6

Velocity in Mechanisms (Instantaneous Centre Method)

Features

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6.1. Introduction

Sometimes, a body has simultaneously a motion of rotation as well as translation, such as wheel of a car, a sphere rolling (but not slipping) on the ground. Such a motion will have the combined effect of rotation and translation.

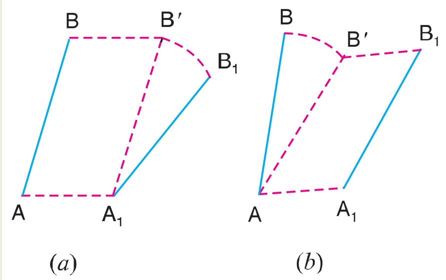
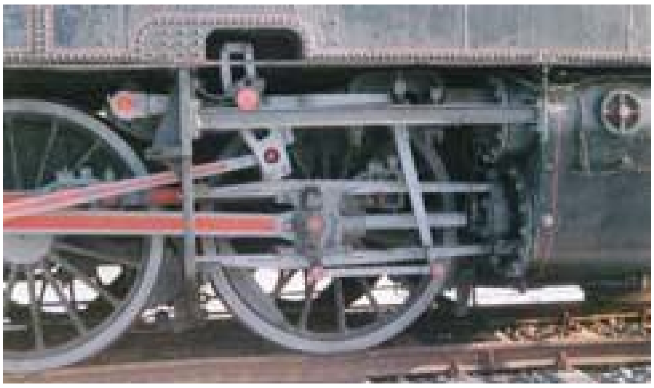


Fig. 6.1. Motion of a link.

Consider a rigid link AB, which moves from its initial position AB to $A_1 B_1$ as shown in Fig. 6.1 (*a*). A little consideration will show that the link neither has wholly a motion of translation nor wholly rotational, but a combination of the two motions. In Fig. 6.1 (*a*), the link has first the motion of translation from AB to A_1B' and then the motion of rotation about A_1, till it occupies the final position $A_1 B_1$. In Fig. 6.1 (*b*), the link AB has first the motion of rotation from AB to $A B'$ about A and then the motion of translation from $A B'$ to

$A_1 B_1$. Such a motion of link AB to $A_1 B_1$ is an example of combined motion of rotation and translation, it being immaterial whether the motion of rotation takes first, or the motion of translation.



Mechanisms on a steam automobile engine.

In actual practice, the motion of link AB is so gradual that it is difficult to see the two separate motions. But we see the two separate motions, though the point B moves faster than the point A. Thus, this combined motion of rotation and translation of the link AB may be assumed to be a motion of pure rotation about some centre I, known as the ***instantaneous centre of rotation (also called centro or virtual centre).*** The position of instantaneous centre may be located as discussed below:

Since the points A and B of the link has moved to A_1 and B_1 respectively under the motion of rotation (as assumed above), therefore the position of the centre of rotation must lie on the intersection of the right bisectors of chords $A A_1$ and $B B_1$. Let these bisectors intersect at I as shown in Fig. 6.2, which is the instantaneous centre of rotation or virtual centre of the link AB.

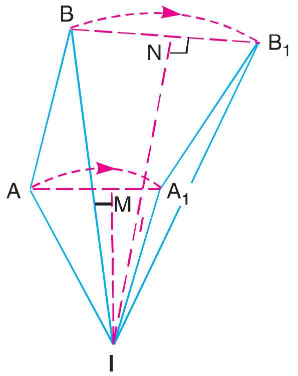


Fig. 6.2. Instantaneous centre of rotation.

From above, we see that the position of the link AB goes on changing, therefore the centre about which the motion is assumed to take place (*i.e.* the instantaneous centre of rotation) also goes on changing. Thus the instantaneous centre of a moving body may be defined as ***that centre which goes on changing from one instant to another***. The locus of all such instantaneous centres is known as ***centrode.*** A line drawn through an instantaneous centre and perpendicular to the plane of motion is called ***instantaneous axis.*** The locus of this axis is known as ***axode.***

6.2. Space and Body Centrodes

A rigid body in plane motion relative to a second rigid body, supposed fixed in space, may be assumed to be rotating about an instantaneous centre at that particular moment. In other words, the instantaneous centre is a point in the body which may be considered fixed at any particular moment. The locus of the instantaneous centre in space during a definite motion of the body is called the ***space centrode*** and the locus of the instantaneous centre relative to the body itself is called the ***body centrode.*** These two centrodes have the instantaneous centre as a common point at any instant and during the motion of the body, the body centrode rolls without slipping over the space centrode.

