Now the acceleration diagram, as shown in Fig. 8.25 (*c*), is drawn as discussed below:

1. Since *A* and *C* are fixed points, therefore these points are marked as one point in the acceleration diagram. Draw vector *a'b'* parallel to *A B*, to some suitable scale, to represent the radial component of the acceleration of *B* with respect to *A* or the acceleration of *B*, such that

$$\text{vector } a'b' = a^r_{BA} = a_B = 284.4 \text{ m/s}^2$$

2. Draw vector *c'd'* parallel to *CD* to represent the radial component of the acceleration of *D* with respect to *C* or the acceleration of *D*, such that

$$\text{vector } c'd' = a^r_{DC} = a_D = 568.8 \text{ m/s}^2$$

3. Now from point *b'*, draw vector *b'x* parallel to *BE* to represent the radial component of the acceleration of *E* with respect to *B*, such that

$$\text{vector } b'x' = a^r_{EB} = 437.4 \text{ m/s}^2$$

4. From point *x*, draw vector *xe'* perpendicular to *BE* to represent the tangential component of acceleration of *E* with respect to *B* (*i.e.* a^t_{EB}) whose magnitude is yet unknown.

5. From point *d'*, draw vector *d'y* parallel to *DE* to represent the radial component of the acceleration of *E* with respect to *D*, such that

$$\text{vector } d'y = a^r_{ED} = 0.15 \text{ m/s}^2$$

Note: Since the magnitude of a^r_{ED} is very small (*i.e.* 0.15 m/s^2), therefore the points *d'* and *y* coincide.

6. From point *y*, draw vector *ye'* perpendicular to *DE* to represent the tangential component of the acceleration of *E* with respect to *D* (*i.e.* a^t_{ED}). The vectors *xe'* and *ye'* intersect at *e'*.

7. From point *e'*, draw vector *e'z* parallel to *EP* to represent the radial component of the acceleration of *P* with respect to *E*, such that

$$\text{vector } e'z = a^r_{PE} = 110.45 \text{ m/s}^2$$

8. From point *z*, draw vector *zp'* perpendicular to *EP* to represent the tangential component of the acceleration of *P* with respect to *E* (*i.e.* a^t_{PE}) whose magnitude is yet unknown.

9. From point *a'*, draw vector *a'p'* parallel to the path of motion of *P* (which is horizontal) to represent the acceleration of *P*. The vectors *zp'* and *a'p'* intersect at *p'*.

By measurement, we find that acceleration of the piston *P*,

$$a_p = \text{vector } a'p' = 655 \text{ m/s}^2 \textbf{ Ans.}$$

8.5. Coriolis Component of Acceleration

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of the acceleration must be calculated.

Consider a link *OA* and a slider *B* as shown in Fig. 8.26 (*a*). The slider *B* moves along the link *OA*. The point *C* is the coincident point on the link *OA*.

Let ω = Angular velocity of the link *OA* at time *t* seconds.

v = Velocity of the slider *B* along the link *OA* at time *t* seconds.

$\omega.r$ = Velocity of the slider *B* with respect to *O* (perpendicular to the link *OA*) at time *t* seconds, and

$(\omega + \delta\omega)$, $(v + \delta v)$ and $(\omega + \delta\omega)\,(r + \delta r)$

$\quad = $ Corresponding values at time $(t + \delta t)$ seconds.

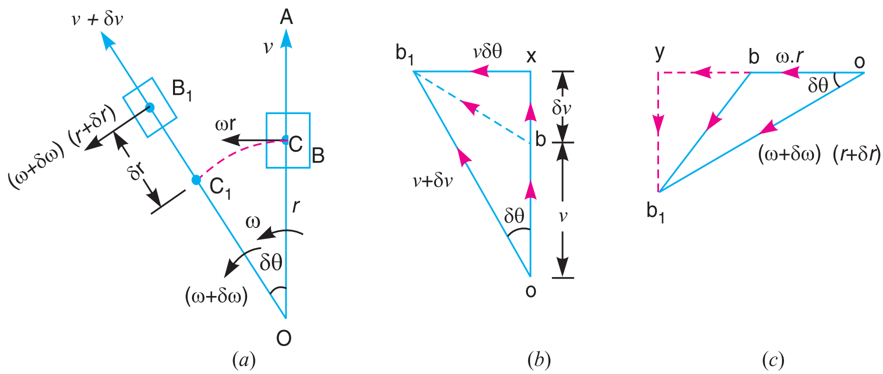


Fig. 8.26. Coriolis component of acceleration.

