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2. Linear Programming

Introduction:

- George B. Dantzig is recognized as the father of linear programming.

- His work was primarily in the search of techniques to solve logistic problems for military planning — US Air Force, Washington.
- His Researcher, J. Von Neumann, L. Hurwicz and T.C. Koopmans working on same subject.
- The original name given to the technique was "programming of Interdependent activities in a linear structure" and that was later shortened to "Linear Programming".
- 1948 - A. Charnes & W.W. Cooper - introducing and applying the technique to Industrial problems.

L.P. Defined:

- Samuelson, Dorfman & Solow:
  "The analysis of problems in which linear function of a number of variables is to be maximized (or minimized) when those variables are subject to a number of restraints in the form of L.P."
- Loomba: "L.P. is only one aspect of what has been called a system approach to management where in all programmes are designed and evaluated in terms of their ultimate effects in the realisation of business objectives."

* Requirements of LPP:

- Decision variables & their relationship
- Well-defined objective function
- Presence of constraints or restrictions
- Alternative courses of action
- Non negative restriction
- Linearity

* Basic Assumptions of LP:

- Proportionality
- Additivity
- Divisibility
- Certainty
- Finiteness

* Application of LP:

a.) Production Management:
- product mix
- Blending problem
- Trim loss
- Production planning

b.) Marketing Management:
- Media selection
- Traveling salesman problem
- Physical distribution

c.) Agricultural applications:
- Farm management

d.) Financial Management!
↳ capital budgeting problem
↳ Profit planning

e.) Miscellaneous problem!
↳ Diet problem
↳ Inspection problem
↳ Military applications.

* General Mathematical Model of LP:

The general LP with 'n' decision variable and 'm' constraints can be stated in the following form

Find the values of decision variables
$x_1, x_2, x_3, ----, x_n$ so as to

optimize (maximize or minimize) $Z = c_1x_1 + c_2x_2 + --- + c_nx_n$ } objective fn

Subject to linear constraints,

$a_{11}x_1 + a_{12}x_2 + --- + a_{1n}x_n \ (\le, =, \ge) \ b_1$
$a_{21}x_1 + a_{22}x_2 + --- + a_{2n}x_n \ (\le, =, \ge) \ b_2$
$\vdots$
$a_{m1}x_1 + a_{m2}x_2 + --- + a_{mn}x_n \ (\le, =, \ge) \ b_m$ } Constraints

and $x_1, x_2, x_3, --- x_n \ge 0$ } non negativity restriction

By using the symbol 'Σ'
in compact form eqn,

Optimize (Max or Min)

$$Z = \sum_{j=1}^{n} c_j x_j \quad \Big\} \text{ objective function}$$

Subject to the linear constraints

$$\sum_{j=1}^{n} a_{ij} x_j \ (\leq, =, \geq)\ b_i \ ; \ i = 1, 2, ---, m \quad \Big\} \text{ constraints}$$

and $x_j \geq 0$, $j = 1, 2, ---, n$ } Non negativity restriction.

Where; $x_1, x_2, --- x_n$ are decision (or choice) variables

- $c_1, c_2, ---, c_n$ are cost or profit coefficients
- a_{ij} $(i = 1, 2, ---, m; j = 1, 2, ---, n)$ – technological or substitution or structural coefficient.
- b_i $(i = 1, 2, ---, m)$ = requirements or available of the ith constant
- $(\leq, =, \geq)$: each ~~constraint~~ constraint may take any one of three possible forms.
- The restriction $x_j \geq 0$ $(j = 1, 2, ---, n)$: x_j's must not negative.

* Product Mix Problems:

Ex.-1. Raju furniture factory produces inexpensive table and chairs. Production process on both are similar in the sense that both require a certain number of carpentry work and a certain number of Labour hours in the painting department. Each table takes 4 hrs in carpentry and 2 hrs in painting department. Each chair needs 3 hrs in carpentry and just 1 hr in painting. During the current production period, only 300 hrs of carpentry time and 120 hrs of painting time are available. Each table sold yields a profit of Rs. 70 and each chair produced may be sold for a profit of Rs. 50.
Raju furniture factory wants to determine the best possible combination of tables and chair to manufacture in order to get the maximum profit.

Ans. → Summarize the data;

Resource/constraints	Hrs required to produce 1 unit		Available hours
	Tables	Chairs	
Carpentry	4	3	300
Painting	2	1	120
Profit contribution (Rs/unit)	70	50	

Steps:

↳ Identify decision variables,

x_1 = number of tables to be produced

x_2 = number of chairs to be produced.

