Layout design II.

Chapter 6

Layout generation

- Pairwise exchange method
- Graph-based method
- CRAFT

Methods for layout design

- Layout evaluation
  - Distance-based scoring
  - Adjacency-based scoring
- Layout generation
  - Construction algorithms
    - Building a block layout by iteratively adding departments
  - Improvement algorithms
    - Incrementally improving an initial block layout

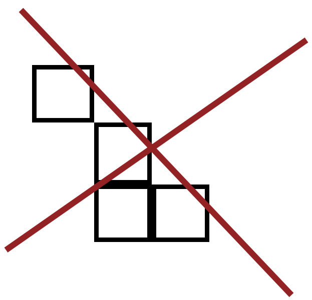
Construction algorithms

- SELECT an activity to be placed
- PLACE the selected activity in the layout

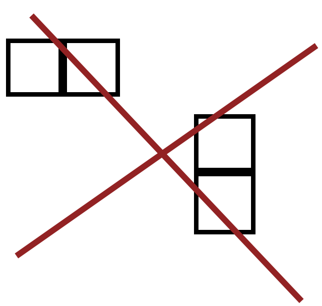
- **Selection rules**
  - Choose the departments based on the importance scores (first place all A, then all E, all I, etc.)
  - Choose the departments having the largest number of A (E, I, etc.) relationships with the departments already in the layout.
  - Supplement the methods with a procedure for choosing first department and breaking ties.
  - Consider costs and user specified placement priorities.

Construction algorithms

- **Placement rules**
  - Contiguity Rule
    - If an activity is represented by more than one unit area square, every unit area square representing the activity must share <u>at least one edge</u> with at least one other unit area square representing the activity.

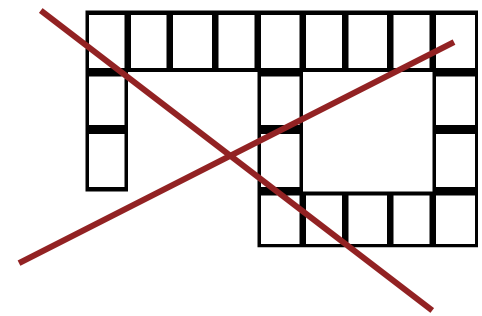


  - Connectedness Rule
    - The perimeter of an activity must be a <u>single closed loop</u> that is always in contact with some edge of some unit area square representing the activity.

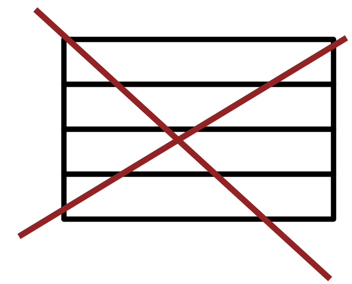


Construction algorithms

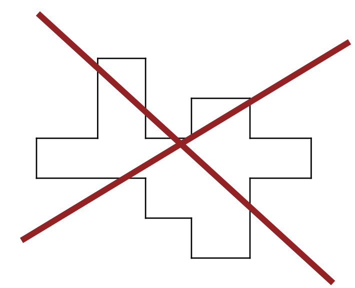
- **Placement Rules**
  - Enclosed Voids Rule
    - No activity shape shall contain an enclosed void.



  - Shape Ratio Rule
    - The ratio of a feasible shape's greatest length to its greatest width shall be constrained to lie between specified limits.



  - Corner Count Rule
    - The number of corners for a feasible shape may not exceed a specified maximum.



Improvement algorithms

- “Move” departments around within the block plan.
- If the shapes of the departments are not fixed
  - Too many degrees of freedom to devise a good method for modifying the block plan.
  - Most of improvement algorithms limit the kinds of changes that are permitted.

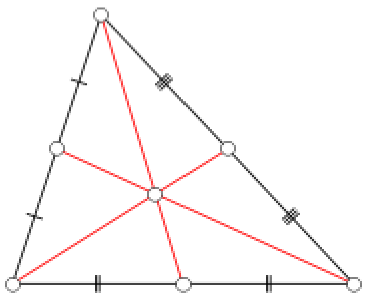
  - Basic procedure
    - CHOOSE a pair (or triple) of activities
    - ESTIMATE the effect of exchanging them
    - EXCHANGE them
    - CHECK to be sure the new layout is better
    - REPEAT until no more improvements are possible

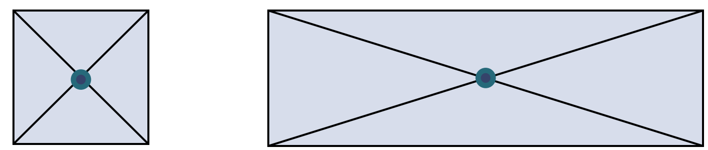
Algorithm classification

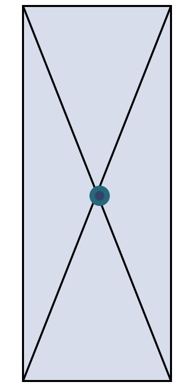
Construction algorithms	Improvement algorithms
Graph-based method ALDEP CORELAP PLANET	Pairwise exchange method CRAFT MCCRAFT MULTIPLE
BLOCPLAN LOGIC Mixed integer programming	