Let us now find out the acceleration of the slider *B* with respect to *O* and with respect to its coincident point *C* lying on the link *OA*.

Fig. 8.26 (*b*) shows the velocity diagram when their velocities v and $(v + \delta v)$ are considered. In this diagram, the vector bb_1 represents the change in velocity in time δt sec ; the vector bx represents the component of change of velocity bb_1 along *OA* (*i.e.* along radial direction) and vector xb_1 represents the component of change of velocity bb_1 in a direction perpendicular to *OA* (*i.e.* in tangential direction). Therefore

$$bx = ox - ob = (v + \delta v)\cos\delta\theta - v \uparrow$$

Since $\delta\theta$ is very small, therefore substituting $\cos\delta\theta = 1$, we have

$$bx = (v + \delta v - v)\uparrow = \delta v \uparrow$$

...(Acting radially outwards)

and $$xb_1 = (v + \delta v)\sin\delta\theta$$

Since $\delta\theta$ is very small, therefore substituting $\sin\delta\theta = \delta\theta$, we have

$$xb_1 = (v + \delta v)\,\delta\theta = v.\delta\theta + \delta v.\delta\theta$$

Neglecting $\delta v.\delta\theta$ being very small, therefore

$$xb_1 = \overleftarrow{v.\delta\theta}$$

...(Perpendicular to *OA* and towards left)

A drill press has a pointed tool which is used for boring holes in hard materials usually by rotating abrasion or repeated bolows.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Fig. 8.26 (*c*) shows the velocity diagram when the velocities $\omega.r$ and $(\omega + \delta\omega)\,(r + \delta r)$ are considered. In this diagram, vector bb_1 represents the change in velocity ; vector yb_1 represents the component of change of velocity bb_1 along *OA* (*i.e.* along radial direction) and vector *by* represents the component of change of velocity bb_1 in a direction perpendicular to *OA* (*i.e.* in a tangential direction). Therefore

$$yb_1 = (\omega + \delta\omega)\,(r + \delta r)\sin\delta\theta \downarrow$$
$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r)\sin\delta\theta$$

Since $\delta\theta$ is very small, therefore substituting $\sin\delta\theta = \delta\theta$ in the above expression, we have

$$yb_1 = \omega.r.\delta\theta + \omega.\delta r.\delta\theta + \delta\omega.r.\delta\theta + \delta\omega.\delta r.\delta\theta$$

$$= \omega.r.\delta\theta \downarrow \text{, acting radially inwards} \quad \text{...(Neglecting all other quantities)}$$

and
$$by = oy - ob = (\omega + \delta\omega)(r + \delta r)\cos\delta\theta - \omega.r$$

$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r)\cos\delta\theta - \omega.r$$

Since $\delta\theta$ is small, therefore substituting $\cos\delta\theta = 1$, we have

$$by = \omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r - \omega.r = \omega.\delta r + r.\delta\omega \quad \text{...(Neglecting } \delta\omega.\delta r\text{)}$$

...(Perpendicular to OA and towards left)

Therefore, total component of change of velocity along radial direction

$$= bx - yb_1 = (\delta v - \omega.r.\delta\theta)\uparrow \quad \text{...(Acting radially outwards from } O \text{ to } A\text{)}$$

∴ Radial component of the acceleration of the slider B with respect to O on the link OA, acting radially outwards from O to A,

$$a_{BO}^{r} = \text{Lt}\,\frac{\delta v - \omega.r.\delta\theta}{\delta t} = \frac{dv}{dt} - \omega.r \times \frac{d\theta}{dt} = \frac{dv}{dt} - \omega^2.r \uparrow \quad \text{...}(i)$$