↳ develope the mathematical relationships to describe the two constraints

i.) carpentry time used ≤ carpentry time available
(4 hrs/table) (No. of tables prod.) + (3 hrs/chair) (No. of chairs prod.) ≤ 300

$4x_1 + 3x_2 \leq 300$ (hrs of carpentry time)

ii) Painting time used ≤ painting time available

$2x_1 + x_2 \leq 120$ (hrs of painting time)

$x_1 \geq 0$, $x_2 \geq 0$

↳ Identify the objective function,

$$Z = 70x_1 + 50x_2$$

the appropriate formulation ~~of~~ as L.P. for

Maximize (total profit),

$$Z = 70x_1 + 50x_2$$

Subject to the constraints,

$$4x_1 + 3x_2 \leq 300$$

$$2x_1 + x_2 \leq 120$$

$$x_1 \geq 0, \; x_2 \geq 0$$

Ex-2: A company has three operational departments (weaving, processing and packing) with capacity to produce three different types of clothes namely suitings, shirtings and woollens yielding the profit Rs. 2, Rs. 4 and Rs. 3 per m respectively. One meter suiting requires 3 mint in weaving, 2 mint in processing and 1 min in packing. Similarly 1 miter of shirting requires

4 minutes in weaving, 1 mint in processing and 3 mint. in packing while one meter woollen requires 3 mint in each department. In a week, total run times of each department are 60, 40 and 80 hrs of weaving, processing and packing department respectively. Formulate the LP problem to find the product mix to maximize the profit.

Ans. →

Resource/const.	Product			Total availability (mint.)
	Suiting	Shirting	woollen	
Weaving dept.	3	4	3	60 X 60
processing dept.	2	1	3	40 X 60
packing dept.	1	3	3	80 X 60
Contribution per meter (Rs.)	2	4	3	

objective function,

$$Z = 2x_1 + 4x_2 + 3x_3$$

Subject to the constraints.

$$3x_1 + 4x_2 + 3x_3 \leq 3600$$

$$2x_1 + x_2 + 3x_3 \leq 2400$$

$$x_1 + 3x_2 + 3x_3 \leq 4800$$

$$x_1, x_2, x_3 \geq 0$$

*Diet Problems:

Ex - 3. Vitamins V and W are found in two different foods F_1 and F_2. One unit of food F_1 contains 2 units of vitamin V and 5 units of vitamin W. One unit of food F_2 contains 4 units of vitamin V and 2 units of vitamin W. One unit of food F_1 and F_2 cost Rs. 30 and 25 respectively. The minimum daily requirements (for a person) of vitamin V and W is 40 and 50 units respectively. Assuming that anything in excess of daily minimum requirement of vitamin V and W is not harmful, find out the optimal mixture of food F_1 and F_2 at the minimum cost which meets the daily minimum requirements of vitamins V and W. Formulate this as LPP.

Ans. →

Resources/constraints	Food F_1	Food F_2	Min. daily requirement
Vitamin V (units)	2	4	40
Vitamin W (units)	5	2	50
Cost per unit	30	25	

Optimize (minimize total cost) } objective function

$$Z = 30x_1 + 25x_2$$

Subject to the constraints } constraints

$$2x_1 + 4x_2 \geq 40$$

$$5x_1 + 2x_2 \geq 50$$

$$x_1, x_2 \geq 0$$

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* Manpower (or Personnel) Schedule Problems:

Ex. 4. A 24-hour supermarket has the following minimum requirement for security officers:

Table 4: Staffing Req.

Time of day	Mini. no. of cashiers req.
Midnight - 4am	7
4am - 8am	20
8am - Noon	14
Noon - 4pm	20
4pm - 8pm	10
8pm - Midnight	5

Table 6: Shift schedule

Shift	Starting time	Ending time
1	Midnight	8 am
2	4am	Noon
3	8 am	4 Pm
4	Noon	8 pm
5	4pm	Midnight
6	8pm	4am

Shift 1 follows immediately after shift 6. An officer works eight consecutive hours, starting at the begining of one of the six periods. The Personnel manager wants to determine how many officers should work each shift in order to minimize the total number of officers employed while still satisfying the staffing requirements. Formulate the problem by $^{\text{ques 4}}$ LPP.

Ans. → decision variables;

x_1 = no. of ~~workers~~ officers working in shift 1.

x_2 = " " " " shift 2

⋮

x_6 = " " " shift 6

Personnel manager wants to minimize this sum.