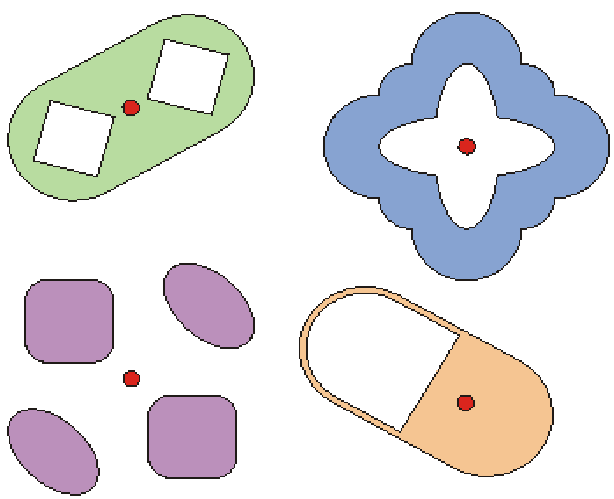
Distance Calculations

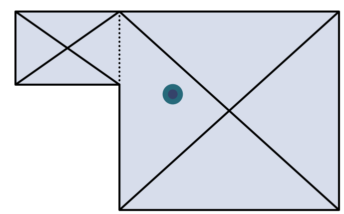
- Centroid is a center of mass

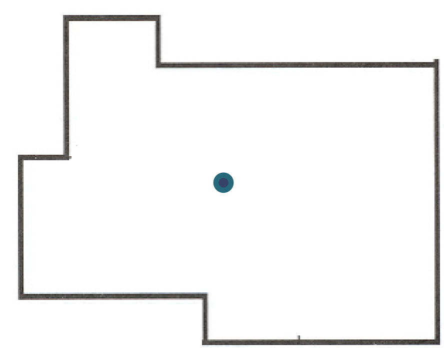












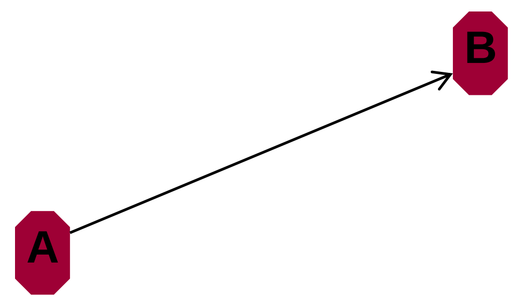
Distance Calculations

- If (x_i,y_i) and (x_j,y_j) represent the coordinates of two locations *i* and *j* then the distance model measures can be:
  - **Rectilinear:**
    - distance between *i* and *j* is

$$D = |x_i - x_j| + |y_i - y_j|$$

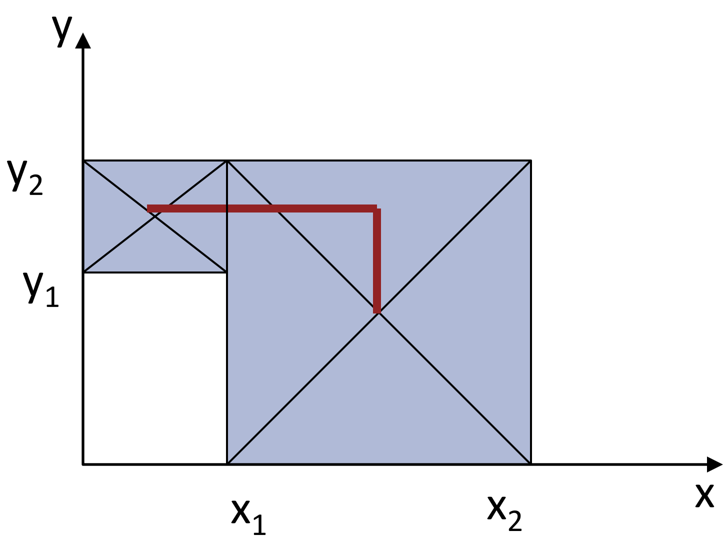
  - **Euclidean:**
    - distance between *i* and *j* is

$$D = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2}$$

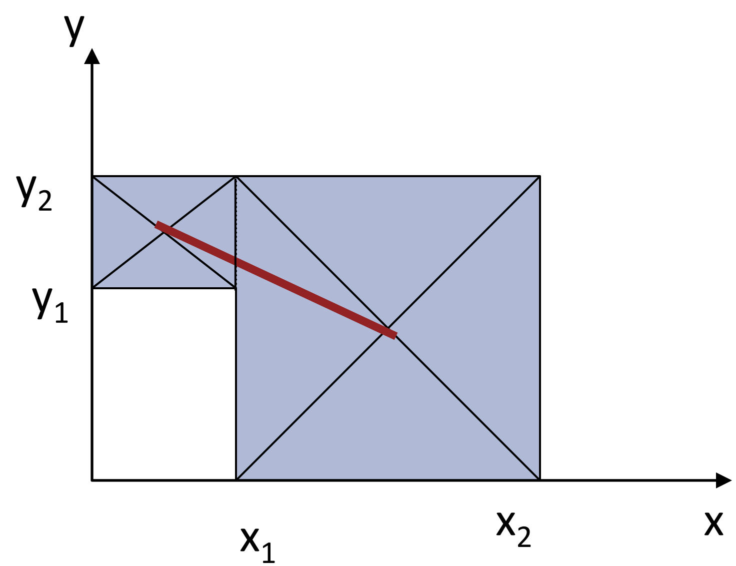


Distance Calculations

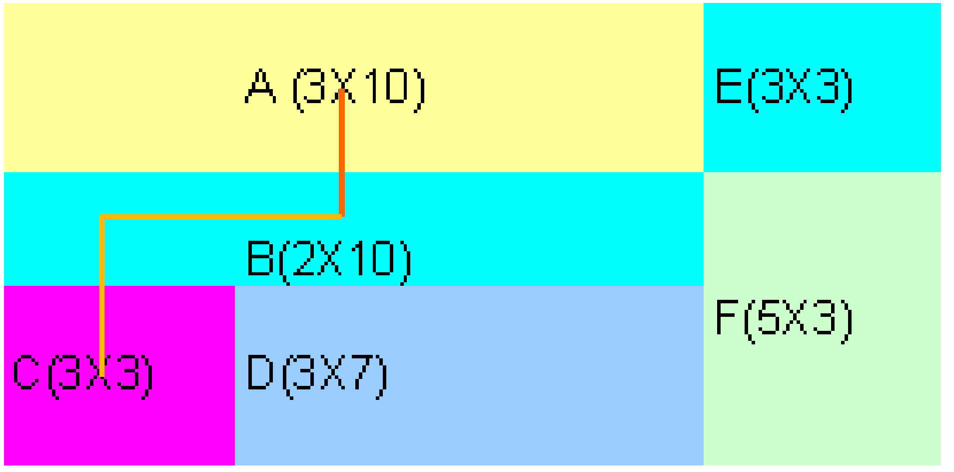
- Rectilinear distance from centroid to centroid



- Euclidean distance from centroid to centroid



Distance Calculations



Rectilinear distance from A to B:

D (AB) = 1.5 + 1 = 2.5

Rectilinear distance from B to C:

D (BC) = (5-1.5) + (1+1.5) = 3.5 + 2.5 = 6

Pairwise exchange method

- For layout improvement
- Distance-based objective
- The objective is to minimize the total cost of transporting materials among all departments in a facility
    - Based on rectilinear distance from centroid to centroid
- Since the final outcome is based on the initial layout, **we might not converge to the optimal solution**

Pairwise exchange method

Procedure

- Calculate total cost for the existing layout
- For each iteration evaluate all feasible exchanges in the locations of department pairs
- Select the pair that results in the largest reduction in total cost
- Recompute the distance matrix each time an exchange is performed
- If the lowest total cost for your next iteration is worse than the total cost for the previous iteration, the procedure is terminated

Pairwise exchange method

Example

Initial layout

1	2	3	4

- Four departments with equal sizes
- From-To Chart

From Department	To Department 1	2	3	4
1	—	10	15	20
2		—	10	5
3			—	5
4				—

- Distance matrix (based on existing layout)