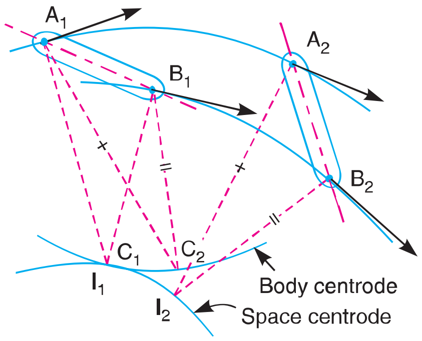


Fig. 6.3. Space and body centrode.

Let I_1 and I_2 be the instantaneous centres for the two different positions $A_1 B_1$ and $A_2 B_2$ of the link $A_1 B_1$ after executing a plane motion as shown in Fig. 6.3. Similarly, if the number of positions of the link $A_1 B_1$ are considered and a curve is drawn passing through these instantaneous centres (I_1, I_2....), then the curve so obtained is called the space centrode.

Now consider a point C_1 to be attached to the body or link A_1B_1 and moves with it in such a way that C_1 coincides with I_1 when the body is in position A_1B_1. Let C_2 be the position of the point C_1 when the link A_1B_1 occupies the position A_2B_2. A little consideration will show that the point C_2 will coincide with I_2 (when the link is in position A_2B_2) only if triangles $A_1B_1C_1$ and $A_2B_2C_2$ are identical.

$$\therefore \qquad A_1C_2 = A_2I_2 \quad \text{and} \quad B_1C_2 = B_2I_2$$

In the similar way, the number of positions of the point C_1 can be obtained for different positions of the link A_1B_1. The curve drawn through these points (C_1, C_2....) is called the body centrode.

6.3. Methods for Determining the Velocity of a Point on a Link

Though there are many methods for determining the velocity of any point on a link in a mechanism whose direction of motion (*i.e.* path) and velocity of some other point on the same link is known in magnitude and direction, yet the following two methods are important from the subject point of view.

1. Instantaneous centre method, and **2.** Relative velocity method.

The instantaneous centre method is convenient and easy to apply in simple mechanisms, whereas the relative velocity method may be used to any configuration diagram. We shall discuss the relative velocity method in the next chapter.

6.4. Velocity of a Point on a Link by Instantaneous Centre Method

The instantaneous centre method of analysing the motion in a mechanism is based upon the concept (as discussed in Art. 6.1) that any displacement of a body (or a rigid link) having motion in one plane, can be considered as a pure rotational motion of a rigid link as a whole about some centre, known as instantaneous centre or virtual centre of rotation.

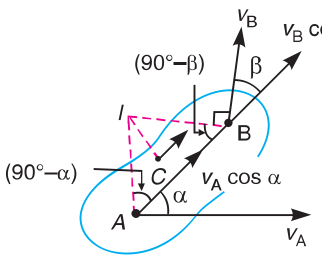


Fig. 6.4. Velocity of a point on a link.

Consider two points A and B on a rigid link. Let v_A and v_B be the velocities of points A and B, whose directions are given by angles α and β as shown in Fig. 6.4. If v_A is known in magnitude and direction and v_B in direction only, then the magnitude of v_B may be determined by the instantaneous centre method as discussed below :

Robots use various mechanisms to perform jobs.

Draw AI and BI perpendiculars to the directions v_A and v_B respectively. Let these lines intersect at I, which is known as instantaneous centre or virtual centre of the link. The complete rigid link is to rotate or turn about the centre I.

Since A and B are the points on a rigid link, therefore there cannot be any relative motion between them along the line AB.

Now resolving the velocities along AB,

$$v_A \cos\alpha = v_B \cos\beta$$

or
$$\frac{v_A}{v_B} = \frac{\cos\beta}{\cos\alpha} = \frac{\sin(90° - \beta)}{\sin(90° - \alpha)} \quad ...(i)$$

Applying Lami's theorem to triangle ABI,

$$\frac{AI}{\sin(90° - \beta)} = \frac{BI}{\sin(90° - \alpha)}$$

or
$$\frac{AI}{BI} = \frac{\sin(90° - \beta)}{\sin(90° - \alpha)} \quad ...(ii)$$

From equation (*i*) and (*ii*),

$$\frac{v_A}{v_B} = \frac{AI}{BI} \quad \text{or} \quad \frac{v_A}{AI} = \frac{v_B}{BI} = \omega \quad ...(iii)$$

where ω = Angular velocity of the rigid link.

If C is any other point on the link, then

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI} \quad ...(iv)$$

From the above equation, we see that

1. If v_A is known in magnitude and direction and v_B in direction only, then velocity of point B or any other point C lying on the same link may be determined in magnitude and direction.

2. The magnitude of velocities of the points on a rigid link is inversely proportional to the distances from the points to the instantaneous centre and is perpendicular to the line joining the point to the instantaneous centre.