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

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Shift	Time Interval					
	midnight to 4am	4am to 8am	8am to Noon	Noon to 4pm	4pm to 8pm	8pm to Midni
1	x_1	x_1				
2		x_2	x_2			
3			x_3	x_3		
4				x_4	x_4	
5					x_5	x_5
6	x_6					x_6
Req.	7	20	14	20	10	5

↳ Objective function:

Minimize

$$Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

↳ Subject to the constraints;

$$x_1 + x_6 \geq 7$$
$$x_1 + x_2 \geq 20$$
$$x_2 + x_3 \geq 14$$
$$x_3 + x_4 \geq 20$$
$$x_4 + x_5 \geq 10$$
$$x_5 + x_6 \geq 5$$

↳ non negative constraint;

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Ex:5: An advertising company wishes to plan an (Dec-11) advertising campaign in three different media: (Tut-1.1) television, radio and a magazine. The purpose of the advertising is to reach as many potential customers as possible. Following are the result of a market study:

	Television		Radio Rs.	Magazine Rs.
	Prime Day Rs.	Prime Time Rs.		
• cost of an adv. unit	40,000	75,000	30,000	15,000
• Number of potential custmers reached / unit	4,00,000	9,00,000	5,00,000	2,00,000
• Number of women customers reached / unit	3,00,000	4,00,000	2,00,000	1,00,000

The company does not want to spend more than Rs. 8,00,000 on advertising. It is further required that.

i) At least 20,00,000 exposures take place among women
ii) Advertising on television be limited to Rs. 5,00,000
iii.) At least 3 advertising unit be bought on prime day and two units during prime time, and
iv) The number of advertising units on radio and magazine should each be between 5 and 10.

Formulate this problem as an L.P. model to maximize potential customer reach.

Ans. → x_1 = no. of adv. units bought in prime day } on television
x_2 = " " " " prime time
x_3 = " " " " on ~~television~~ radio
x_4 = " " " " on magazine

↳ objective function:

Maximize total customer reach:

$Z = 400000x_1 + 900000x_2 + 500000x_3 + 200$

↳ Subject to the constraints:

$40000x_1 + 75000x_2 + 30000x_3 + 15000x_4 \le 800000$

i.) No. of women customer reach by adv. compaig constraint

$300000x_1 + 400000x_2 + 200000x_3 + 100000x_4 \ge 2000$

ii.) Television adv. constraints

$40000x_1 + 75000x_2 \le 500000$

iii.) $x_1 \ge 3$

$x_2 \ge 2$

iv) Radio & magazine adv. constraints

$5 \le x_3 \le 10$

$5 \le x_4 \le 10$

↳ Non negative constraints

$x_1, x_2, x_3, x_4 \ge 0$

Ex-6: (May-12) (Tut. 1.3) A small fabrication industry is faced with a problem of scheduling production and subcontracting for three products A, B and C. Each product requires castings machining and assembly operat. Casting operation for product A and B can be subcontracted but product C requires special tooling hence it can not be subcontracted. Each unit of product A, B and C requires 6, 10 and 8 minutes of casting time in the foundry sh of a company. Machining time per unit of products A, B and C are 6, 3 and 8 minute while assembly times are 3, 2 and 2 minute

2 minutes respectively. The time available per week in foundry, machining and assembly shop are 8000, 12000 and 10000 minutes respectively. If product A, B and C are produced completely in the company, the overall profits per unit of product are Rs. 700, Rs 1000 and Rs. 1100 respectively. When castings are obtained from subcontractors, the profit per unit of product A and B are Rs. 500 and 900 respectively. Formulate above problem as LPP so as to maximize the profit for company by scheduling its production and subcontracting.

Ans.→

Process/Product	Production in Company A (min)	B (min)	C (min)	Subcontracting A	B	Time available per week (minute)
Casting	6	10	8	–	–	8000
Machining	6	3	8	6	3	12000
Assembly	3	2	2	3	2	10000
Profits	700	1000	1100	500	900	

decision variables:

x_1 = No. of A in house.

x_2 = " B "

x_3 = " C "

x_4 = No. of A subcontract

x_5 = " B "

↳ Objective function:

Optimize (max); $Z = 700x_1 + 1000x_2 + 1100x_3 + 500x_4 + 900x_5$

↳ Subject to the constraints:

$6x_1 + 10x_2 + 8x_3 \leq 8000$

$6x_1 + 3x_2 + 8x_3 + 6x_4 + 3x_5 \leq 12000$

$3x_1 + 2x_2 + 2x_3 + 3x_4 + 2x_5 \leq 10000$

↳ Non negative constraints $x_1, x_2, x_3, x_4, x_5 \geq 0$..