From Department	To Department 1	2	3	4
1	—	1	2	3
2		—	1	2
3			—	1
4				—

➢ Determine final layout

Pairwise exchange method

Example

From-To Chart

	1	2	3	4
1	—	10	15	20
2		—	10	5
3			—	5
4				—

Distance matrix - initial layout

	1	2	3	4
1	—	1	2	3
2		—	1	2
3			—	1
4				—

Iteration 0

1	2	3	4

Total cost for the existing layout: $z = \sum_{i=1}^{m}\sum_{j=1}^{m} f_{ij}c_{ij}d_{ij}$

TC_{1234}=10*1 + 15*2 + 20*3 + 10*1 + 5 *2 + 5*1 = 125

Evaluate all feasible pairwise exchanges:

Switch departments 1-2, 1-3, 1-4, 2-3, 2-4, and 3-4

Switching:	Layout	Total cost
1-2	2 1 3 4	$TC_{2134}(1-2) = 10(1) + 15(1) + 20(2) + 10(2) + 5(3) + 5(1) = 105$
1-3	3 2 1 4	$\boxed{TC_{3214}(1-3) = 10(1) + 15(2) + 20(1) + 10(1) + 5(2) + 5(3) = 95}$
1-4	4 2 3 1	$TC_{4231}(1-4) = 10(2) + 15(1) + 20(3) + 10(1) + 5(1) + 5(2) = 120$
2-3	1 3 2 4	$TC_{1324}(2-3) = 10(2) + 15(1) + 20(3) + 10(1) + 5(1) + 5(2) = 120$
2-4	1 4 3 2	$TC_{1432}(2-4) = 10(3) + 15(2) + 20(1) + 10(1) + 5(2) + 5(1) = 105$
3-4	1 2 4 3	$TC_{1243}(3-4) = 10(1) + 15(3) + 20(2) + 10(2) + 5(1) + 5(1) = 125$

The lowest cost => Iteration 1

3	2	1	4

Pairwise exchange method
Example

Iteration 1

3	2	1	4

- Repeat the switching between departments 1-2, 1-3, 1-4, 2-3, 2-4, and 3-4.
  - Sequence 2314 with total score of 90 is the best option

$TC_{2314} = 10*2 + 15*1 + 20*1 + 10*1 + 5*3 + 5*2 = 90$

Iteration 2

2	3	1	4

- The process is repeated until the lowest total score cannot be reduced anymore.
- In this particular case the best option is 2314.

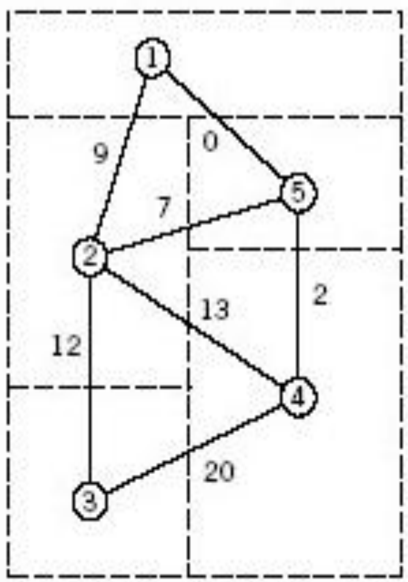
Pairwise exchange method

- The method does not guarantee the optimality, only **local optimum**
- The procedure may cycle back to one alternative
- Symmetric layout may occur
- Pairwise exchange can be easily accomplished only if the departments are of the same size

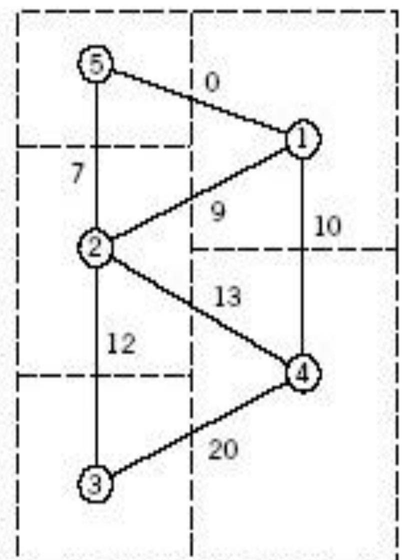
Graph-based method

- For layout construction
- Adjacency-based objective
- Adjacency graphs for block layouts:

$$z = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} f_{ij} x_{ij}$$

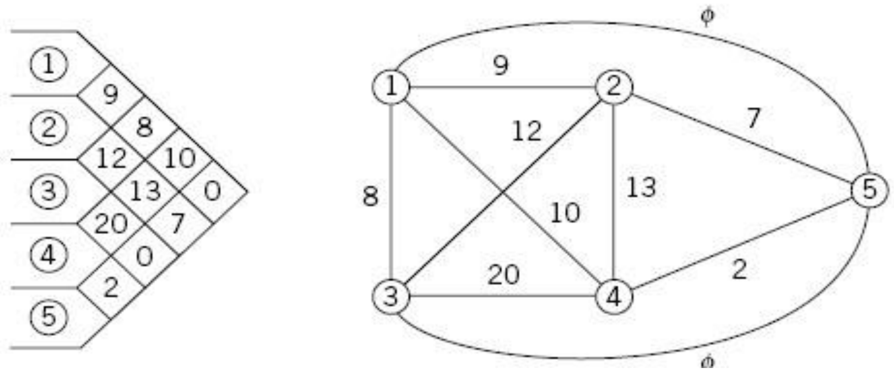


Arc	Weight
1-2	9
1-5	0
2-3	12
2-4	13
2-5	7
3-4	20
4-5	2
	63 (total)



Arc	Weight
1-5	0
2-5	7
1-2	9
1-4	10
2-4	13
2-3	12
3-4	20
	71 (total)

Graph-based method



- Relationships are given by weights, rather than the alphabetic closeness ratings
  - The score is very sensitive to the assignment of weights
- Relationship can have a positive value only when departments are adjacent. Other relationships are ignored
- The adjacency score does not account for:
  - The distance
  - All he relationships (except those between adjacent departments)
  - Dimensional specifications
  - The length of common boundaries between adjacent departments
- The arcs do not intersect (planarity)