6.5. Properties of the Instantaneous Centre

The following properties of the instantaneous centre are important from the subject point of view :

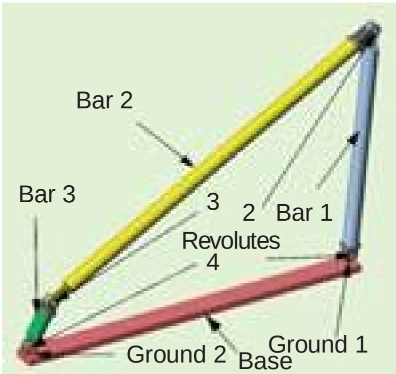
1. A rigid link rotates instantaneously relative to another link at the instantaneous centre for the configuration of the mechanism considered.

2. The two rigid links have no linear velocity relative to each other at the instantaneous centre. At this point (*i.e.* instantaneous centre), the two rigid links have the same linear velocity relative to the third rigid link. In other words, the velocity of the instantaneous centre relative to any third rigid link will be same whether the instantaneous centre is regarded as a point on the first rigid link or on the second rigid link.

6.6. Number of Instantaneous Centres in a Mechanism

The number of instantaneous centres in a constrained kinematic chain is equal to the number of possible combinations of two links. The number of pairs of links or the number of instantaneous centres is the number of combinations of n links taken two at a time. Mathematically, number of instantaneous centres,

$$N = \frac{n(n-1)}{2}, \quad \text{where } n = \text{Number of links.}$$



Four bar mechanisms.

6.7. Types of Instantaneous Centres

The instantaneous centres for a mechanism are of the following three types :

1. Fixed instantaneous centres, **2.** Permanent instantaneous centres, and **3.** Neither fixed nor permanent instantaneous centres.

The first two types *i.e.* fixed and permanent instantaneous centres are together known as ***primary instantaneous centres*** and the third type is known as ***secondary instantaneous centres***.

Consider a four bar mechanism *ABCD* as shown in Fig. 6.5. The number of instantaneous centres (*N*) in a four bar mechanism is given by

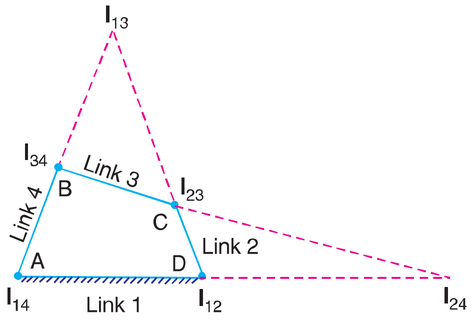


Fig. 6.5. Types of instantaneous centres.

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6 \qquad \ldots (\because n = 4)$$

The instantaneous centres I_{12} and I_{14} are called the ***fixed instantaneous centres*** as they remain in the same place for all configurations of the mechanism. The instantaneous centres I_{23} and I_{34} are the ***permanent instantaneous centres*** as they move when the mechanism moves, but the joints are of permanent nature. The instantaneous centres I_{13} and I_{24} are ***neither fixed nor permanent instantaneous centres*** as they vary with the configuration of the mechanism.

Note: The instantaneous centre of two links such as link 1 and link 2 is usually denoted by I_{12} and so on. It is read as *I* one two and not *I* twelve.

6.8. Location of Instantaneous Centres

The following rules may be used in locating the instantaneous centres in a mechanism :

1. When the two links are connected by a pin joint (or pivot joint), the instantaneous centre

Computer disk drive mechanisms.

Note : This picture is given as additional information and is not a direct example of the current chapter.

lies on the centre of the pin as shown in Fig. 6.6 (*a*). Such a instantaneous centre is of permanent nature, but if one of the links is fixed, the instantaneous centre will be of fixed type.

2. When the two links have a pure rolling contact (*i.e.* link 2 rolls without slipping upon the fixed link 1 which may be straight or curved), the instantaneous centre lies on their point of contact, as shown in Fig. 6.6 (*b*). The velocity of any point *A* on the link 2 relative to fixed link 1 will be perpendicular to $I_{12}\,A$ and is proportional to $I_{12}\,A$. In other words

$$\frac{v_A}{v_B} = \frac{I_{12}\,A}{I_{12}\,B}$$

3. When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact. We shall consider the following three cases :

(*a*) When the link 2 (slider) moves on fixed link 1 having straight surface as shown in Fig. 6.6 (*c*), the instantaneous centre lies at infinity and each point on the slider have the same velocity.

(*b*) When the link 2 (slider) moves on fixed link 1 having curved surface as shown in Fig. 6.6 (*d*),the instantaneous centre lies on the centre of curvature of the curvilinear path in the configuration at that instant.