Ex. - 7: A cofee company mixes Brazilian, Columbian and
(Nov.-13) African coffee to make two brands of coffee
plains A and B. The characteristics used in blen-
ding the coffee include strength, acidity and ca...
The test result of the available supply of Brazilian,
Columbian and African coffee.

	Price/kg	Strength	Acidity	% coffee	Supply available
Brazilian	60	6	4	2	50000
Columbian	70	8	3	2.5	30000
African	65	5	3.5	1.5	25000

The requirement for A and B coffee are given as below

Plain coffee	Price/kg	Min Strength	Max acidity	Max % coffee	Quantity Demanded
A	75	6.5	3.8	2.2	65000
B	85	6.0	3.5	2	55000

Assume that 35000 kg of plain A and 25000 kg of plain B are to be sold formulate LPP.

Ans. →

	SP A	cost A	Profit A	SP B	Cost B	Profit B
Brazilian	75	60	15	85	60	25
Columbian	75	70	5	85	70	15
African	75	65	10	85	65	20

Let for plain coffee A:

x_{11} = qty in kg of Brazilian coffee
x_{12} = qty " Columbian "
x_{13} = " " African "

& for plain coffee B:

x_{21} = qty in kg of Brazilian coffee
x_{22} = " " Columbian "
x_{23} = " " African "

↳ Objective function:

Maximize:

$$Z = 75(x_{11}+x_{12}+x_{13}) - (60x_{11}+70x_{12}+65x_{13})$$
$$+ 85(x_{21}+x_{22}+x_{23}) - (60x_{21}+70x_{22}+65x_{2}$$
$$Z = 15x_{11} + 5x_{12} + 10x_{13} + 25x_{21} + 15x_{22} + 20x_{2}$$

↳ Subject to the constraints:

$$6x_{11} + 8x_{12} + 5x_{13} \geq 6.5(x_{11}+x_{12}+x_{13})$$
$$6x_{21} + 8x_{22} + 5x_{23} \geq 6(x_{21}+x_{22}+x_{23})$$
$$4x_{11} + 3x_{12} + 3.5x_{13} \leq 3.8(x_{11}+x_{12}+x_{13})$$
$$4x_{21} + 3x_{22} + 3.5x_{23} \leq 3.8(x_{21}+x_{22}+x_{23})$$
$$2x_{11} + 2.5x_{12} + 1.5x_{13} \leq 2.2(x_{11}+x_{12}+x_{13})$$
$$2x_{21} + 2.5x_{22} + 1.5x_{23} \leq 2(x_{21}+x_{22}+x_{23})$$
$$x_{11} + x_{12} \leq 50000$$
$$x_{12} + x_{22} \leq 30000$$
$$x_{13} + x_{23} \leq 25000$$
$$x_{11} + x_{12} + x_{13} \geq 65000$$
$$x_{21} + x_{22} + x_{23} \geq 55000$$

↳ Non negative constraint:

$$x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geq 0$$

* GRAPHICAL METHOD :

↳ Solving two variable LPP.

Steps: 1.) Formulate the problem:

2.) Plot each of the constrains on the gr

$x_2 = 0 \Rightarrow x_1$ } plot x_1 as x-axis

$x_1 = 0 \Rightarrow x_2$ } x_2 as Y-axis.

3.) Identify the feasible region:

↳ constraints are (≤) type,

- the area on or below the constraint
- towards origin will be considered.

↳ constraints are (≥) type,

- the area on or above the constraint li
- away from the origin is considered.

∴ " The area common to all constraints is called feasible region "

4.) Graphical solution techniques:

a.) Extreme Corner Point Method

b.) Iso-profit or Iso-cost method.