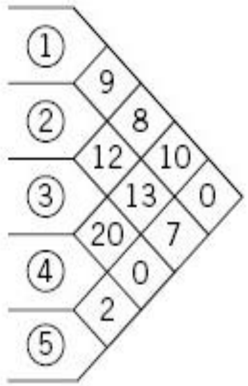
Graph-based method

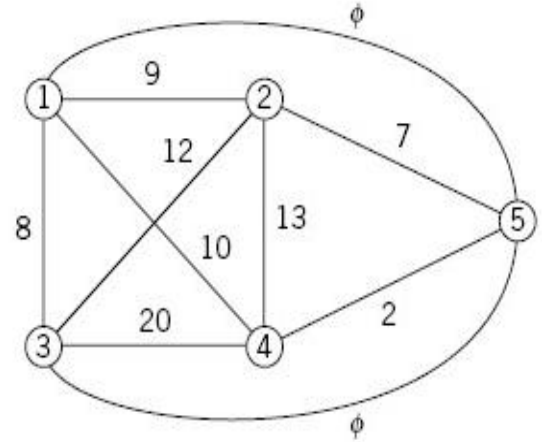
Procedure

1. Select the department pair with the largest weight
   - Ties are broken arbitrarily
2. Select the third department to enter based on the largest sum of the weights with the first two departments
3. Pick the fourth department to enter on a face of the graph (a bounded region of a graph)
4. Determine **an adjacency graph**
5. Construct a corresponding block layout

Graph-based method

Example

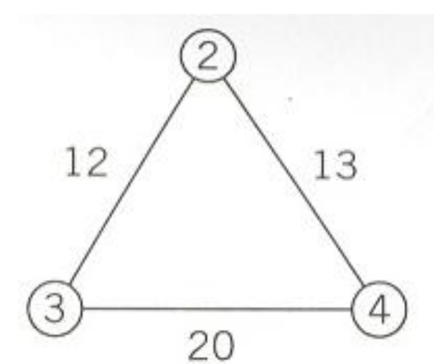


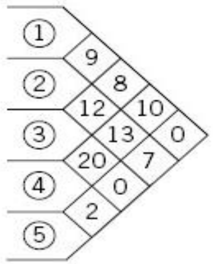


- Step 1: Select the department pair which has the highest weight (3-4)
- Step 2: Select the third department based on the sum of the relationship with both selected two departments in step 1 (2)

a. Step 2

	3	4	Total
1	8	10	18
2	12	13	25 (best)
5	0	2	2





Graph-based method

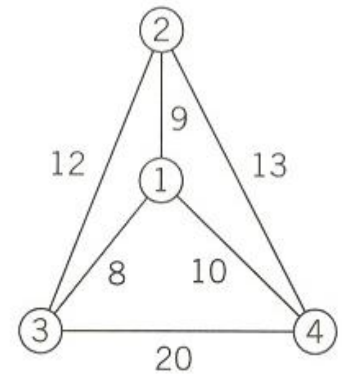
Example

- Step 3: Select the next department and place it on a face - inside the triangle (1)

b. Step 3

	2	3	4	Total
1	9	8	10	27 (best)
5	7	0	9	9

→



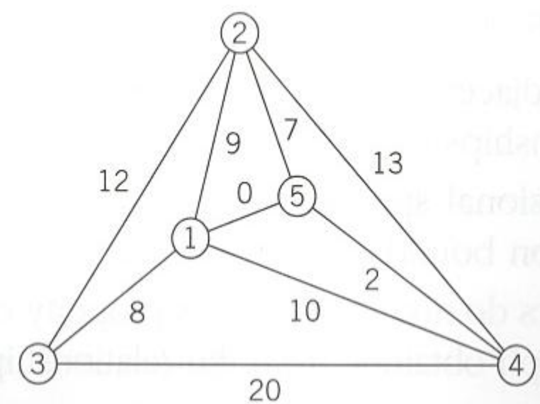
- Step 4: Finally, for the fifth department we look inside of all the formed triangles. The sum of their weights which gets the highest value is chosen as the location (1-2-4)

c. Step 4

	1	2	3	4	Total
5	0	7	0	2	9

Faces	Total
1-2-3	7
1-2-4	9 (best)
1-3-4	2

→

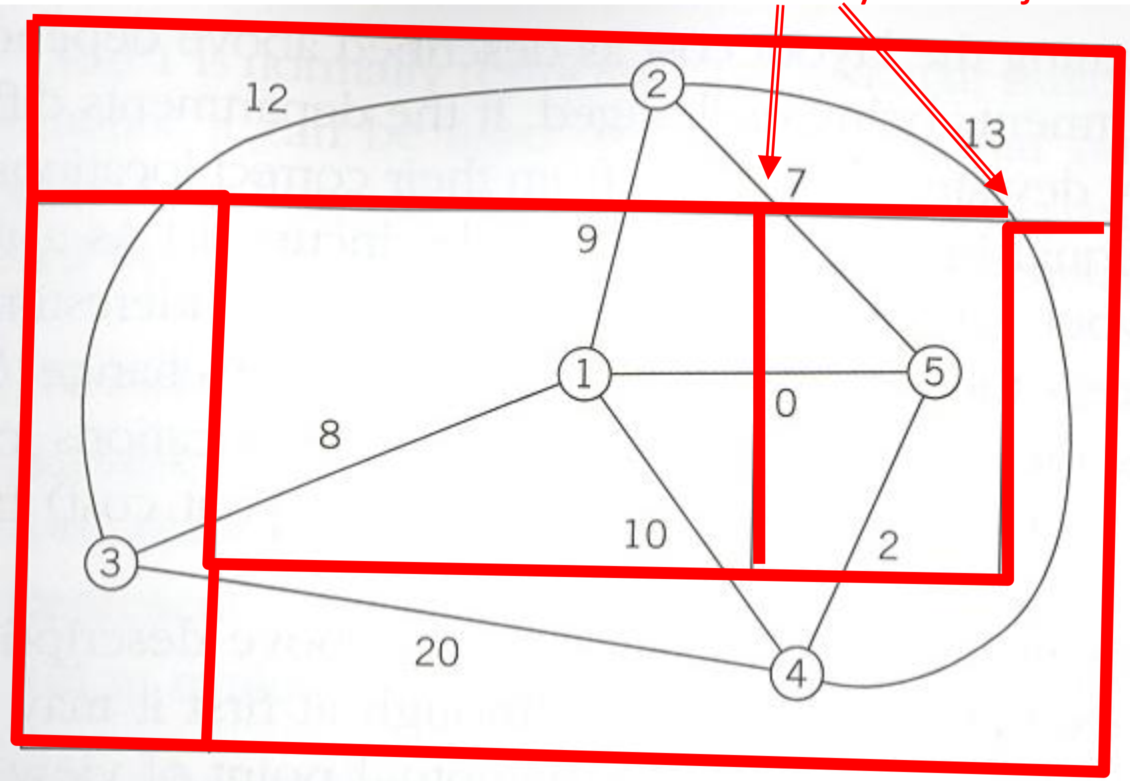


Graph-based method
Example

- Step 5: Based on the final adjacency graph a corresponding block design is constructed (based on the required and available dimensions)

Connected departments are adjacent

Each wall should cross only one adjacency line



Arc	Weight
1-2	9
1-3	8
1-4	10
1-5	0
2-3	12
2-4	13
2-5	7
3-4	20
4-5	2
	81 (tot

Figure 6.14 Block layout from the final adjacency graph.