(*c*) When the link 2 (slider) moves on fixed link 1 having constant radius of curvature as shown in Fig. 6.6 (*e*), the instantaneous centre lies at the centre of curvature *i.e.* the centre of the circle, for all configuration of the links.

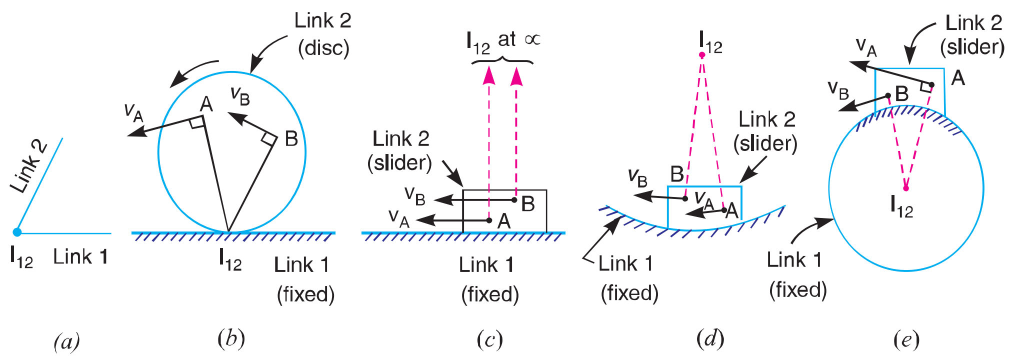


Fig. 6.6. Location of instantaneous centres.

6.9. Aronhold Kennedy (or Three Centres in Line) Theorem

The Aronhold Kennedy's theorem states that ***if three bodies move relatively to each other, they have three instantaneous centres and lie on a straight line.***

Consider three kinematic links *A*, *B* and *C* having relative plane motion. The number of instantaneous centres (*N*) is given by

$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

where n = Number of links = 3

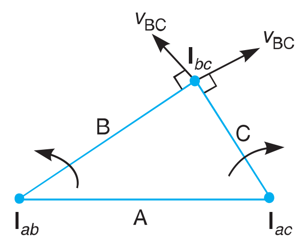
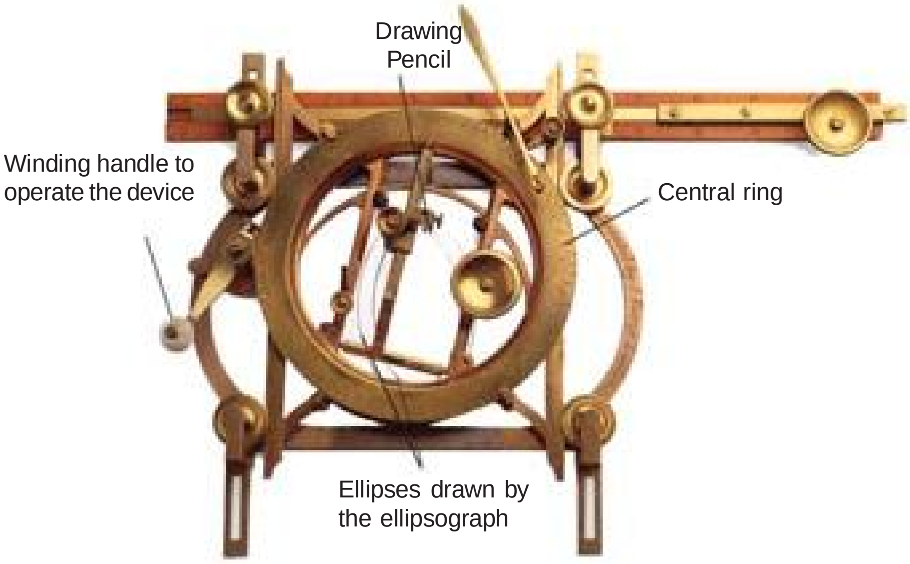


Fig. 6.7. Aronhold Kennedy's theorem.

The two instantaneous centres at the pin joints of *B* with *A*, and *C* with *A* (*i.e.* I_{ab} and I_{ac}) are the permanent instantaneous centres. According to Aronhold Kennedy's theorem, the third instantaneous centre I_{bc} must lie on the line joining I_{ab} and I_{ac}. In order to prove this, let us consider that the instantaneous centre I_{bc} lies outside the line joining I_{ab} and I_{ac} as shown in Fig. 6.7. The point I_{bc} belongs to both the links *B* and *C*. Let us consider the point I_{bc} on the link *B*. Its velocity v_{BC} must be perpendicular to the line joining I_{ab} and I_{bc}. Now consider the point I_{bc} on the link *C*. Its velocity v_{BC} must be perpendicular to the line joining I_{ac} and I_{bc}.

