Chapter – 4
Exergy

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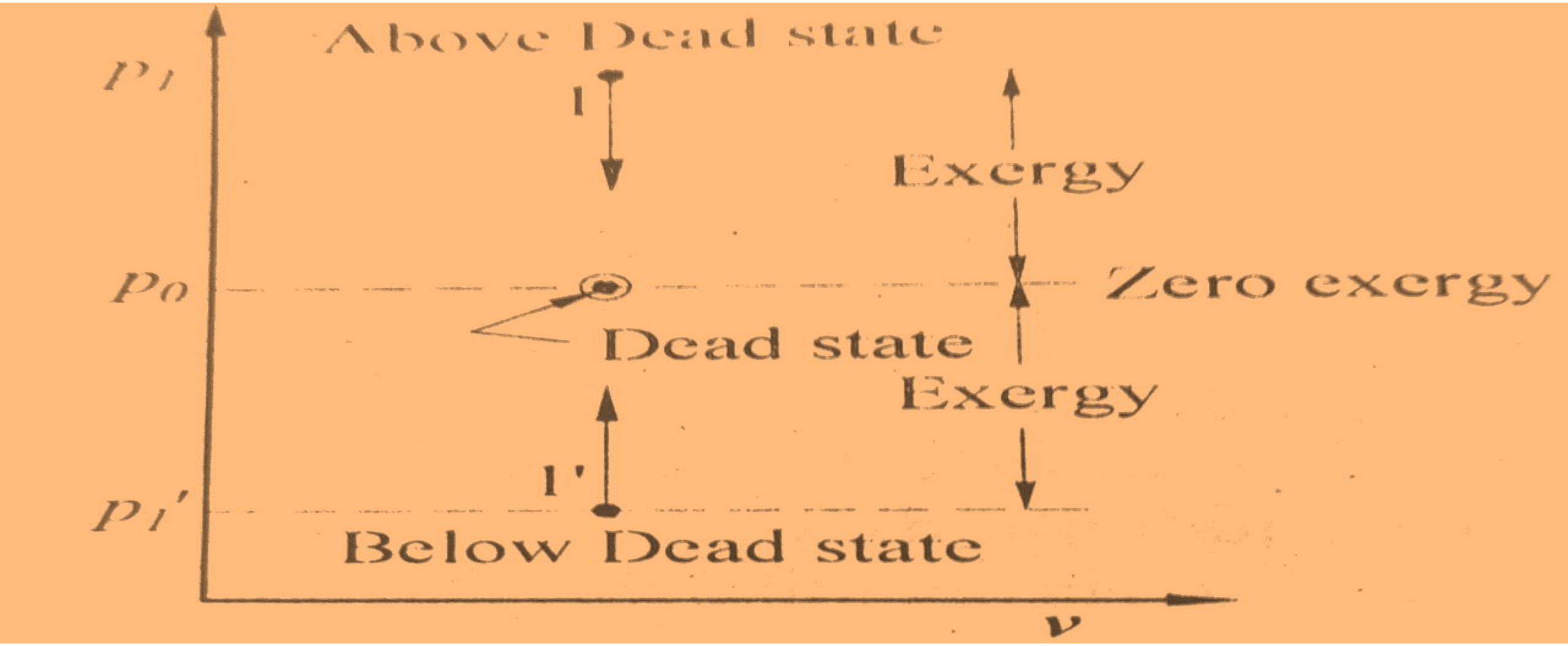
- ***4.1 Introduction***
- "The maximum useful work that could be obtained from the system at a given state in a specified environment".
- The exergy of a system is "the maximum useful work possible during a process that brings the system into equilibrium with a heat reservoir, reaching maximum entropy".
- When the surroundings are the reservoir, exergy is the potential of a system to cause a change as it achieves equilibrium with its environment.
- Exergy is the energy that is available to be used.

- ***4.1 Introduction***
- Therefore it is also called the availability or available energy.
- After the system and surroundings reaches to equilibrium the exergy becomes zero.
- "the energy which is not utilizable and rejected to the surroundings" is called Anergy (unavailable energy).
- For a thermodynamic system:

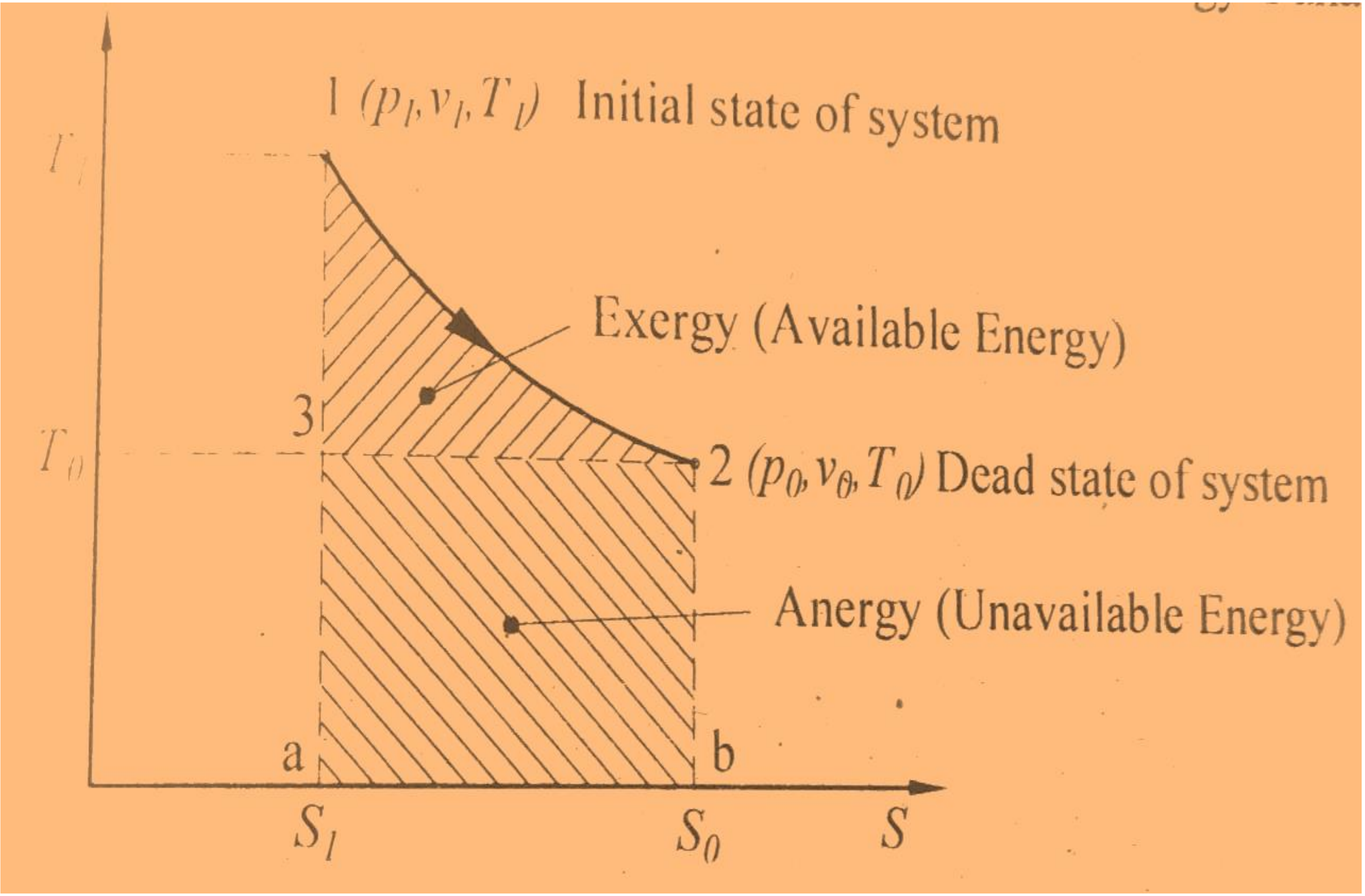
$$\text{Energy} = \text{Exergy} + \text{Anergy}$$

- ***4.1 Introduction***
- Exergy is always destroyed when a process is irreversible, for example loss of heat to the environment.
- This destruction is proportional to the entropy increase of the system together with its surroundings. The destroyed exergy has been called Anergy.
- The exergy cannot be calculated without defining appropriate parameters for the environment, where the target system operates in terms of temperature, pressure, chemical composition.

- ***4.1 Introduction***
- At this reference state system comes to complete equilibrium with the environment, and no energy difference will exist to cause further work. Such state is called as DEAD STATE.



- ## <u>*4.1 Introduction*</u>



Available & Unavailable Energy

4.2 Exergy of heat input in a cycle

(1) Exergy (available energy) referred to an infinite heat source (heat reservoir)

Consider a system working on Carnot cycle (reversible heat engine) as shown in Fig 5.3 (a). Engine withdraws heat energy Q_1 from infinite heat source (at constant temperature T) and rejects Q_2 heat at surroundings (at constant temperature T_0).