CRAFT

Computerized Relative Allocation of Facilities Technique

- For improvement of an existing facility
- Attempts to minimize transportation cost, where

  Transportation cost = flow * unit cost * distance

$$Min \quad z = \sum_{i=1}^{m} \sum_{j=1}^{m} f_{ij} c_{ij} d_{ij}$$

- Assumptions
  - Moving costs are not dependent on the equipment utilization.
  - Moving costs are linearly related to the length of the move.
- Distance metric used is the rectilinear distance between department centroids.
- Input is FT Chart (From-To chart)
- Department shapes are not restricted to the rectangular ones

CRAFT

Procedure

1. Determine department centroids.
2. Calculate rectilinear distance between centroids.
3. Calculate transportation cost for the layout.
4. Consider department exchanges of either **equal area departments** or **departments sharing a common border**.
5. Determine the estimated change in transportation cost of each possible exchange.
6. Select and implement the departmental exchange that offers the greatest reduction in transportation cost.
7. Repeat the procedure for the new layout until no interchange is able to reduce the transportation cost.

CRAFT

Example

- A facility with 7 departments
- Cost of carrying any material $c_{ij} = 1$ for all i and j pairs.
- Each grid size is 20×20, total $72{,}000\ m^2$ is available
- Total requirement is *70,000* m^2
- Location of receiving (A) and shipping (G) departments are fixed

Dept. Name	Area	No of Grids	Flow							
			A	B	C	D	E	F	G	H
A: Receiving	12,000	30	0	45	15	25	10	5	0	0
B: Milling	8,000	20	0	0	0	30	25	15	0	0
C: Press	6,000	15	0	0	0	0	5	10	0	0
D :Screw	12,000	30	0	20	0	0	35	0	0	0
E: Assembly	8,000	20	0	0	0	0	0	65	35	0
F: Plating	12,000	30	0	5	0	0	25	0	65	0
G: Shipping	12,000	30	0	0	0	0	0	0	0	0
H: Dummy	2,000	5	0	0	0	0	0	0	0	0

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	A	A	A	A	A	A	A	A	A	A	G	G	G	G	G	G	G	G
2	A				Receiving D.					A	G							G
3	A	A	A	A	A	A	A	A	A	A	G	G	G		Shipping D.			G
4	B	B	B	B	B	C	C	C	C	C	E	E	G	G	G	G	G	G
5	B				B	C				C	E	E	E	E	E	E	E	E
6	B				B	C	C	C	C	C	E	E	E	E	E	E	E	E
7	B	B	B	B	B	D	D	D	D	F	F	F	F	F	F	F	E	E
8	D	D	D	D	D	D			D	F							F	F
9	D								D	F	F	F	F	F				F
10	D	D	D	D	D	D	D	D	H	H	H	H	H	F	F	F	F	F

Dummy Department

➢ Improve the layout

- Total available space > total required space:
  therefore we use a dummy department (H) with the size of 2,000 m^2

CRAFT Example

1. Determine department centroids.
2. Calculate rectilinear distance between centroids.
3. Calculate transportation cost for the layout.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	A	A	A	A	A	A	A	A	A	A	G	G	G	G	G	G	G	G
2	A									A	G							G
3	A	A	A	A	A	A	A	A	A	A	G	G	G					G
4	B	B	B	B	B	C	C	C	C	C	E	E	G	G	G	G	G	G
5	B				B	C				C	E	E	E	E	E	E	E	E
6	B				B	C	C	C	C	C	E	E	E	E	E	E	E	E
7	B	B	B	B	B	D	D	D	D	F	F	F	F	F	F	F	E	E
8	D	D	D	D	D	D			D	F						F	F	F
9	D							D	D	F	F	F	F	F				F
10	D	D	D	D	D	D	D	D	H	H	H	H	H	F	F	F	F	F

Figure 6.15 Initial CRAFT layout and department centroids for Example 6.1 ($z = 2974 \times 20 = 59{,}480$ units).

Distance between A and B is 6 units (illustrated by the red line above)

$$z = \sum_{i=1}^{m} \sum_{j=1}^{m} f_{ij} c_{ij} d_{ij}$$

Dept. Name	Area	No of Grids	Flow							
			A	B	C	D	E	F	G	H
A: Receiving	12,000	30	0	45	15	25	10	5	0	0
B: Milling	8,000	20	0	0	0	30	25	15	0	0
C: Press	6,000	15	0	0	0	0	5	10	0	0
D :Screw	12,000	30	0	20	0	0	35	0	0	0
E: Assembly	8,000	20	0	0	0	0	0	65	35	0
F: Plating	12,000	30	0	5	0	0	25	0	65	0
G: Shipping	12,000	30	0	0	0	0	0	0	0	0
H: Dummy	2,000	5	0	0	0	0	0	0	0	0

- **Distance Matrix**

	A	B	C	D	E	F	G	H
A		6	5	6	13	16		
B				6	11	14		
C					7	10		
D		6			12			
E						3	4	
F		14			3		7	
G								
H								

- **Cost Matrix**

	A	B	C	D	E	F	G	H	Total
A		270	75	150	130	80			705
B				180	275	210			665
C					35	100			135
D		120			420				540
E						195	140		335
F		70			75		455		600
G									0
H									0
									2980

CRAFT Example

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	A	A	A	A	A	A	A	A	A	A	G	G	G	G	G	G	G	G
2	A			Receiving D.						A	G							G
3	A	A	A	A	A	A	A	A	A	A	G	G	G	Shipping D.				G
4	B	B	B	B	B	C	C	C	C	C	E	E	G	G	G	G	G	G
5	B				B	C				C	E	E	E	E	E	E	E	E
6	B				B	C	C	C	C	C	E	E	E	E	E	E	E	E
7	B	B	B	B	B	D	D	D	D	F	F	F	F	F	F	F	E	E
8	D	D	D	D	D	D			D	F							F	F
9	D								D	F	F	F	F	F				F
10	D	D	D	D	D	D	D	D	H	H	H	H	H	F	F	F	F	F

Dummy Department

Which departments to exchange?