Ex.-9. Use the graphical method to solve the following LPP.
(Nov.-11) Maximize, $Z = 2x_1 + x_2$

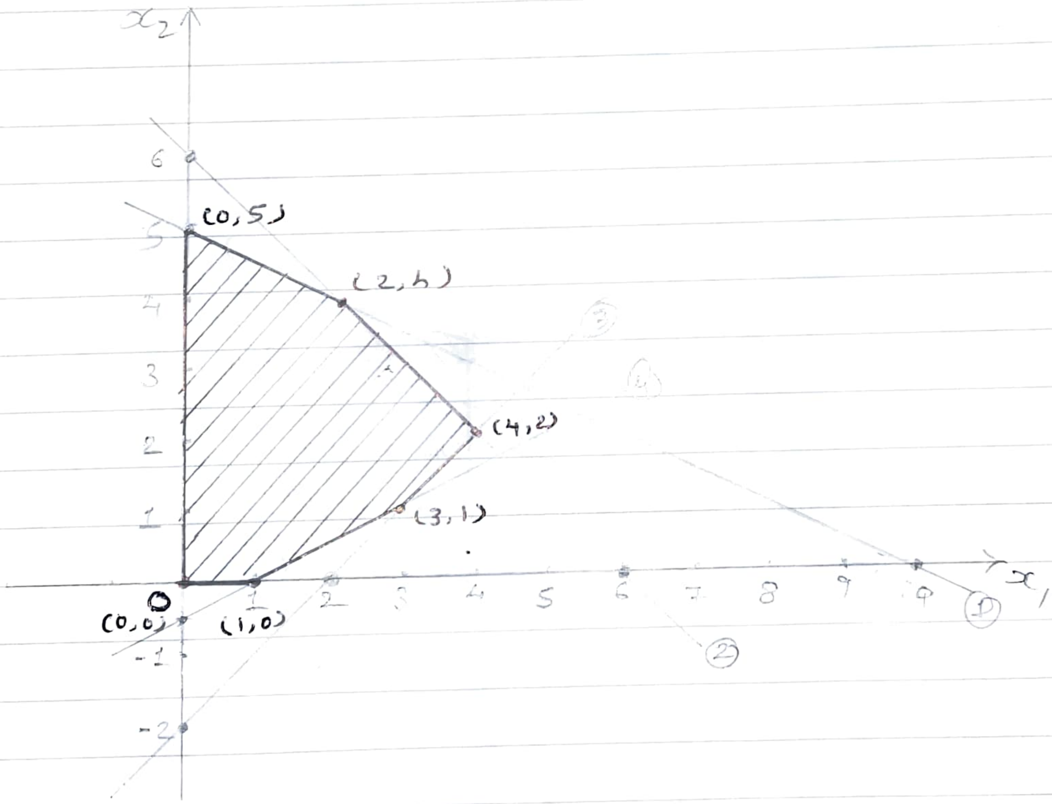
Subject to, $x_1 + 2x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$ and $x_1, x_2 \geq 0$

Ans.→



	$Z = 2x_1 + x_2$
(0,0)	0
(0,5)	5
(2,4)	8
$\boxed{(4,2)}$	$\boxed{10}$
(3,1)	7
(1,0)	2

$\therefore x_1 = 4, x_2 = 2$

max, $Z = 10$

Ex.-10. Find the maximum value of following LPP using
(Nov.-12) graphical approach.
(Nov.-17)