...($\because d\theta/dt = \omega$)

Also, the total component of change of velocity along tangential direction,

$$= xb_1 + by = \overleftarrow{v.\delta\theta} + (\overleftarrow{\omega.\delta r + r.\delta\omega})$$

...(Perpendicular to OA and towards left)

∴ Tangential component of acceleration of the slider B with respect to O on the link OA, acting perpendicular to OA and towards left,

$$a_{BO}^{t} = \text{Lt}\,\frac{v.\delta\theta + (\omega.\delta r + r.\delta\omega)}{\delta t} = v\frac{d\theta}{dt} + \omega\frac{dr}{dt} + r\frac{d\omega}{dt}$$

$$= v.\omega + \omega.v + r.\alpha = (\overleftarrow{2v.\omega + r.\alpha}) \quad \text{...}(ii)$$

...($\because dr/dt = v$, and $d\omega/dt = \alpha$)

Now radial component of acceleration of the coincident point C with respect to O, acting in a direction from C to O,

$$a_{CO}^{r} = \omega^2.r \uparrow \quad \text{...}(iii)$$

and tangential component of acceleraiton of the coincident point C with respect to O, acting in a direction perpendicular to CO and towards left,

$$a_{CO}^{t} = \overleftarrow{\alpha.r} \uparrow \quad \text{...}(iv)$$

Radial component of the slider B with respect to the coincident point C on the link OA, acting radially outwards,

$$a_{BC}^{r} = a_{BO}^{r} - a_{CO}^{r} = \left(\frac{dv}{dt} - \omega^2.r\right) - \left(-\omega^2.r\right) = \frac{dv}{dt}\uparrow$$

and tangential component of the slider B with respect to the coincident point C on the link OA acting in a direction perpendicular to OA and towards left,

$$a_{BC}^{t} = a_{BO}^{t} - a_{CO}^{t} = (2\omega.v + \alpha.r) - \alpha.r = \overleftarrow{2\omega.v}$$

This tangential component of acceleration of the slider B with respect to the coincident point C on the link is known as ***coriolis component of acceleration*** and is always perpendicualr to the link.

∴ Coriolis component of the acceleration of B with respect of C,

$$a_{BC}^{c} = a_{BC}^{t} = 2\,\omega.v$$

where ω = Angular velocity of the link OA, and

v = Velocity of slider B with respect to coincident point C.

In the above discussion, the anticlockwise direction for ω and the radially outward direction for v are taken as ***positive.*** It may be noted that the direction of coriolis component of acceleration changes sign, if either ω or v is reversed in direction. But the direction of coriolis component of acceleration will not be changed in sign if both ω and v are reversed in direction. It is concluded that the direction of coriolis component of acceleration is obtained by rotating v, at 90°, about its origin in the same direction as that of ω.

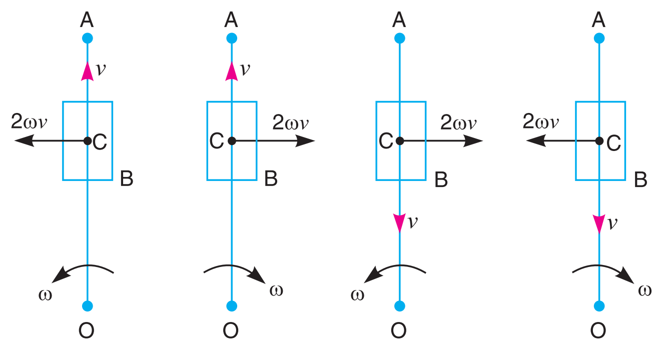


Fig. 8.27. Direction of coriolis component of acceleration.

The direction of coriolis component of acceleration (2 ω.v) for all four possible cases, is shown in Fig. 8.27. The directions of ω and v are given.