- Cost Matrix

	A	B	C	D	E	F	G	H	Total
A		270	75	150	130	80			705
B				180	275	210			665
C					35	100			135
D		120			420				540
E						195	140		335
F		70			75		455		600
G									0
H									0
									2980

1. Bringing the departments **E and D** closer might help to reduce total material flow
2. Bringing the departments **F and G** closer might help to reduce total material flow

Exchange E and F

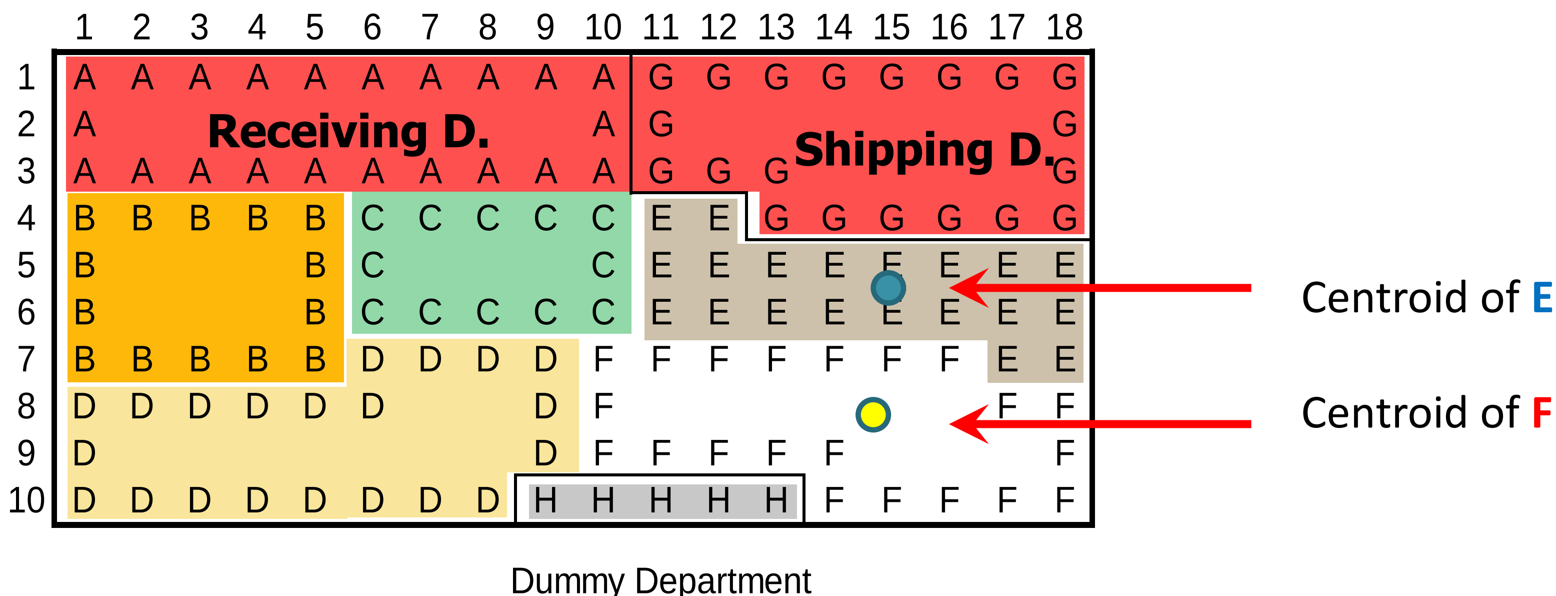
Departments E and F can be reorganized only if they have **the same areas OR** they have **common border**

CRAFT
Selection Criterion for Exchange

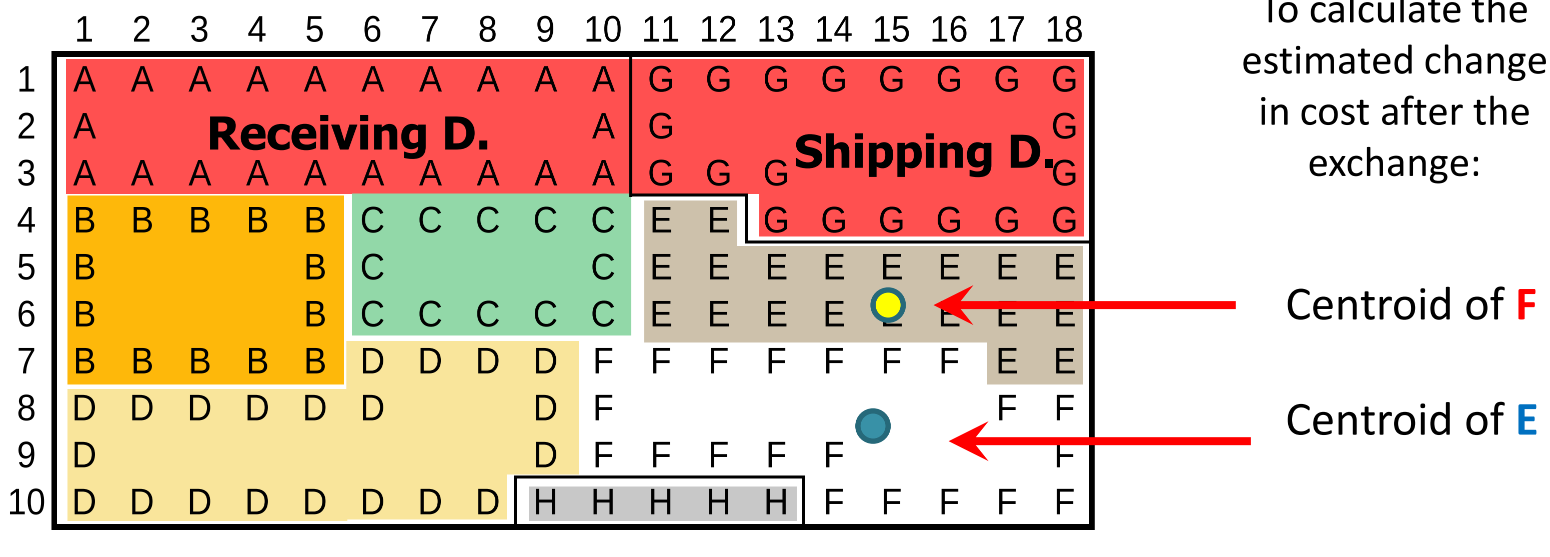
- Estimated change in the transportation cost:
  - Consider two departments ***i*** and ***j***:
    - Let the centroids of each location be L_i and L_j
    - Assume that after the exchange, the new centroid of *i* becomes L_j and the centroid of *j* becomes L_i.
    - Compute the change in the total transportation cost by using the new **estimated centroids**
      - Centroids of the two departments are temporarily swapped
      - The actual size of cost reduction can be overestimated or underestimated

