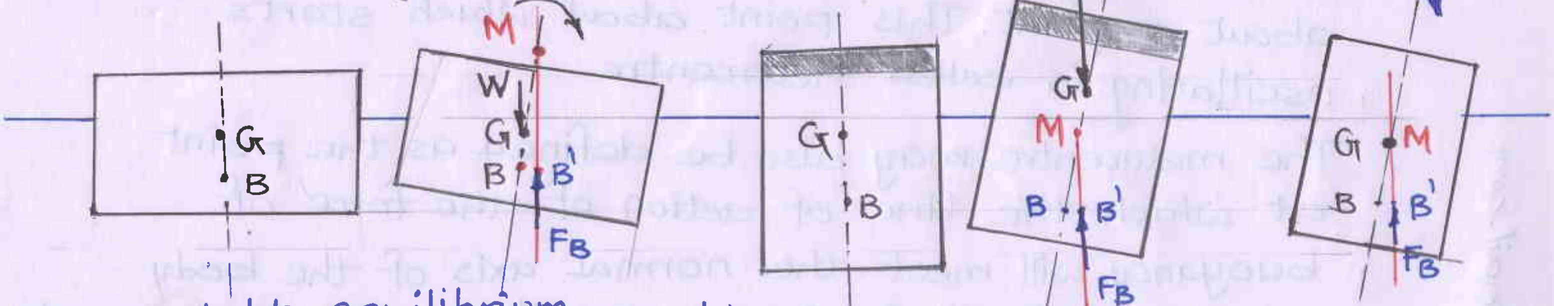
$$GM = BM - BG$$

$$= \frac{I}{\forall} - BG$$

where $\forall$ Volume of the fluid displaced by the body

3.12 Conditions of equilibrium of a floating & Sub-merged Bodies

- Stability of Floating body

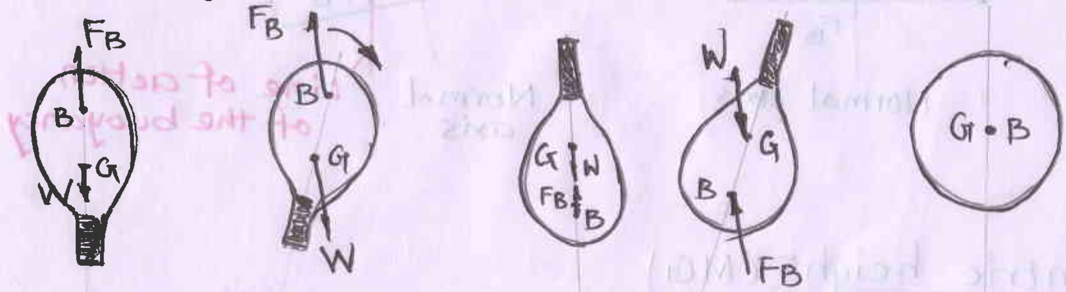


a) stable equilibrium
$GM > 0$ (M is above G)

b) Unstable equilibrium
$GM < 0$ (M is below G)

c) Neutral equilibrium
$GM = 0$
(M coinciding with G)

- Stability of sub-merged body

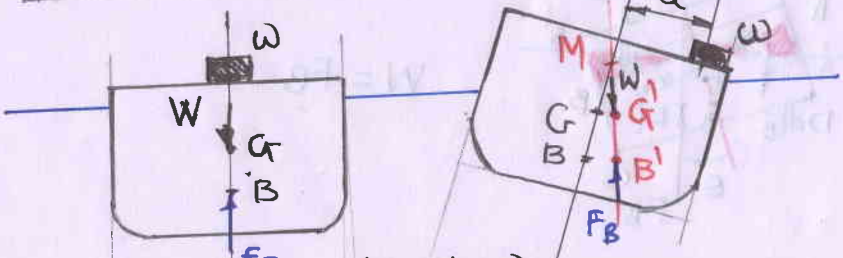


a) stable equilibrium
$W = F_B$
(B is above G)

b) Unstable equilibrium
$W = F_B$
(B is below G)

c) Neutral equilibrium
$W = F_B$
(B and G at the same point)

3.13 Experimental Method of determination of Metacentric height



$W = W_s + \omega$

where W_s = weight of ship
ω = applied weight
W = Total weight

$$\boxed{GM = \frac{\omega \cdot x}{W \tan\theta}}$$

3.14 Oscillation of a floating Body

$$T = 2\pi \sqrt{\frac{K^2}{GM \cdot g}}$$

K = Radius of gyration