We have already discussed in Art. 6.5, that the velocity of the instantaneous centre is same whether it is regarded as a point on the first link or as a point on the second link. Therefore, the velocity of the point I_{bc} cannot be perpendicular to both lines $I_{ab}\,I_{bc}$ and $I_{ac}\,I_{bc}$ unless the point I_{bc} lies on the line joining the points I_{ab} and I_{ac}. Thus the three instantaneous centres (I_{ab}, I_{ac} and I_{bc}) must lie on the same straight line. The exact location of I_{bc} on line $I_{ab}\,I_{ac}$ depends upon the directions and magnitudes of the angular velocities of *B* and *C* relative to *A*.



The above picture shows ellipsograph which is used to draw ellipses.

Note : This picture is given as additional information and is not a direct example of the current chapter.

6.10. Method of Locating Instantaneous Centres in a Mechanism

Consider a pin jointed four bar mechanism as shown in Fig. 6.8 (*a*). The following procedure is adopted for locating instantaneous centres.

1. First of all, determine the number of instantaneous centres (*N*) by using the relation

$$N = \frac{n(n-1)}{2}, \text{ where } n = \text{Number of links.}$$

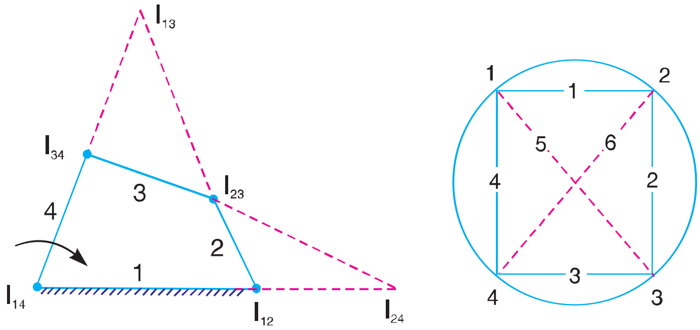
In the present case, $$N = \frac{4(4-1)}{2} = 6 \qquad \ldots(\because\ n = 4)$$

2. Make a list of all the instantaneous centres in a mechanism. Since for a four bar mechanism, there are six instantaneous centres, therefore these centres are listed as shown in the following table (known as book-keeping table).

Links	1	2	3	4
Instantaneous centres (6 in number)	12 13 14	23 24	34	–

3. Locate the fixed and permanent instantaneous centres by inspection. In Fig. 6.8 (*a*), I_{12} and I_{14} are fixed instantaneous centres and I_{23} and I_{34} are permanent instantaneous centres.

Note. The four bar mechanism has four turning pairs, therefore there are four primary (*i.e.* fixed and permanent) instantaneous centres and are located at the centres of the pin joints.



(*a*) Four bar mechanism. (*b*) Circle diagram.

Fig. 6.8. Method of locating instantaneous centres.

4. Locate the remaining neither fixed nor permanent instantaneous centres (or secondary centres) by Kennedy's theorem. This is done by circle diagram as shown in Fig. 6.8 (*b*). Mark points on a circle equal to the number of links in a mechanism. In the present case, mark 1, 2, 3, and 4 on the circle.

5. Join the points by solid lines to show that these centres are already found. In the circle diagram [Fig. 6.8 (*b*)] these lines are 12, 23, 34 and 14 to indicate the centres I_{12}, I_{23}, I_{34} and I_{14}.

6. In order to find the other two instantaneous centres, join two such points that the line joining them forms two adjacent triangles in the circle diagram. The line which is responsible for completing two triangles, should be a common side to the two triangles. In Fig. 6.8 (*b*), join 1 and 3 to form the triangles 123 and 341 and the instantaneous centre* I_{13} will lie on the intersection of I_{12} I_{23} and I_{14} I_{34}, produced if necessary, on the mechanism. Thus the instantaneous centre I_{13} is located. Join 1 and 3 by a dotted line on the circle diagram and mark number 5 on it. Similarly the instantaneous centre I_{24} will lie on the intersection of I_{12} I_{14} and I_{23} I_{34}, produced if necessary, on the mechanism. Thus I_{24} is located. Join 2 and 4 by a dotted line on the circle diagram and mark 6 on it. Hence all the six instantaneous centres are located.

Note: Since some of the neither fixed nor permanent instantaneous centres are not required in solving problems, therefore they may be omitted.