Example 8.13. *A mechanism of a crank and slotted lever quick return motion is shown in Fig. 8.28. If the crank rotates counter clockwise at 120 r.p.m., determine for the configuration shown, the velocity and acceleration of the ram D. Also determine the angular acceleration of the slotted lever.*

Crank, AB = 150 mm ; Slotted arm, OC = 700 mm and link CD = 200 mm.

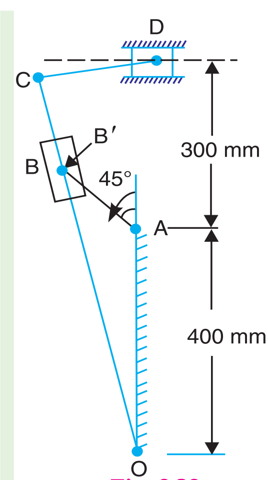


Fig. 8.28

Solution. Given : N_{BA} = 120 r.p.m or $\omega_{BA} = 2\pi \times 120/60$ = 12.57 rad/s ; AB = 150 mm = 0.15 m; OC = 700 mm = 0.7 m; CD = 200 mm = 0.2 m

We know that velocity of B with respect to A,

$$v_{BA} = \omega_{BA} \times AB$$

$$= 12.57 \times 0.15 = 1.9 \text{ m/s}$$

...(Perpendicular to AB)

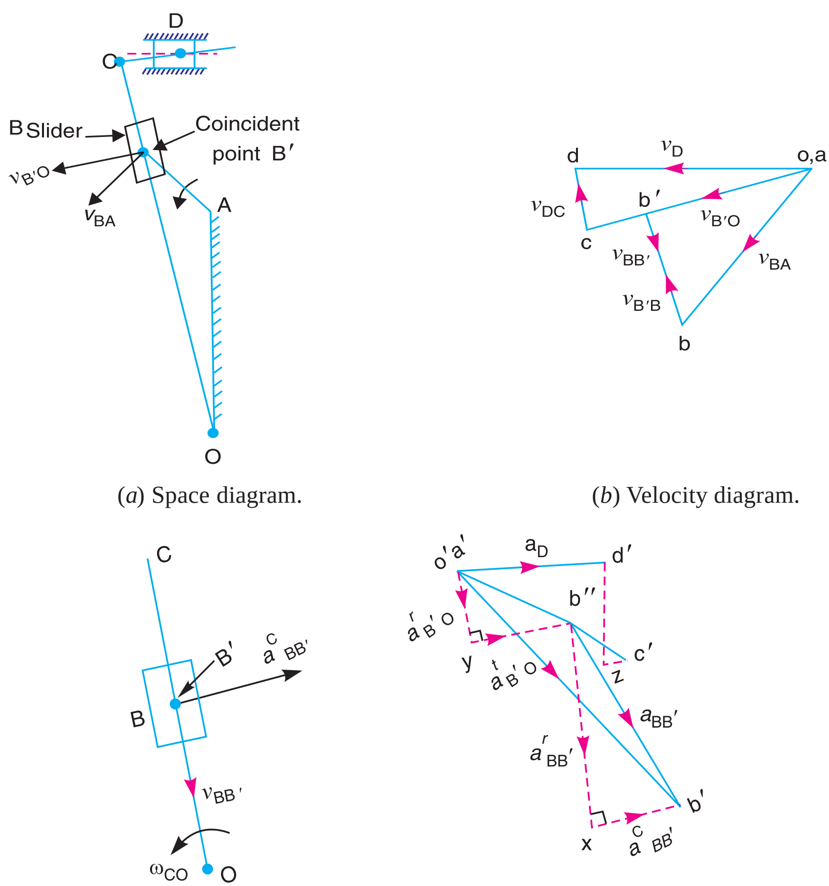
Velocity of the ram D

First of all draw the space diagram, to some suitable scale, as shown in Fig. 8.29 (a). Now the velocity diagram, as shown in Fig. 8.29

(*b*), is drawn as discussed below:

1. Since *O* and *A* are fixed points, therefore these points are marked as one point in velocity diagram. Now draw vector *ab* in a direction perpendicular to *AB*, to some suitable scale, to represent the velocity of slider *B* with respect to *A* *i.e.* v_{BA}, such that

$$\text{vector } ab = v_{BA} = 1.9 \text{ m/s}$$



(*a*) Space diagram.