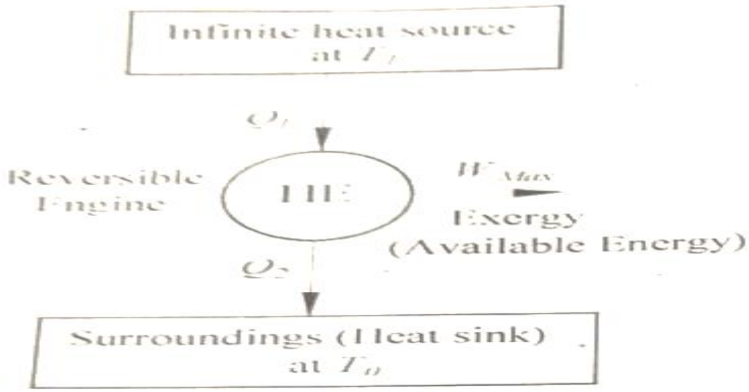


Fig. 5.3(a) Reversible cycle with infinite heat source

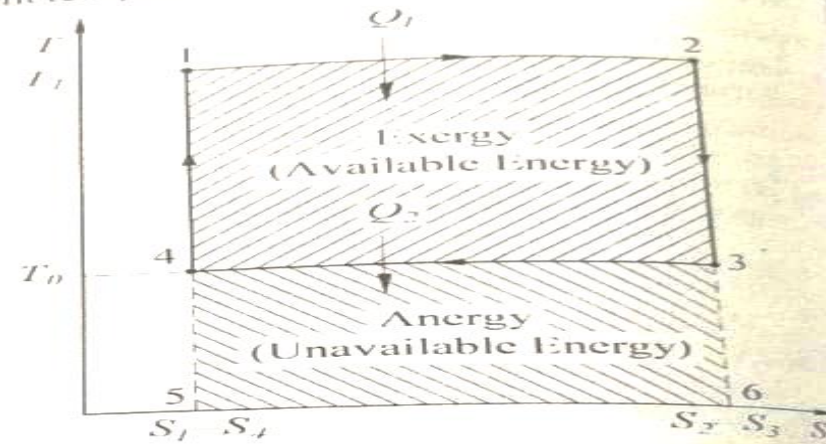


Fig. 5.3(b) T-S diagram of reversible cycle with infinite heat source

For given values of the temperature T_1 and T_2, the efficiency of reversible engine is given by

$$\eta_{rev} = 1 - \frac{T_2}{T_1}$$

For given temperature $T_1 = T$, η_{rev} will increase with the decrease of T_2. The lowest practicable temperature of heat rejection is the temperature of surroundings T_0.

$$\therefore \eta_{max} = 1 - \frac{T_0}{T}$$

and maximum work

$$W_{max} = \text{Heat supplied} \times \text{maximum efficiency}$$

$$= Q_1 \times \eta_{max} \quad \left(\because \eta_{max} = \frac{W_{max}}{Q_1}\right)$$

$$= Q_1\left(1 - \frac{T_0}{T}\right) = Q_1 - Q_1\frac{T_0}{T} = Q_1 - \frac{Q_1}{T}\cdot T_0$$

4.2 Exergy of heat input in a cycle

$\therefore W_{max} = Q_1 - T_0\, dS \qquad \left(\because \frac{Q_1}{T} = dS\right)$ ...(5.1)

W_{max} is the Exergy or availability or available energy and it is represented on T-S diagram by the area 1 – 2 – 3 – 4 is shown in Fig. 5.3(b). The Anergy or unavailable energy is represented on T – S diagram by area 3 – 4 – 5 – 6. The unavailable energy is the energy rejected from engine and hence it is portion of heat supplied that can not be converted into work.

$\therefore$ Heat supplied = Available energy + Unavailable energy

$\therefore Q_1 = AE + UE$ or Q_1 = Exergy + Anergy

$\therefore Q_1 = W_{max} + UE$

$\therefore W_{max} = Q_1 - UE$ ...(5.2)

From equation (4.1) and (4.2)

Unavailable energy $UE = T_0 dS = T_0(S_2 - S_1)$ ...(5.3)

Available energy $AE = W_{max} = Q_1 - T_0(S_2 - S_1)$ ...(5.4)

(2) Exergy (available energy) referred to finite heat source or finite heat capacity body **[Dec. '15]**

Consider a system working on reversible cycle, as shown in Figure 5.4(a). The heat engine withdraws heat Q_1 from finite heat source where the temperature of which decreases with the withdrawal of heat from it. And engine rejects heat Q_2 to surroundings at constant temperature T_0. The process 1—2 can be broken up for dividing number of Carnot cycles working between higher temperature T (different for different cycle, it decreases for successive Carnot cycles) and temperature of surroundings T_0 (common for all cycles, all cycles rejects heat to surroundings at constant temperature T_0).

Considering an elemental Carnot engine, as shown in Fig. 5.4(b).

$$\eta = \frac{\delta W}{\delta Q_1} = 1 - \frac{T_0}{T}$$

$$\therefore \delta W = \delta Q_1\left(1 - \frac{T_0}{T}\right)$$

For cycle 1 – 2 – 3 – 4 as a whole,

$$W_{max} = \int \delta Q_1\left(1 - \frac{T_0}{T}\right) = \int \delta Q_1 - T_0 \int \frac{\delta Q_1}{T} = Q_1 - T_0\, dS \quad ...($$

Available energy $= Q_1 - T_0\, dS$ ...

Unavailable energy $= T_0\, dS$

where dS = entropy change for the process 1 – 2 or entropy difference (1) are as

4.2 Exergy of heat input in a cycle

$$S_2 - S_1 = mR\ln\frac{V_2}{V_1} + mC_v\ln\frac{T_2}{T_1} = mR\ln\frac{p_1}{p_2} + mC_p\ln\frac{T_2}{T_1} = mC_p\ln\frac{V_2}{V_1} + mC_v\ln\frac{p_2}{p_1}$$