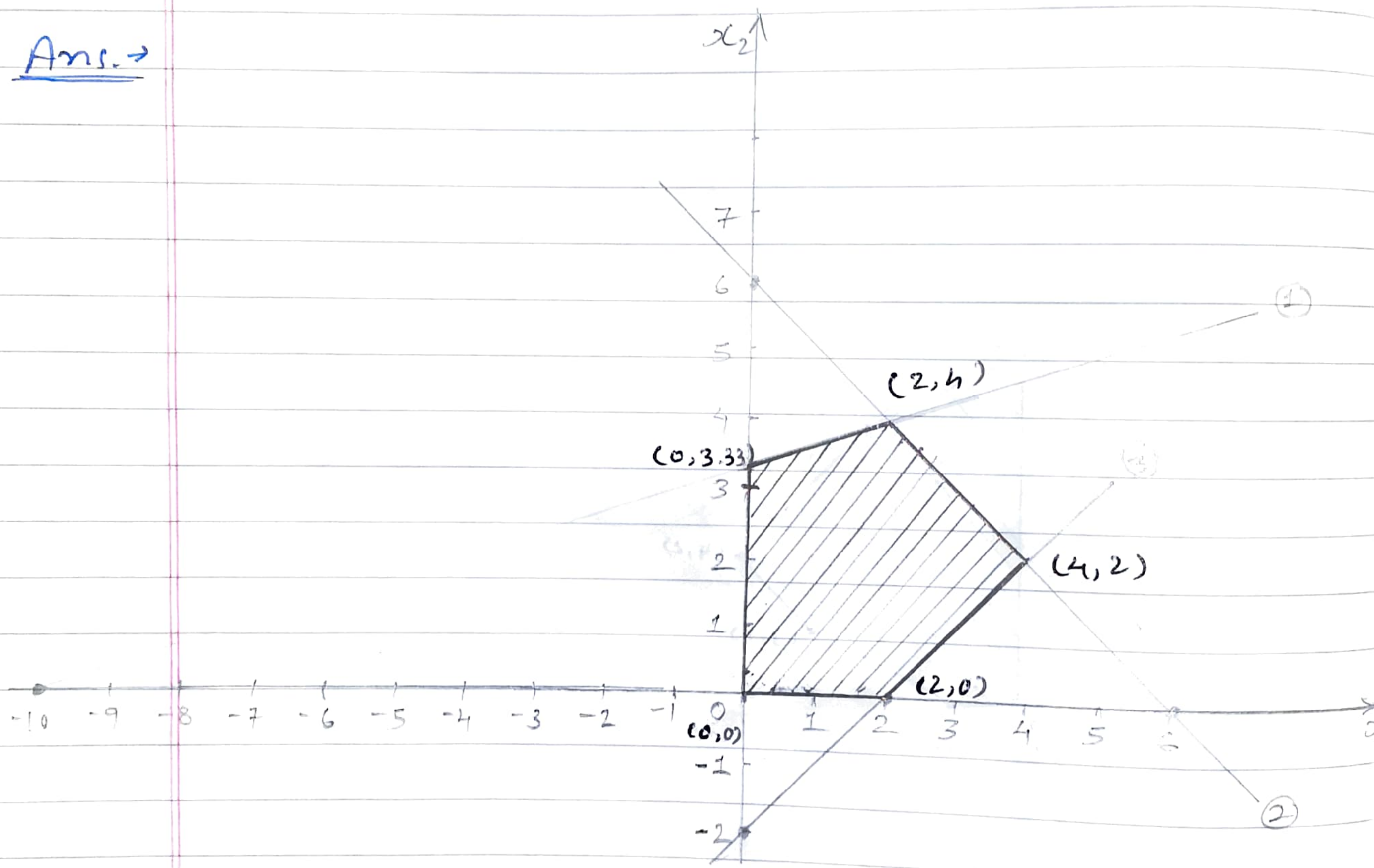
$$Z = -x_1 + 2x_2$$

subject to, $-x_1 + 3x_2 \le 10$

$x_1 + x_2 \le 6$

$x_1 - x_2 \le 2$ & $x_1, x_2 \ge 0$

Ans.→



	$Z = -x_1 + 2x_2$
(0,0)	0
(0,3.33)	6.67
(2,4)	6
(4,2)	0
(2,0)	-2

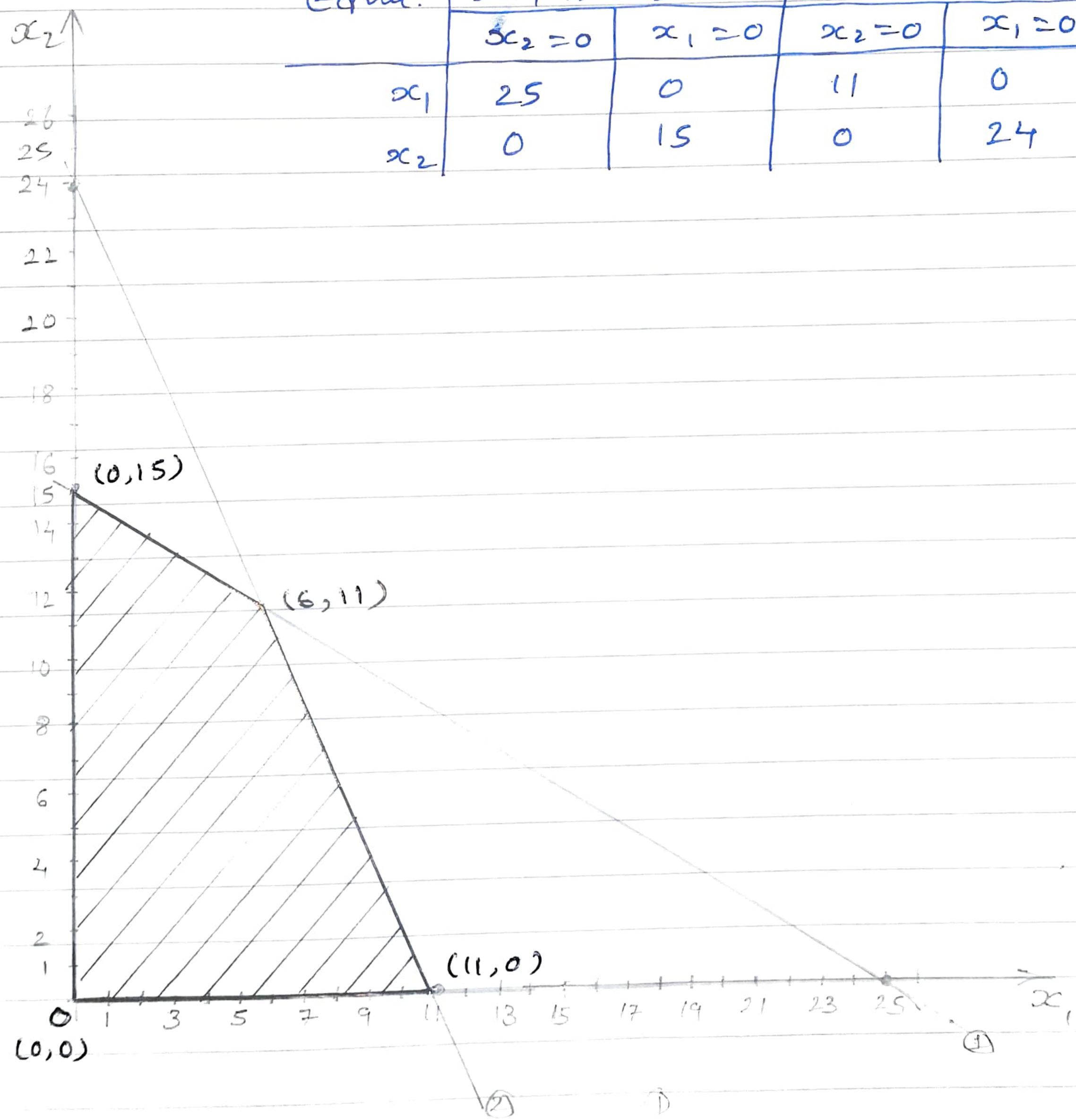
$\therefore x_1 = 0, \; x_2 = 3.33$

max. $Z = 6.67$

Ex.-11 Use graphical method to solve the following LPP.
(May-15) Maximize, $Z = 17x_1 + 15x_2$
Subject to, $15x_1 + 25x_2 \le 375$
$24x_1 + 11x_2 \le 265$ and $x_1, x_2 \ge 0$

Ans.→

Equn.	$15x_1 + 25x_2 = 375$		$24x_1 + 11x_2 = 265$	
	$x_2 = 0$	$x_1 = 0$	$x_2 = 0$	$x_1 = 0$
x_1	25	0	11	0
x_2	0	15	0	24



	$Z = 17x_1 + 15x_2$
(0,0)	0
(0,15)	225
(6,11)	267
(11,0)	187

$\therefore x_1 = 6, \; x_2 = 11$

max $Z = 267$

* Simplex Method:

↳ The "Simplex Method" was developed by George B. Dantzig in 1947.

↳ When LPP can have more than two variables, to solve such kind of LPPs simplex Method is used.

- **Slack Variables:**

A variable added to the left hand side of a "less than or equal to ($\leq$)" constraints to convert the constraint into an equality is called a 'Slack Variable'.

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n + \underbrace{S_i}_{\text{Slack variable}} \leq b_i \;;\; i = 1, 2, \dots$$

$$\boxed{\text{Slack} = \text{Requirement} - \text{Production}}$$

- **Surplus Variables:**

A variable subtracted from the left hand side of the "greater than or equal to ($\geq$)" constraint, to convert the constraint into an equality is called a 'Surplus variable'.

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n - S_i \geq b_i \;;\; i = 1, 2, \dots$$

$$\boxed{\text{Surplus} = \text{Production} - \text{Requirement}}$$

* Steps to solve LPP by Simplex Method:

Step 1.) Formulate the LP model

Maximize

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

subject to constraints

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_m$$

$$x_1, x_2, \cdots, x_n \ge 0$$

2.) Introduce Slack variables in objective function and in constraints.