(*b*) Velocity diagram.

(*c*) Direction of coriolis component.

(*d*) Acceleration diagram.

Fig. 8.29

2. From point *o*, draw vector *ob′* perpendicular to *OB′* to represent the velocity of coincident point *B′* (on the link *OC*) with respect to *O* *i.e.* $v_{B'O}$ and from point *b* draw vector *bb′* parallel to the path of motion of *B′* (which is along the link *OC*) to represent the velocity of coincident point *B′* with respect to the slider *B* *i.e.* $v_{B'B}$. The vectors *ob′* and *bb′* intersect at *b′*.

Note: Since we have to find the coriolis component of acceleration of the slider *B* with respect to the coincident point *B′*, therefore we require the velocity of *B* with respect to *B′* *i.e.* $v_{BB'}$. The vector *b′b* will represent $v_{BB'}$ as shown in Fig. 8.29 (*b*).

3. Since the point *C* lies on *OB′* produced, therefore, divide vector *ob′* at *c* in the same ratio as *C* divides *OB′* in the space diagram. In other words,

$$ob'/oc = OB'/OC$$

The vector *oc* represents the velocity of *C* with respect to *O* *i.e.* v_{CO}.

4. Now from point *c*, draw vector *cd* perpendicular to *CD* to represent the velocity of *D* with respect to *C i.e.* v_{DC} ,and from point *o* draw vector *od* parallel to the path of motion of *D* (which is along the horizontal) to represent the velocity of *D i.e.* v_D.The vectors *cd* and *od* intersect at *d*.

By measurement, we find that velocity of the ram *D*,

$$v_D = \text{vector } od = 2.15 \text{ m/s} \quad \textbf{Ans.}$$

From velocity diagram, we also find that

Velocity of *B* with respect to *B′*,

$$v_{BB'} = \text{vector } b'b = 1.05 \text{ m/s}$$

Velocity of *D* with respect to *C*,

$$v_{DC} = \text{vector } cd = 0.45 \text{ m/s}$$

Velocity of *B′* with respect to *O*

$$v_{B'O} = \text{vector } ob' = 1.55 \text{ m/s}$$

Velocity of *C* with respect to *O*,

$$v_{CO} = \text{vector } oc = 2.15 \text{ m/s}$$

∴ Angular velocity of the link *OC* or *OB′*,

$$\omega_{CO} = \omega_{B'O} = \frac{v_{CO}}{OC} = \frac{2.15}{0.7} = 3.07 \text{ rad/s (Anticlockwise)}$$

Acceleration of the ram D

We know that radial component of the acceleration of *B* with respect to *A*,

$$a^r_{BA} = \omega^2_{BA} \times AB = (12.57)^2 \times 0.15 = 23.7 \text{ m/s}^2$$

Coriolis component of the acceleration of slider *B* with respect to the coincident point *B′*,

$$a^c_{BB'} = 2\omega.v = 2\omega_{CO}.v_{BB'} = 2 \times 3.07 \times 1.05 = 6.45 \text{ m/s}^2$$

...($\because$ $\omega = \omega_{CO}$ and $v = v_{BB'}$)

Radial component of the acceleration of *D* with respect to *C*,

$$a^r_{DC} = \frac{v^2_{DC}}{CD} = \frac{(0.45)^2}{0.2} = 1.01 \text{ m/s}^2$$

Radial component of the acceleration of the coincident point *B′* with respect to *O*,

$$a^r_{B'O} = \frac{v^2_{B'O}}{B'O} = \frac{(1.55)^2}{0.52} = 4.62 \text{ m/s}^2 \qquad \text{...(By measurement } B'O = 0.52 \text{ m)}$$

Now the acceleration diagram, as shown in Fig. 8.29 (*d*), is drawn as discussed below:

1. Since *O* and *A* are fixed points, therefore these points are marked as one point in the acceleration diagram. Draw vector *a′b′* parallel to *A B*, to some suitable scale, to represent the radial component of the acceleration of *B* with respect to *A i.e.* a^r_{BA} or a_B, such that

$$\text{vector } a'b' = a^r_{BA} = a_B = 23.7 \text{ m/s}^2$$

2. The acceleration of the slider *B* with respect to the coincident point *B′* has the following two components :

(*i*) Coriolis component of the acceleration of *B* with respect to *B′ i.e.* $a^c_{BB'}$, and

(*ii*) Radial component of the acceleration of *B* with respect to *B′ i.e.* $a^r_{BB'}$.

These two components are mutually perpendicular. Therefore from point *b′* draw vector *b′x* perpendicular to *B′O i.e.* in a direction as shown in Fig. 8.29 (*c*) to represent $a^c_{BB'} = 6.45 \text{ m/s}^2$. The direction of $a^c_{BB'}$ is obtained by rotating $v_{BB'}$ (represented by vector *b'b* in velocity diagram) through 90° in the same sense as that of link *OC* which rotates in the counter clockwise direction. Now from point *x*, draw vector *xb''* perpendicular to vector *b'x* (or parallel to *B'O*) to represent $a^r_{BB'}$ whose magnitude is yet unknown.

3. The acceleration of the coincident point *B'* with respect to *O* has also the following two components:

(*i*) Radial component of the acceleration of coincident point *B'* with respect to *O i.e.* $a^r_{B'O}$, and

(*ii*) Tangential component of the acceleration of coincident point *B'* with respect to *O*, *i.e.* $a^t_{B'O}$.

These two components are mutually perpendicular. Therefore from point *o'*, draw vector *o'y* parallel to *B'O* to represent $a^r_{B'O} = 4.62$ m/s^2 and from point *y* draw vector *yb''* perpendicular to vector *o'y* to represent $a^t_{B'O}$. The vectors *xb''* and *yb''* intersect at *b''*. Join *o'b''*. The vector *o'b''* represents the acceleration of *B'* with respect to *O*, *i.e.* $a_{B'O}$.

4. Since the point *C* lies on *OB'* produced, therefore divide vector *o'b''* at *c'* in the same ratio as *C* divides *OB'* in the space diagram. In other words,

$$o'b''/o'c' = OB'/OC$$

5. The acceleration of the ram *D* with respect to *C* has also the following two components:

(*i*) Radial component of the acceleration of *D* with respect to *C i.e.* a^r_{DC}, and

(*ii*) Tangential component of the acceleration of *D* with respect to *C*, *i.e.* a^t_{DC}.

The two components are mutually perpendicular. Therefore draw vector *c'z* parallel to *CD* to represent $a^r_{DC} = 1.01$ m/s^2 and from *z* draw *zd'* perpendicular to vector *zc'* to represent a^t_{DC}, whose magnitude is yet unknown.

6. From point *o'*, draw vector *o'd'* in the direction of motion of the ram *D* which is along the horizontal. The vectors *zd'* and *o'd'* intersect at *d'*. The vector *o'd'* represents the acceleration of ram *D i.e.* a_D.

By measurement, we find that acceleration of the ram *D*,

$$a_D = \text{vector } o'd' = 8.4 \text{ m/s}^2$$ **Ans.**

Angular acceleration of the slotted lever

By measurement from acceleration diagram, we find that tangential component of the coincident point *B'* with respect to *O*,

$$a^t_{B'O} = \text{vector } yb'' = 6.4 \text{ m/s}^2$$

We know that angular acceleration of the slotted lever,

$$= \frac{a^t_{B'O}}{OB'} = \frac{6.4}{0.52} = 12.3 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

Example 8.14. *The driving crank AB of the quick-return mechanism, as shown in Fig. 8.30, revolves at a uniform speed of 200 r.p.m. Find the velocity and acceleration of the tool-box R, in the position shown, when the crank makes an angle of 60° with the vertical line of centres PA. What is the acceleration of sliding of the block at B along the slotted lever PQ ?*