CRAFT

Swapping the centroids



To calculate the estimated change in cost after the exchange:



CRAFT Example

- **Estimation** of the change in transportation cost

Trial distance matrix

	A	B	C	D	E	F	G	H
A		6	5	6	16	13		
B				6	14	11		
C					10	7		
D		6			9			
E						3	7	
F		11			3		4	
G								
H								

Initial cost matrix

	A	B	C	D	E	F	G	H	Total
A		270	75	150	130	80			705
B				180	275	210			665
C					35	100			135
D		120			420				540
E						195	140		335
F		70			75		455		600
G									0
H									0
									2980

Trial cost matrix

	A	B	C	D	E	F	G	H	Total
A		270	75	150	160	65			720
B				180	350	165			695
C					50	70			120
D		120			315				435
E						195	245		440
F		55			75		260		390
G									0
H									0
									2800

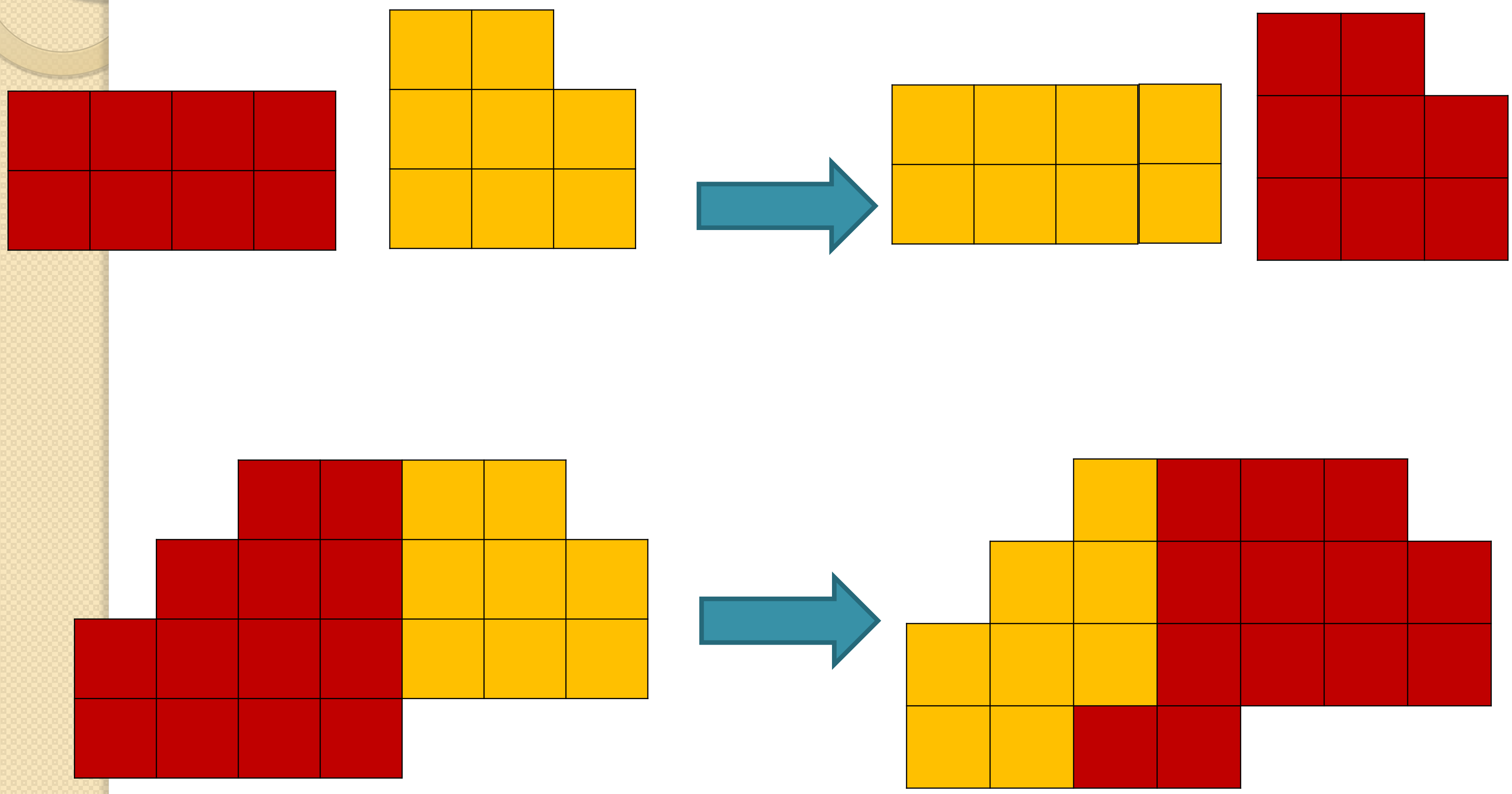
CRAFT
Exchanging two departments

- If the areas of the two departments are of **equal sizes** one department takes the shape of the other.

- If the areas **are not identical**:
  - Draw a box enclosing the two departments (this enclosed shaped includes the grids of the two departments only)
  - Count the number of grids of the smaller department. Let this count be *k*
  - Count *k* grids from the non-adjacent side of the larger department. These grids now become the new location of the smaller department. The space emptied by the smaller department now becomes part of the larger department’s new territory

CRAFT

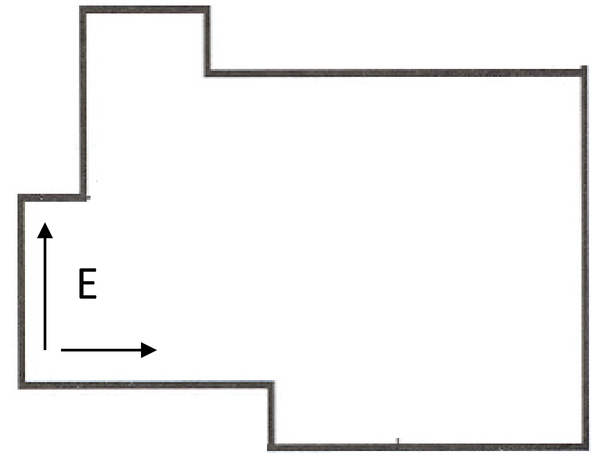
Exchanging two departments



CRAFT **Example** – exchanging E and F

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	A	A	A	A	A	A	A	A	A	A	G	G	G	G	G	G	G	G
2	A			**Receiving D.**						A	G				**Shipping D.**			G
3	A	A	A	A	A	A	A	A	A	A	G	G	G					G
4	B	B	B	B	B	C	C	C	C	C	E	E	G	G	G	G	G	G
5	B				B	C				C	E	E	E	E	E	E	E	E
6	B				B	C	C	C	C	C	E	E	E	E	E	E	E	E
7	B	B	B	B	B	D	D	D	D	F	F	F	F	F	F	F	E	E
8	D	D	D	D	D	D			D	F							F	F
9	D								D	F	F	F	F	F				F
10	D	D	D	D	D	D	D	D	H	H	H	H	H	F	F	F	F	F

