LUKHDHIRJI ENGINEERING COLLEGE, MORBI

Subject: Mathematics - 1 (3110014)

Tutorial-4 **Branch: ALL** **Semester: 1**

1. Verify $f_{xy} = f_{yx}$ for $f(x, y) = \ln(2x + 3y)$
2. If $u = e^{3xyz}$ show that $\dfrac{\partial^3 u}{\partial x \partial y \partial z} = \left(3 + 27xyz + 27x^2y^2z^2\right)e^{3xyz}$
3. If $\theta = t^n e^{\frac{-r^2}{4t}}$ then find n so that $\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial \theta}{\partial r}\right) = \dfrac{\partial \theta}{\partial t}$
4. If $u = e^{ax}\sin by$, prove that $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$
5. If $z = e^{xy}$, $x = t\cos t$, $y = t\sin t$, find $\dfrac{dz}{dt}$ at $t = \dfrac{\pi}{2}$
6. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$
7. If $u = f\left(\dfrac{y-x}{xy}, \dfrac{z-x}{xz}\right)$, prove that $x^2\dfrac{\partial u}{\partial x} + y^2\dfrac{\partial u}{\partial y} + z^2\dfrac{\partial u}{\partial z} = 0$
8. If $x^4 + 4y^2 + 7x^2y + 3xy^2 + y^4 = 4$, find dy/dx
9. If $u = \cos^{-1}\left(\dfrac{x+y}{\sqrt{x} + \sqrt{y}}\right)$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = -\dfrac{1}{2}\cot u$
10. If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$
11. If $u = \tan^{-1}\left(x^2 + 2y^2\right)$ show that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 2\sin u\cos 3u$
12. If $u = \operatorname{cosec}^{-1}\left(\sqrt{\dfrac{\left(x^{1/2} + y^{1/2}\right)}{\left(x^{1/3} + y^{1/3}\right)}}\right)$, show that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \dfrac{1}{144}\tan u\left(13 + \tan^2 u\right)$
13. Find the value of $\dfrac{\partial u}{\partial s}$ at $r = 1, s = 1, t = 0$; if $u = 2x^3y + y^3z^2$; $x = rse^t$, $y = rs^2e^{-t}$,

    $z = r^2 s\sin t$

Partial Derivatives Applications

1. Expand $e^x \cos y$ at $\left(1, \frac{\pi}{4}\right)$.
2. Expand e^{x+y} using Taylor's series at origin.
3. Expand $\tan^{-1}\left(\frac{y}{x}\right)$ in powers of x-1 and y-1 using Taylor's expansion.
4. Expand $x^2 + xy + y^2$ in powers of x-1 and y-2 up to second degree.
5. Expand $xy^2 + 2xy + 5$ in powers of $x - 2$ and $y + 1$.
6. Find the equation of tangent plane and normal line to the surface $x^3 + 2xy^2 - 7z^2 + 3y + 1 = 0$ at $(1,1,1)$.
7. Show that function $x^3 + y^3 - 63(x + y) + 12xy$ is maximum at $(-7,-7)$ and minimum at $(3,3)$.
8. Find all the local maxima, local minima and saddle points of the function $x^2 - 2xy + 2y^2 - 2x + 2y + 1$.
9. Find maxima and minima of $xy - x^2 - y^2 - 2x - 2y + 4$.
10. Find the minimum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$ using the method of Lagrange multiplier method.
11. A soldier placed at a point $(3,4)$ wants to shoot the fighter plane of an enemy which is flying along the curve $y = x^2 + 4$ when it is nearest to him. Find such distance.
12. A rectangular box, open at the top, is to have volume 32c.c. Find the dimensions of the box requiring lest material for its construction.
13. Find $\frac{\partial(u, v)}{\partial(x, y)}$ if $u = 3x + 5y$, $v = 4x - 3y$.
14. If $x = e^u \cos v$, $y = e^u \sin v$, find Jacobian.
15. Verify $JJ' = 1$, if $x = u$, $y = u \tan v$ and $z = w$.