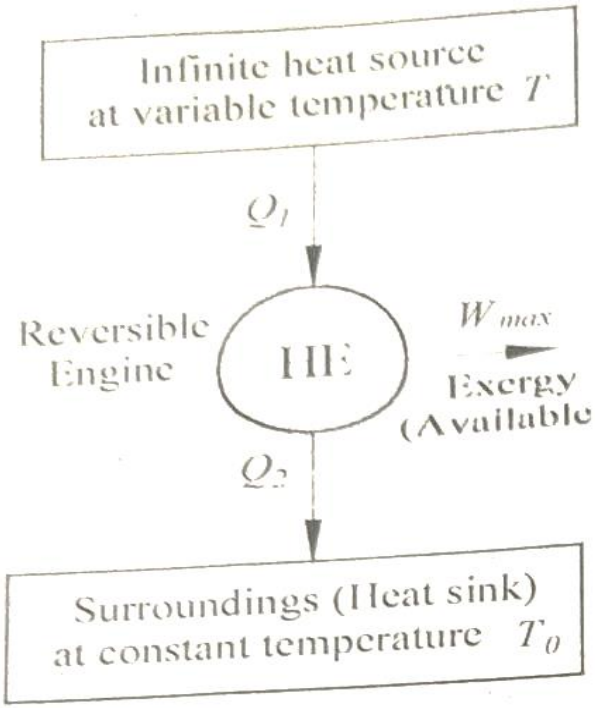


Fig. 5.4(a) Reversible cycle with finite heat source

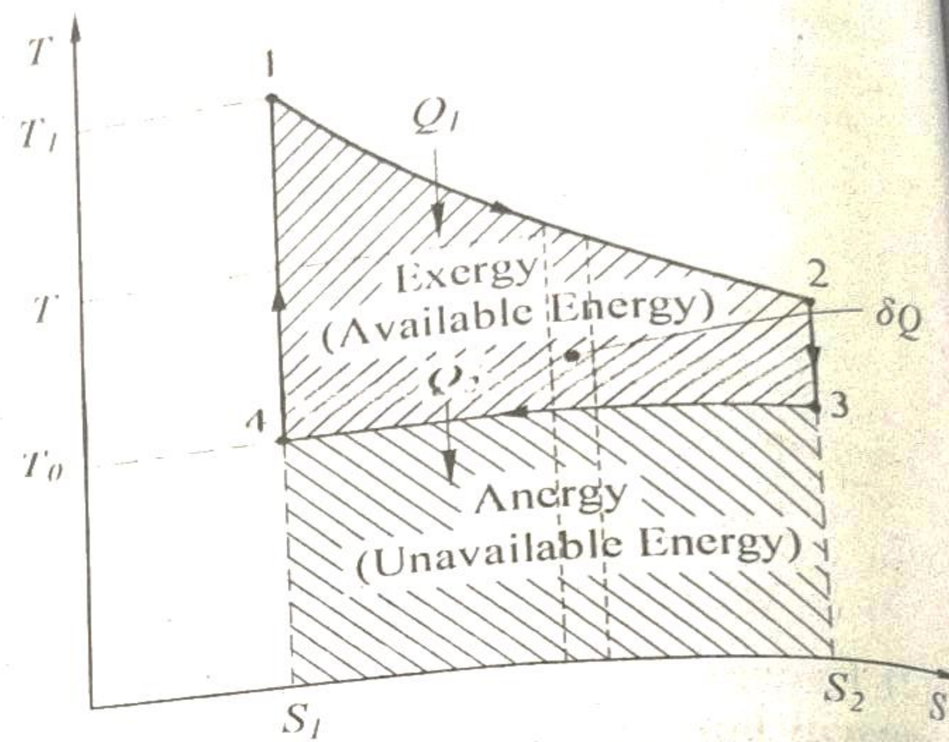


Fig. 5.4(b) T-S diagram of reversible cycle with finite heat source

... heat from a infinite heat source at 800 K. The

4.3 Loss Work - Exergy destruction in heat transfer process

Whenever heat transferred through a finite temperature difference, there is always loss of the availability of energy due to irreversible heat transfer.

Consider heat engine working on a reversible cycle between temperature T_1 and T_0. The reversible heat engine, absorbs heat Q from the reservoir at temperature T_1 and rejects heat Q_0 to the surroundings at temperature T_0 as shown in Fig. 5.8(a). And heat engine cycle is represented by 1 2 3 4 on T-S diagram.

The maximum work or Exergy or available energy corresponding to the heat Q is

$$W_{max} = AE = Q \cdot \eta = Q\left(1 - \frac{T_0}{T_1}\right)$$

$$= Q - \frac{Q}{T_1}T_0 = Q - T_0 \Delta S = \text{Area under } 1\text{-}2\text{-}3\text{-}4$$