Dummy Department



Department E needs less space than department F. Then:

Starting from the non-adjacent side of department F, locate all the cells for department E

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	A	A	A	A	A	A	A	A	A	A	G	G	G	G	G	G	G	G
2	A			**Receiving D.**						A	G				**Shipping D.**			G
3	A	A	A	A	A	A	A	A	A	A	G	G	G					G
4	B	B	B	B	B	C	C	C	C	C	F	F	G	G	G	G	G	G
5	B				B	C				C	F	F	F	F	F	F	F	F
6	B				B	C	C	C	C	C	F	F	F	F	F	F		F
7	B	B	B	B	B	D	D	D	D	E	E	E	E	E	E	F		F
8	D	D	D	D	D	D			D	E					E	F		F
9	D								D	E	E	E	E	E	E	F		F
10	D	D	D	D	D	D	D	D	H	H	H	H	H	E	E	F	F	F

New Layout – after exchanging E and F

CRAFT **Example**

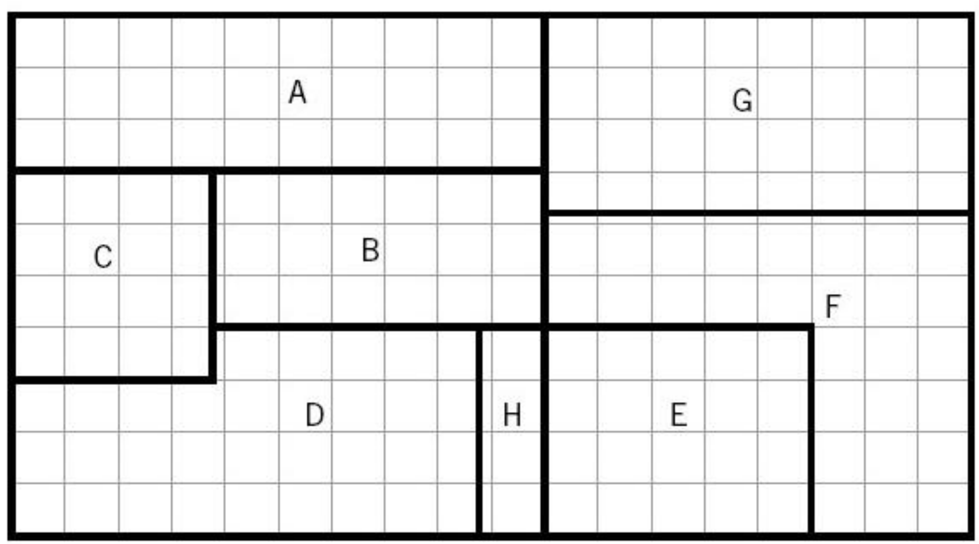
Final Layout – after exchanging B and C

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	A	A	A	A	A	A	A	A	A	A	G	G	G	G	G	G	G	G
2	A				**Receiving D.**					A	G							G
3	A	A	A	A	A	A	A	A	A	A	G	G	G		**Shipping D.**			G
4	C	C	C	B	B	B	B	B	B	B	F	F	G	G	G	G	G	G
5	C		C	C	B					B	F	F	F	F	F	F	F	F
6	C			C	B	B	B	B	B	B	F	F	F	F	F	F		F
7	C	C	C	C	B	D	D	D	D	E	E	E	E	E	E	F		F
8	D	D	D	D	D	D			D	E					E	F		F
9	D								D	E	E	E	E	E	E	F		F
10	D	D	D	D	D	D	D	D	H	H	H	H	H	E	E	F	F	F

CRAFT Example

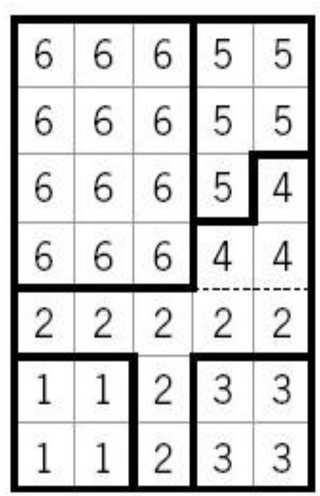
Manual Adjustment on CRAFT output

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	A	A	A	A	A	A	A	A	A	A	G	G	G	G	G	G	G	G
2	A									A	G							G
3	A	A	A	A	A	A	A	A	A	A	G	G	G					G
4	C	C	C	B	B	B	B	B	B	B	F	F	G	G	G	G	G	G
5	C		C	C	B					B	F	F	F	F	F	F	F	F
6	C			C	B	B	B	B	B	B	F	F	F	F	F	F		F
7	C	C	C	C	B	D	D	D	D	E	E	E	E	E	E	F		F
8	D	D	D	D	D	D			D	E					E	F		F
9	D							D	D	E	E	E	E	E	E	F		F
10	D	D	D	D	D	D	D	D	H	H	H	H	H	E	E	F	F	F



CRAFT Insufficiency of Adjacency for Exchange

- If 2 departments are not equal in area, then adjacency is a necessary but not sufficient condition for an exchange



CRAFT is unable to exchange departments 2 and 4 without splitting the department 2 or shifting other departments

CRAFT - **Pros**

- CRAFT is flexible with respect to department shapes.
  - In theory, CRAFT is applicable only to rectangular facilities, yet using dummy extensions, we can still apply CRAFT algorithm to non-rectangular shapes.
- **Dummy departments**
  - Have no flows or interaction with other departments
  - Require certain area
  - Can be fixed
  - Used for:
    - Non-rectangular facilities
    - Fixed areas in the layout (obstacles, unusable areas, etc.)
    - Aisle locations
    - Extra space
    - Building irregularities
- CRAFT captures the initial layout with reasonable accuracy

CRAFT - **Cons**

- **Locally optimal solution** only
  - CRAFT is a path-oriented method so the final layout is dependent on the initial layout. Therefore, a number of different initial layouts should be used as input to the CRAFT procedure.
- CRAFT may lead to **irregular shapes** both for individual departments and the facility itself.
  - Most of the time, a manual “finishing” must be done before presenting the CRAFT output.
- It is not always possible to exchange two unequal size, adjacent departments without splitting the larger one.

Next lecture

- Layout generation
  - MCRAFT
  - BLOCPLAN
  - LOGIC