Example 6.1. *In a pin jointed four bar mechanism, as shown in Fig. 6.9, AB = 300 mm, BC = CD = 360 mm, and AD = 600 mm. The angle BAD = 60°. The crank AB rotates uniformly at 100 r.p.m. Locate all the instantaneous centres and find the angular velocity of the link BC.*

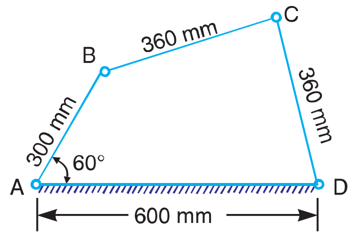


Fig. 6.9

Solution. Given : N_{AB} = 100 r.p.m or

$$\omega_{AB} = 2\pi \times 100/60 = 10.47 \text{ rad/s}$$

Since the length of crank AB = 300 mm = 0.3 m, therefore velocity of point B on link AB,

* We may also say as follows: Considering links 1, 2 and 3, the instantaneous centres will be I_{12}, I_{23} and I_{13}. The centres I_{12} and I_{23} have already been located. Similarly considering links 1, 3 and 4, the instantaneous centres will be I_{13}, I_{34} and I_{14}, from which I_{14} and I_{34} have already been located. Thus we see that the centre I_{13} lies on the intersection of the lines joining the points I_{12} I_{23} and I_{14} I_{34}.

$$v_B = \omega_{AB} \times AB = 10.47 \times 0.3 = 3.141 \text{ m/s}$$

Location of instantaneous centres

The instantaneous centres are located as discussed below:

1. Since the mechanism consists of four links (*i.e.* $n = 4$), therefore number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

2. For a four bar mechanism, the book keeping table may be drawn as discussed in Art. 6.10.

3. Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12}, I_{23}, I_{34} and I_{14}, as shown in Fig. 6.10.

4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. This is done by circle diagram as shown in Fig. 6.11. Mark four points (equal to the number of links in a mechanism) 1, 2, 3, and 4 on the circle.

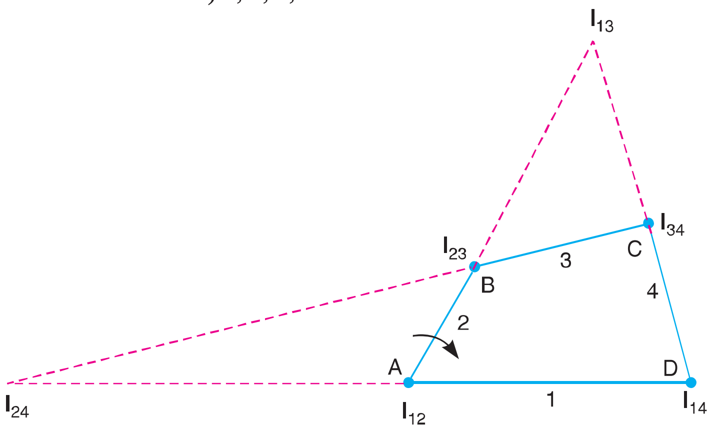


Fig. 6.10

5. Join points 1 to 2, 2 to 3, 3 to 4 and 4 to 1 to indicate the instantaneous centres already located *i.e.* I_{12}, I_{23}, I_{34} and I_{14}.

6. Join 1 to 3 to form two triangles 1 2 3 and 3 4 1. The side 13, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{13} lies on the intersection of the lines joining the points I_{12} I_{23} and I_{34} I_{14} as shown in Fig. 6.10. Thus centre I_{13} is located. Mark number 5 (because four instantaneous centres have already been located) on the dotted line 1 3.

7. Now join 2 to 4 to complete two triangles 2 3 4 and 1 2 4. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore centre I_{24} lies on the intersection of the lines joining the points I_{23} I_{34} and I_{12} I_{14} as shown in Fig. 6.10. Thus centre I_{24} is located. Mark number 6 on the dotted line 2 4. Thus all the six instantaneous centres are located.

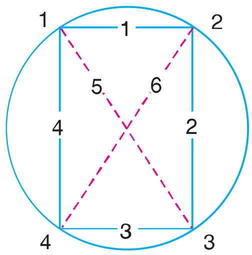


Fig. 6.11

Angular velocity of the link BC

Let ω_{BC} = Angular velocity of the link *BC*.

Since *B* is also a point on link *BC*, therefore velocity of point *B* on link *BC*,

$$v_B = \omega_{BC} \times I_{13}B$$

By measurement, we find that $I_{13} B = 500 \text{ mm} = 0.5 \text{ m}$

$$\therefore \quad \omega_{BC} = \frac{v_B}{I_{13} B} = \frac{3.141}{0.5} = 6.282 \text{ rad/s} \quad \textbf{Ans.}$$

Example 6.2. *Locate all the instantaneous centres of the slider crank mechanism as shown in Fig. 6.12. The lengths of crank OB and connecting rod AB are 100 mm and 400 mm respectively. If the crank rotates clockwise with an angular velocity of 10 rad/s, find:* **1.** *Velocity of the slider A, and* **2.** *Angular velocity of the connecting rod AB.*