4.3 Loss Work - Exergy destruction in heat transfer process

Unavailable energy $T_0 \Delta S$ area under 4 3 b a

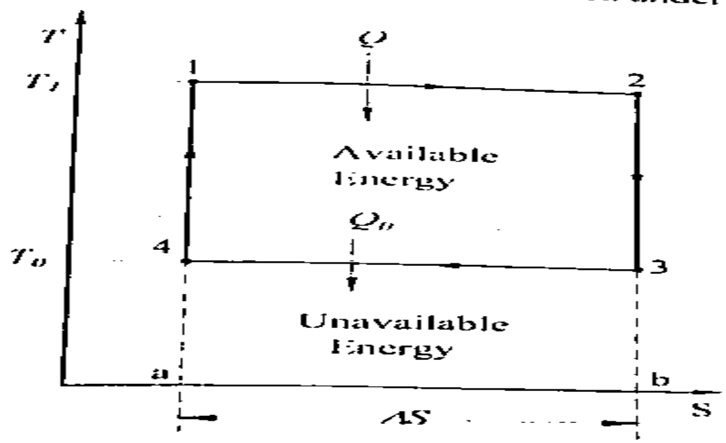


Fig. 5.8(a) T-S diagram of reversible heat engine

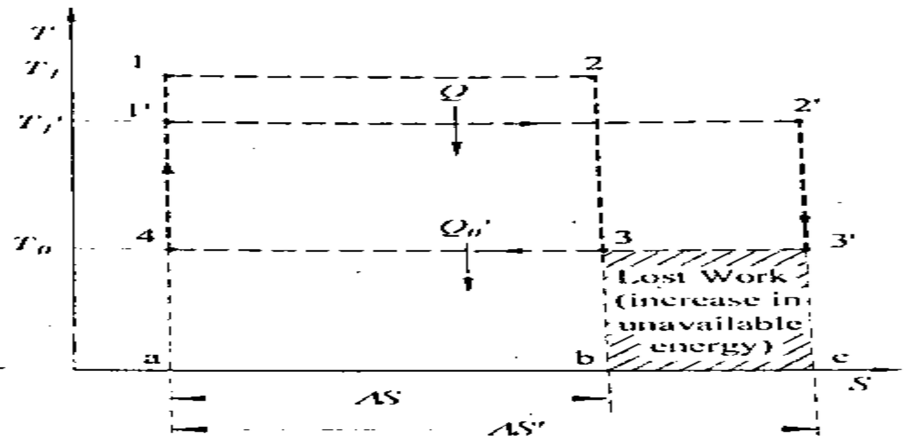


Fig. 5.8(b) Lost work (increase in unavailable energy)

Now consider, heat Q is absorbed from the reservoir at T_1 is source of another heat engine. working between temperature T_1' and T_0 as sink (surrounding) as shown in Fig. 5.8(b). And heat engine cycle is represented by 1'–2'–3'–4, since heat transferred Q is the same.
∴ Maximum work or available energy for second heat engine is given by

$$W'_{max} = (AE)' = Q\eta' = Q\left(1 - \frac{T_0}{T_1'}\right) \quad \text{Area under } 1' - 2' \quad 3' \quad 4$$

Since T_1' is less than T_1, therefore the work W'_{max} will be less than W_{max}. Thus transfer of heat to a lower temperature level involves a loss of work, i.e. a degradation of energy. This loss is known as **lost work or exergy destruction** or **loss of available energy or gain in unavailable energy** (as shown in Fig. 5.8(b) by area 3 - 3' - c - b).
So, loss of available energy (decrease in AE) or lost work is

$$W_{lost} = W_{max} - W'_{max}$$

$$= Q\left(1 - \frac{T_0}{T_1}\right) - Q\left(1 - \frac{T_0}{T_1'}\right) = QT_0\left(\frac{1}{T_1'} - \frac{1}{T_1}\right) = T_0\left(\frac{Q}{T_1'} - \frac{Q}{T_1}\right)$$

4.3 Loss Work - Exergy destruction in heat transfer process

$= T_0(\Delta S' - \Delta S) = \text{Area } 3-3'-c-b$

The above loss of available energy is due to irreversible heat transferred through finite temperature difference between the source and the working fluid during the heat addition process. The effect of such processes is decrease in available energy. And entropy of the universe changes due to the such irreversible process. Greater is the temperature difference $(T_1 - T_1')$ greater is heat rejection Q_0' and greater will be the unavailable energy.

The entropy of the combination of the two interacting systems increases as a result of the heat transfer Q from a system at temperature T_1 to a system at T_1' as given by

$$(\Delta S)_{net} = (\Delta S)_{system\,1} + (\Delta S)_{system\,2}$$

$$= \frac{Q}{T_1'} + \left(-\frac{Q}{T_1}\right)$$

$$= \Delta S' - \Delta S$$

$(\Delta S)_{net}$ or total entropy change is called **entropy of universe**. Since the heat transfer has been through a finite temperature difference, the process is irreversible. $\therefore \ (\Delta S)_{net} > 0$.

The concept of available energy provides a useful measure of the quality of energy. Energy is said to be degraded each time it is transferred through a finite temperature difference. Therefore, the second law may be referred to as **law of degradation of energy**.

4.4 Exergy of closed system

[May 12, March/April 10]

Consider a piston-cylinder arrangement (closed system) in which the fluid expanding reversibly from initial state of p_1, V_1, T_1 to final atmospheric state of p_0, V_0, T_0. During this process, fluid produces work W_{fluid} and rejects heat Q to atmosphere. To find the maximum work done, we will assume that the heat Q rejected by the system is utilized in reversible Carnot engine producing work W_{engine} and rejecting part of heat Q_0 to atmosphere at temperature T_0, as shown in Fig. 5.16 (a), practically this would be possible by having infinite number of reversible engines arranged in parallel, each one receiving heat at a different constant temperature but each one rejecting heat at atmospheric temperature T_0 as shown in Fig. 5.16(b).