Maximize

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n + 0s_1 + 0s_2 + \cdots + 0s_m$$

constraints

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + S_1 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + S_2 = b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + S_m = b_m$$

3.) Design the Initial Feasible Solution:

$$x_1 = x_2 = \cdots = x_n = 0$$

$$\therefore S_1 = b_1, S_2 = b_2, \cdots, S_m = b_m$$

4.) Set up the Initial Simplex Tableau:

C_j		c_1	c_2 --- c_n	0	0 --- 0		
Basic variables 'B'	C_B	Coefficient Matrix x_1	x_2 --- x_n	Identity Mat. S_1	S_2 --- S_m	value of Basic Vari. b_j	Mi ru 'θ'
S_1	0	a_{11}	a_{12} --- a_{1n}	1	0 --- 0	b_1	
S_2	0	a_{21}	a_{22} --- a_{2n}	0	1 --- 0	b_2	
⋮	⋮	⋮	⋮ ⋮	⋮		⋮	
S_m	0	a_{m1}	a_{m2} --- a_{mn}	0	0 --- 1	b_m	
$Z = \Sigma C_{Bj} \cdot a_{ij}$		0	0 --- 0	0	0 --- 0		
$C_j - Z_j$		c_1	c_2 --- c_n	0	0 --- 0		

Note: The basic variables will always have unit matrix according to their row position in the table.

5.) Calculate the 'Z' & '$C_j - Z_j$' for Initial basic variables:

Z for x_1 column $= C_{B1} \cdot a_{11} + C_{B2} \cdot a_{12} + \dots + C_{Bj} \cdot a$

6.) Select the key column:

↳ max. positive value → maximization
↳ max. negative value → minimization

7.) Select the key row:

↳ min. positive value is selected as key row.

↳ key number → intersection of key column & key row

↳ min. ratio 'θ' $= \dfrac{b_j}{\text{key number}}$

8.) Revision/Prepare the new Simplex Table:

↳ For key row elements, divide all row elements by key number.

↳ New number for every cell
= Old number − (F.R. × corresponding key row number)

Fraction ratio (F.R.) = $\frac{\text{Corresponding key column No.}}{\text{key number}}$

9.) Evaluate the new solution:

↳ Repeat step 5 to 8 until the values of $C_j - Z_j$ to all variables are,

- coming as zero or negative for maximization problem
- coming as zero or positive for minimization problem.

Ex.-13. Solve the following LPP by simplex method
(May-14) maximize, $Z = 3x_1 + 2x_2$
Constraints, $2x_1 + x_2 \le 5$
$x_1 + x_2 \le 3$ and $x_1; x_2 \ge 0$

Ans.→ Maximize,
$$z = 3x_1 + 2x_2 + 0S_1 + 0S_2$$
subject to constraints,
$$2x_1 + x_2 + S_1 = 5$$
$$x_1 + x_2 + S_2 = 3$$
$$x_1, x_2, S_1, S_2 \ge 0$$

An Initial Basic Feasible Solution is;
$x_1 = 0$, $x_2 = 0$
$S_1 = 5$, $S_2 = 3$ at $z = 0$

C_j		3	2				
Basic	C_B	x_1	x_2	S_1	S_2	b_j	min. ratio 'θ'
S_1	0	2 key number	1	1	0	5	5/2 ← key row
S_2	0	1	1	0	1	3	3
	Z_j	0	0	0	0		$Z = 0$
	$C_j - Z_j$	3	2	0	0		

↑ key column

Z_1 for x_1 column $= \Sigma C_{Bj} a_{ij}$
$= C_{B1} \cdot a_{11} + C_{B2} a_{12}$
$= 0 \cdot 2 + 0 \cdot 3 = 0$

for x_1 column, $C_j - Z_j = 3 - 0 = 3$

↳ Prepare the new simplex table :

↳ New a_{21} = old no. − (F.R.X corresponding key row no.
$= 1 - (\frac{1}{2} \times 2) = 0$
$a_{22} = 1 - (\frac{1}{2} \times 1) = \frac{1}{2}$

C_j		3	2	0	0		
Basic	C_B	x_1	x_2	s_1	s_2	b_j	θ
x_1	3	1	$1/2$	$\frac{1}{2}$	0	$5/2$	5
s_2	0	0	$1/2$	$-1/2$	1	$1/2$	1 ← key row
	Z_j	3	$3/2$	$3/2$	0		
	$C_j - Z_j$	0	$1/2$	$-3/2$	0		

↑
key column

C_j		3	2	0	0	
Basic	C_B	x_1	x_2	s_1	s_2	b_j
x_1	3	1	0	1	-1	2
x_2	2	0	1	-1	2	1
	Z_j	3	2	1	1	$Z = 8$
	$C_j - Z_j$	0	0	-1	-1	

Hence, solution for this LPP is,

$x_1 = 2$, $x_2 = 1$

$Z = 8$