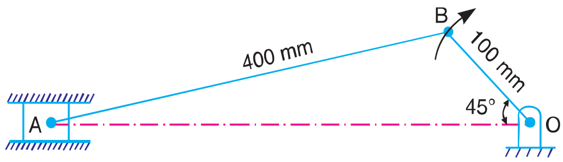


Fig. 6.12

Solution. Given : $\omega_{OB} = 10$ rad/ s; $OB = 100$ mm $= 0.1$ m

We know that linear velocity of the crank OB,

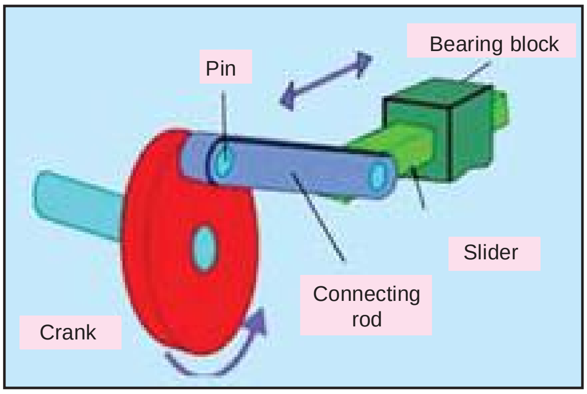
$$v_{OB} = v_B = \omega_{OB} \times OB = 10 \times 0.1 = 1 \text{ m/s}$$

Location of instantaneous centres

The instantaneous centres in a slider crank mechanism are located as discussed below:

1. Since there are four links (*i.e.* $n = 4$), therefore the number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$



Slider crank mechanism.

2. For a four link mechanism, the book keeping table may be drawn as discussed in Art. 6.10.

3. Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12}, I_{23} and I_{34} as shown in Fig. 6.13. Since the slider (link 4) moves on a straight surface (link 1), therefore the instantaneous centre I_{14} will be at infinity.

Note: Since the slider crank mechanism has three turning pairs and one sliding pair, therefore there will be three primary (*i.e.* fixed and permanent) instantaneous centres.

4. Locate the other two remaining neither fixed nor permanent instantaneous centres, by Aronhold Kennedy's theorem. This is done by circle diagram as shown in Fig. 6.14. Mark four points 1, 2, 3 and 4 (equal to the number of links in a mechanism) on the circle to indicate I_{12}, I_{23}, I_{34} and I_{14}.

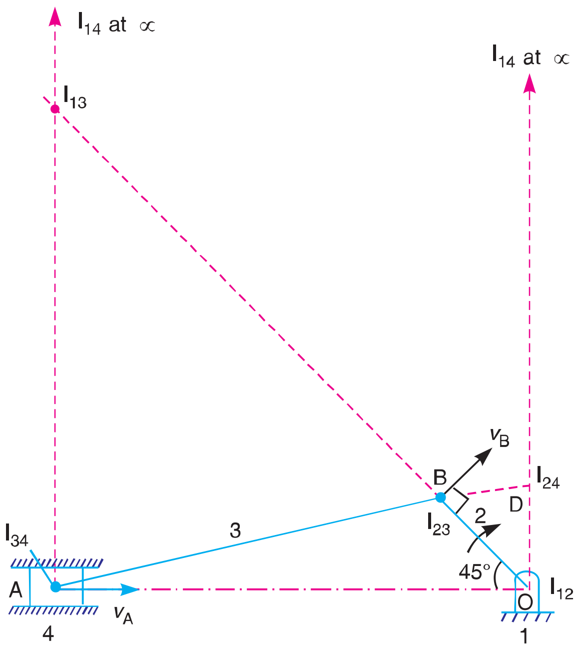


Fig. 6.13

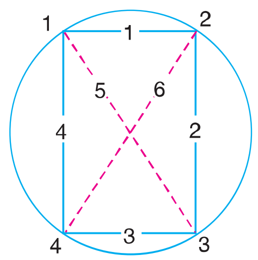


Fig. 6.14

5. Join 1 to 3 to form two triangles 1 2 3 and 3 4 1 in the circle diagram. The side 1 3, common to both triangles, is responsible for completing the two triangles. Therefore the centre I_{13} will lie on the intersection of $I_{12}\, I_{23}$ and $I_{14}\, I_{34}$, produced if necessary. Thus centre I_{13} is located. Join 1 to 3 by a dotted line and mark number 5 on it.

6. Join 2 to 4 by a dotted line to form two triangles 2 3 4 and 1 2 4. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore the centre I_{24} lies on the intersection of $I_{23}\, I_{34}$ and $I_{12}\, I_{14}$. Join 2 to 4 by a dotted line on the circle diagram and mark number 6 on it. Thus all the six instantaneous centres are located.