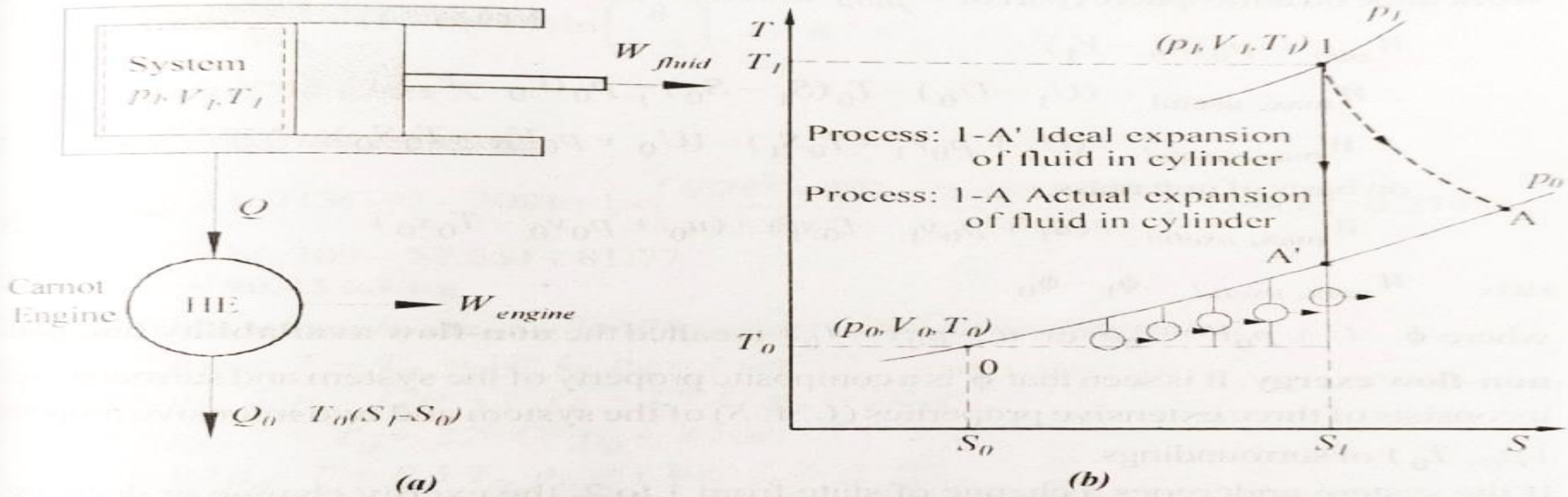


Fig. 5.16 Availability in closed system

Work done by Carnot engine

W_{engine} = heat supplied − heat rejected

$= Q - Q_0$

$$= Q - T_0(S_1 - S_0) \quad \left(\because \frac{Q_0}{T_0} = S_1 - S_0,\ T_0 = \text{constant}\right) \qquad ...(5.7)$$

Work done by fluid

$W_{fluid} = -\Delta U - Q$ ($\because$ first law of thermodynamics for closed system $Q = \Delta U + W$)

(Q is negative as it is rejected from system)

4.4 Exergy of closed system

$$= (U_0 - U_1) - Q \quad ...(5.8)$$

$$\therefore W_{fluid} = (U_1 - U_0) - Q$$

The total maximum work done by whole system or availability of the system

$$W_{max} = W_{fluid} + W_{engine}$$

$$= (U_1 - U_0) - Q + Q - T_0(S_1 - S_0) \quad ...(5.9)$$

$$= (U_1 - U_0) - T_0(S_1 - S_0)$$

All the work W_{max} of the system not be available for delivery, since certain portion of it would be spent in pushing out the atmosphere. So, maximum useful work is defined as

$$W_{max,\ useful} = W_{max} - W_{atm}$$

Work done on atmosphere (part of W_{fluid} is utilized) in compressing the atmosphere

$$W_{atm} = p_0(V_0 - V_1)$$

$$\therefore W_{max,\ useful} = (U_1 - U_0) - T_0(S_1 - S_0) - p_0(V_0 - V_1)$$

$$\therefore W_{max,\ useful} = (U_1 + p_0V_1 - T_0S_1) - (U_0 + p_0V_0 - T_0S_0)$$

on basis of unit mass,

$$W_{max,\ useful} = (u_1 + p_0v_1 - T_0s_1) - (u_0 + p_0v_0 - T_0s_0) \quad ...(5.10)$$

$$W_{max,\ useful} = \phi_1 - \phi_0 \quad ...(5.11)$$

where $\phi = U + p_0V - T_0S$ or $u + p_0v - T_0s$ is called the **non-flow availability function** or **non-flow exergy**. It is seen that ϕ is a composite property of the system and surroundings as it consists of three extensive properties (U, V, S) of the system and two intensive properties (p_0, T_0) of surroundings.

If the system undergoes a change of state from 1 to 2, the exergy change or decrease in availability or loss of availability will be

$$W_{max,useful} = a = (\phi_1 - \phi_0) - (\phi_2 - \phi_0) \quad ...(5.12)$$

$$= \phi_1 - \phi_2$$

$$= (u_1 - u_2) + p_0(v_1 - v_2) - T_0(s_1 - s_2) \quad ...(5.13)$$

4.4 Exergy of steady flow open system

5.5 Exergy (Availability) of steady ...

Consider flow of fluid through a open system as shown in Fig. 5.17. The working fluid enter the system at section 1 (p_1, V_1, T_1, U_1, C_1) and leave the system at section 2 (p_0, V_0, T_0, U_0, C_0) and passing at a steady rate. Let the system rejects heat Q, which for getting maximum work should be passed through a reversible heat engine. This reversible (Carnot) engine absorbs heat Q from system at temperature T_1 and rejects $T_0(S_1 - S_0)$ heat to surroundings.

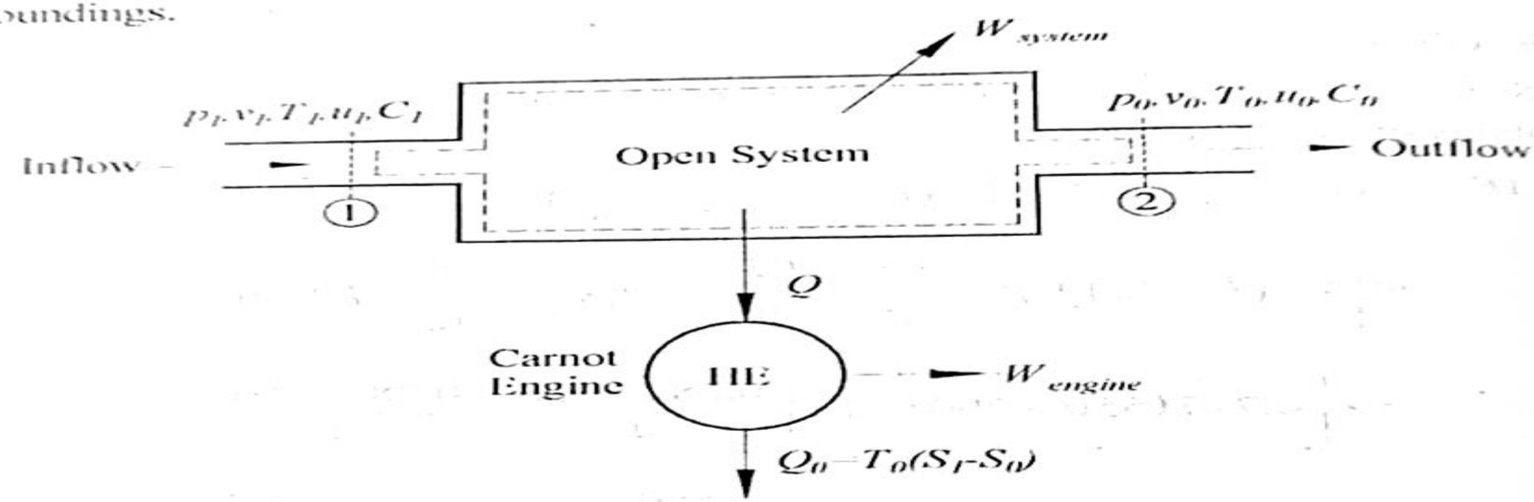


Fig 5.17 Availability of steady flow open system.

The steady flow energy equation for system may be written as

$$U_1 + p_1V_1 + m\frac{C_1^2}{2} + mgz_1 - Q = U_0 + p_0V_0 + \frac{mC_0^2}{2} + mgz_0 + W_{system}$$

where Q is negative (heat rejected by system) and W_{system} is positive, work produced ... system.

Steady flow work by the system

$$W_{system} = (U_1 - U_0) + (p_1V_1 - p_0V_0) + m\left(\frac{C_1^2 - C_0^2}{2}\right) + mg(z_1 - z_0) - Q$$

4.4 Exergy of steady flow open system

$$[(U_1 + p_1V_1) - (U_0 + p_0V_0)] + m\left(\frac{C_1^2 - C_0^2}{2}\right) + mg(z_1 - z_0) - Q$$

or $$W_{system} = (H_1 - H_0) + m\left(\frac{C_1^2 - C_0^2}{2}\right) + mg(z_1 - z_0) - Q \quad (\because H = U + pV)$$

...(5.14)

The heat Q rejected by the system is absorbed by reversible Carnot engine, therefore maximum work output from engine,

$$W_{engine} = Q - Q_0$$

$$= Q - T_0(S_1 - S_0) \quad \left(\because dS = \frac{Q_0}{T_0},\ T_0 = \text{constant}\right)$$

Total maximum available useful work, or availability

$$W_{max} = W_{system} + W_{engine}$$

$$= (H_1 - H_0) + m\left(\frac{C_1^2 - C_0^2}{2}\right) + mg(z_1 - z_0) - Q + Q - T_0(S_1 - S_0)$$

$$= (H_1 - H_0) + m\left(\frac{C_1^2 - C_0^2}{2}\right) + mg(z_1 - z_0) - T_0(S_1 - S_0)$$

$$W_{max} = \left(H_1 - T_0S_1 + \frac{mC_1^2}{2} + mg\,z_1\right) - \left(H_0 - T_0S_0 + \frac{mc_0^2}{2} + mg\,z_0\right) \quad ...(5.15)$$

or $$W_{max} = m\left[\left(h_1 - T_0s_1 + \frac{C_1^2}{2} + gz_1\right) - \left(h_0 - T_0s_0 + \frac{C_0^2}{2} + gz_0\right)\right]$$