By measurement, we find that

$$I_{13} A = 460 \text{ mm} = 0.46 \text{ m} \text{ ; and } I_{13} B = 560 \text{ mm} = 0.56 \text{ m}$$

1. ***Velocity of the slider A***

Let v_A = Velocity of the slider A.

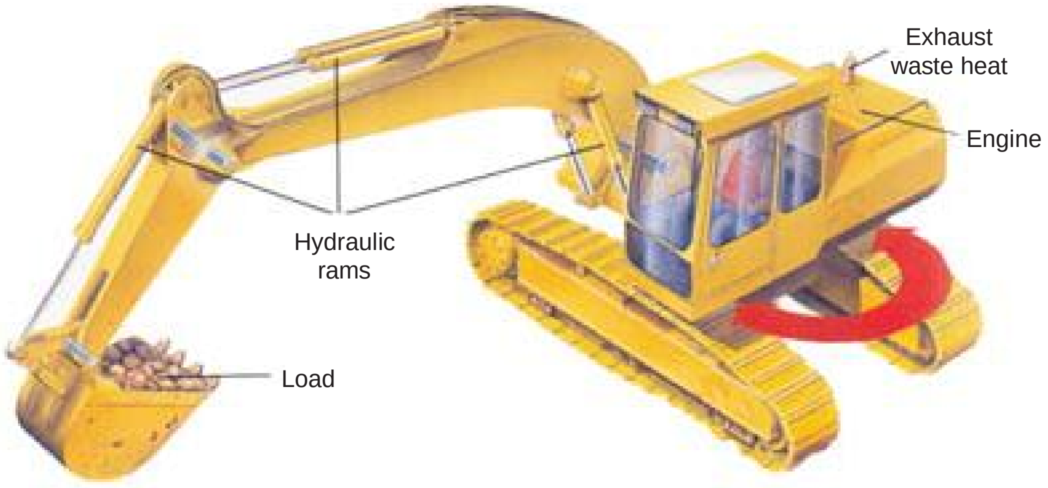
We know that
$$\frac{v_A}{I_{13} A} = \frac{v_B}{I_{13} B}$$

or
$$v_A = v_B \times \frac{I_{13} A}{I_{13} B} = 1 \times \frac{0.46}{0.56} = 0.82 \text{ m/s} \quad \textbf{Ans.}$$

2. ***Angular velocity of the connecting rod AB***

Let ω_{AB} = Angular velocity of the connecting rod AB.

We know that
$$\frac{v_A}{I_{13} A} = \frac{v_B}{I_{13} B} = \omega_{AB}$$



The above picture shows a digging machine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

$$\therefore \qquad \omega_{AB} = \frac{v_B}{I_{13}B} = \frac{1}{0.56} = 1.78 \text{ rad/s} \quad \textbf{Ans.}$$

Note: The velocity of the slider A and angular velocity of the connecting rod AB may also be determined as follows :

From similar triangles $I_{13}\,I_{23}\,I_{34}$ and $I_{12}\,I_{23}\,I_{24}$,

$$\frac{I_{12}\,I_{23}}{I_{13}\,I_{23}} = \frac{I_{23}\,I_{24}}{I_{23}\,I_{34}} \qquad \text{...}(i)$$

and

$$\frac{I_{13}\,I_{34}}{I_{34}\,I_{23}} = \frac{I_{12}\,I_{24}}{I_{23}\,I_{24}} \qquad \text{...}(ii)$$

We know that

$$\omega_{AB} = \frac{v_B}{I_{13}B} = \frac{\omega_{OB} \times OB}{I_{13}B} \qquad \text{...}(\because\ v_B = \omega_{OB} \times OB)$$

$$= \omega_{OB} \times \frac{I_{12}\,I_{23}}{I_{13}\,I_{23}} = \omega_{OB} \times \frac{I_{23}\,I_{24}}{I_{23}\,I_{34}} \qquad \text{...[From equation } (i)\text{] ...}(iii)$$

Also

$$v_A = \omega_{AB} \times I_{13}\,A = \omega_{OB} \times \frac{I_{23}\,I_{24}}{I_{23}\,I_{34}} \times I_{13}\,I_{34}. \qquad \text{...[From equation } (iii)\text{]}$$

$$= \omega_{OB} \times I_{12}\,I_{24} = \omega_{OB} \times OD \qquad \text{...[From equation } (ii)\text{]}$$

Example 6.3. *A mechanism, as shown in Fig. 6.15, has the following dimensions:*

OA = 200 mm; AB = 1.5 m; BC = 600 mm; CD = 500 mm and BE = 400 mm. Locate all the instantaneous centres.

If crank OA rotates uniformly at 120 r.p.m. clockwise, find **1.** *the velocity of B, C and D,* **2.** *the angular velocity of the links AB, BC and CD.*