In steady flow system, the volume of the system does not change, hence maximum useful work is equal to maximum work.

$$\therefore W_{max,\,useful} = W_{max}$$

neglecting changes in Kinetic and Potential energy:

$$W_{max} = m[(h_1 - T_0s_1) - (h_0 - T_0s_0)]$$

$$= (H_1 - T_0S_1) - (H_0 - T_0S_0)$$

$$W_{max} = (U_1 + p_1V_1 - T_0S_1) - (U_0 + p_0V_0 - T_0S_0) \quad ...(5.16)$$

4.5 Irreversibility & Gouy – Stodola Theorem

surroundings.

[Jan 2010, PDDC]

5.7 Irreversibility and Gouy - Stodola Theorem

(I) Irreversibility of closed (no-flow system

Maximum work done by closed system is given by

$$W_{max,\,useful} = (U_1 + p_0V_1 - T_0S_1) - (U_2 + p_0V_2 - T_0S_2)$$

Actual work done by closed system is given by

$$W = Q - \Delta U \quad (\because Q = \Delta U + W \text{ where Q is negative, heat rejected})$$

$$= -Q - (U_2 - U_1)$$

Now

Irreversibility of closed system

$$I = W_{max,useful} - W_{actual}$$

$$= (U_1 + p_0V_1 - T_0S_1) - (U_2 + p_0V_2 - T_0S_2) - [-Q + (U_1 - U_2)]$$

$$= (U_1 - U_2) + p_0(V_1 - V_2) - T_0(S_1 - S_2) + Q - (U_1 - U_2)$$

$$I = T_0(S_2 - S_1) + Q + p_0(V_1 - V_2)$$

neglecting work done on atmosphere (surroundings)

$$I = T_0(S_2 - S_1) + Q$$

Now, change of entropy of surrounding (atmosphere) due to receipt of heat Q at T_0 from system.

$$(\Delta S)_{surr} = \frac{Q}{T_0}$$

$$\therefore Q = T_0(\Delta S)_{surr} \quad ...(5.21)$$

Also entropy change of system due to rejected of heat Q to surroundings at T_0.

$$(\Delta S)_{system} = (S_2 - S_1) \quad ...(5.22)$$

$$\therefore I = T_0(\Delta S)_{system} + T_0(\Delta S)_{surr} \quad (\because \text{ from equation (5.21) \& (5.22)})$$

$$= T_0[(\Delta S)_{system} + (\Delta S)_{surr}]$$

$$I = T_0(\Delta S)_{universe} \quad ...(5.23$$

From above equation, irreversibility of closed process is the product of temperature

4.5 Irreversibility & Gouy – Stodola Theorem

surroundings and entropy change of universe.

(2) Irreversibility of steady flow open system **[Jan 2010]**

Maximum work done by steady flow open system is given by

$$W_{max} = (U_1 + p_1V_1 - T_0S_1) - (U_2 + p_2V_2 - T_0S_2)$$

Actual work done by steady flow open system,

$$W = (U_1 + p_1V_1) - (U_2 + p_2V_2) - Q \text{ (system rejecting heat)}$$

($\because$ steady flow energy equation $U_1 + p_1V_1 + Q = U_2 + p_2V_2 + W$, neglecting K.E. and P.E.)

Now, irreversibility of steady flow open system

$$I = W_{max} - W$$

$$= \left[(U_1 + p_1V_1 - T_0S_1) - (U_2 + p_2V_2 - T_0S_2)\right] - \left[(U_1 + p_1V_1) - (U_2 + p_2V_2) - Q\right]$$

$$= U_1 + p_1V_1 - T_0S_1 - U_2 - p_2V_2 + T_0S_2 - U_1 - p_1V_1 + U_2 + p_2V_2 + Q$$

$$I = T_0(S_2 - S_1) + Q$$

But $(+Q) = T_0(\Delta S)_{Surr}$ (surrounding absorbing heat)

and $(S_2 - S_1) = (\Delta S)_{system}$

$$\therefore I = T_0(\Delta S)_{system} + T_0(\Delta S)_{surr}$$

$$\therefore I = T_0(\Delta S)_{Universe} \quad ...(4.24)$$

The same expression, $T_0(\Delta S)_{universe}$ for irreversibility applies to both flow and and non-flow processes. The quantity $T_0(\Delta S)_{universe}$ represents an increase in unabailable energy or Anergy.

Gouy-Stodola Theorem : The rate of loss of exergy (available energy) in a process is proportional to the rate of entropy generation.

i.e. $I = W_{Lost}$

$= T_0(\Delta S)_{universe}$

$= T_0 S_{gen}$

This is called **Gouy-Stodola equation**

4.6 Second law efficiency / Effectiveness

- The ratio of actual thermal efficiency to the maximum possible (reversible) thermal efficiency under the same conditions.

$$\eta H = \eta th/\eta rev$$