Curves and Surfaces

Chapter 2

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Outline

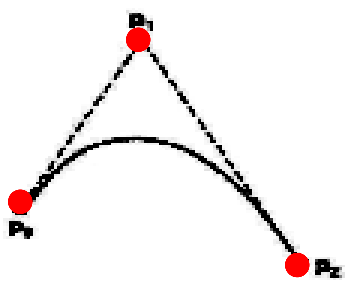
- ✓ Bezier Curve
- ✓ B-Spline Curve
- ✓ Types of Surfaces

Bezier Curves

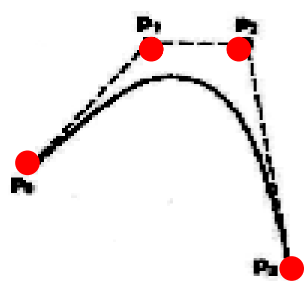
Properties of a Bézier Curve

$$\boxed{\mathbf{C}(u) = \sum_{k=0}^{n} \mathbf{p}_k B_{k,n}(u), \quad 0 \le u \le 1}$$

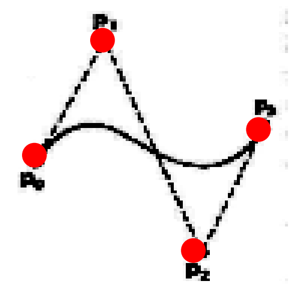
1. The **degree** of a Bézier curve defined by n+1 control points is **n**:



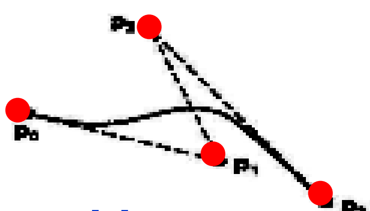
Parabola Curve



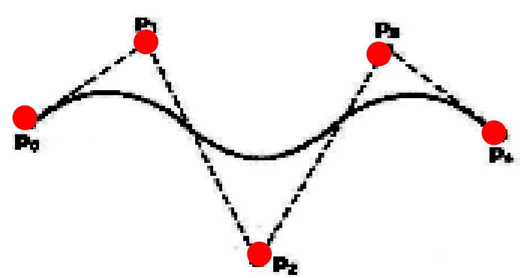
Cubic Curve



Cubic Curve

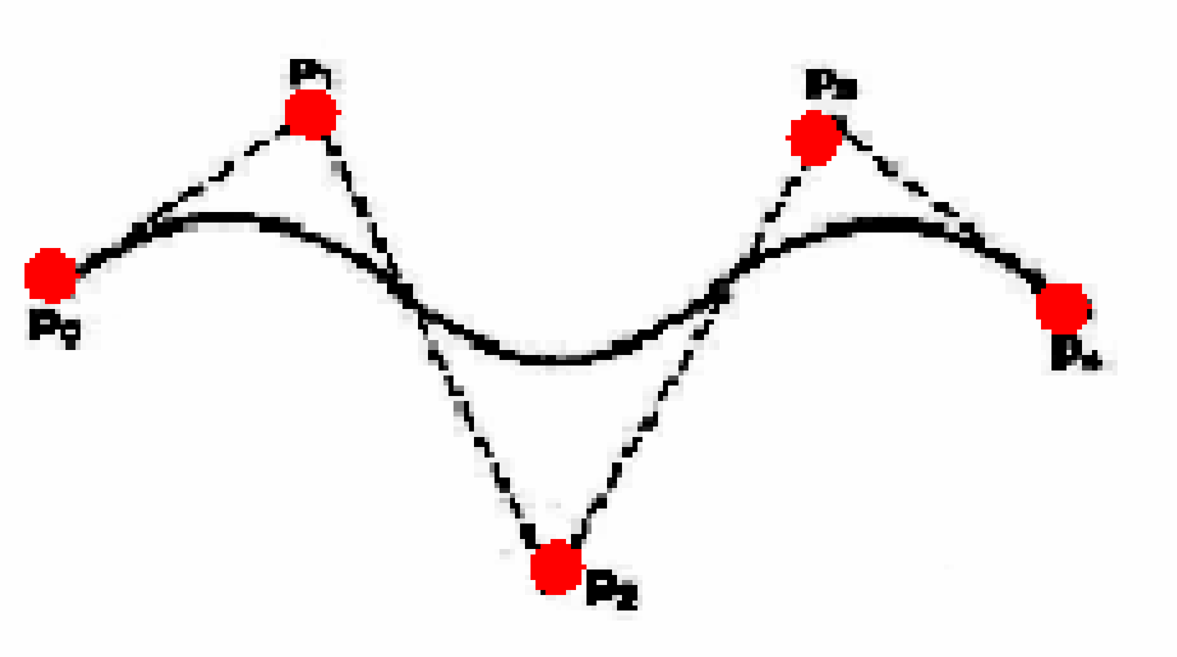


Cubic Curve



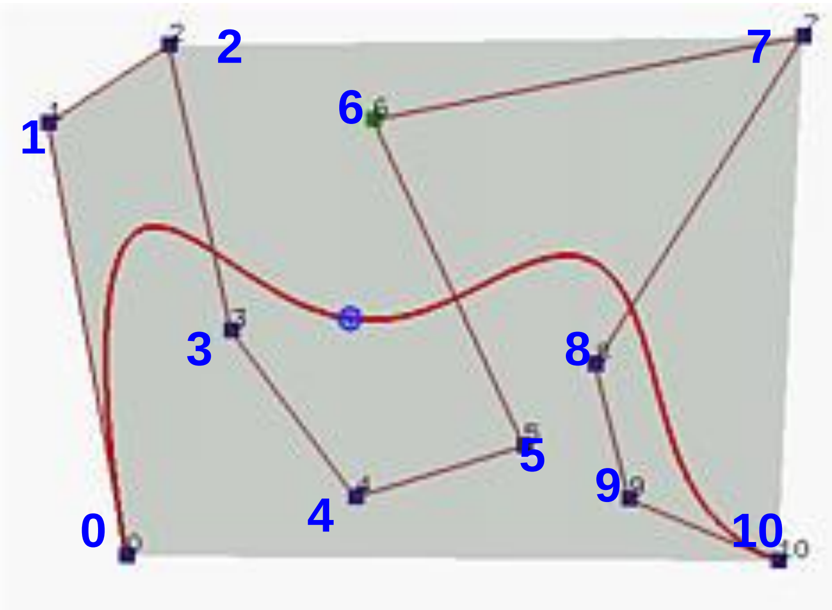
Properties of a Bézier Curve

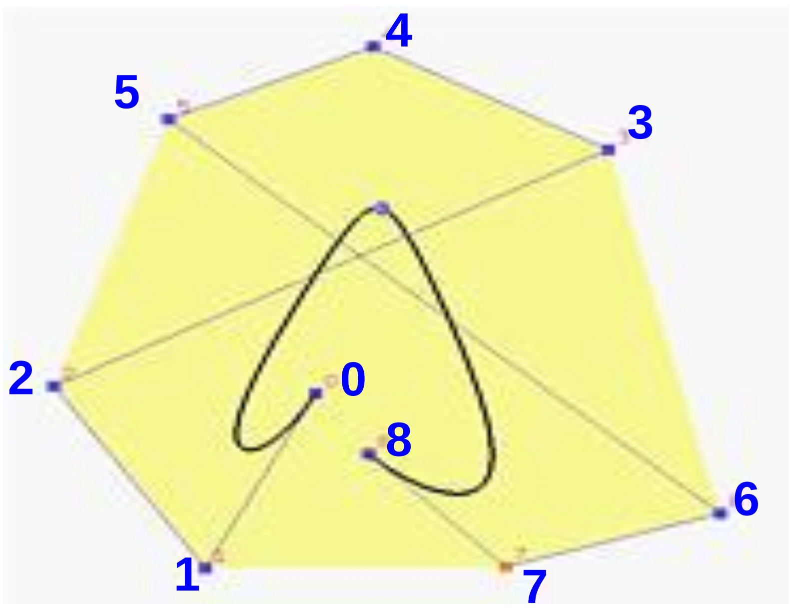
2. The curve passes though the first and the last control point **C**(u) passes through $\mathbf{P}_0$ and $\mathbf{P}_\mathbf{n}$.



Properties of a Bézier Curve

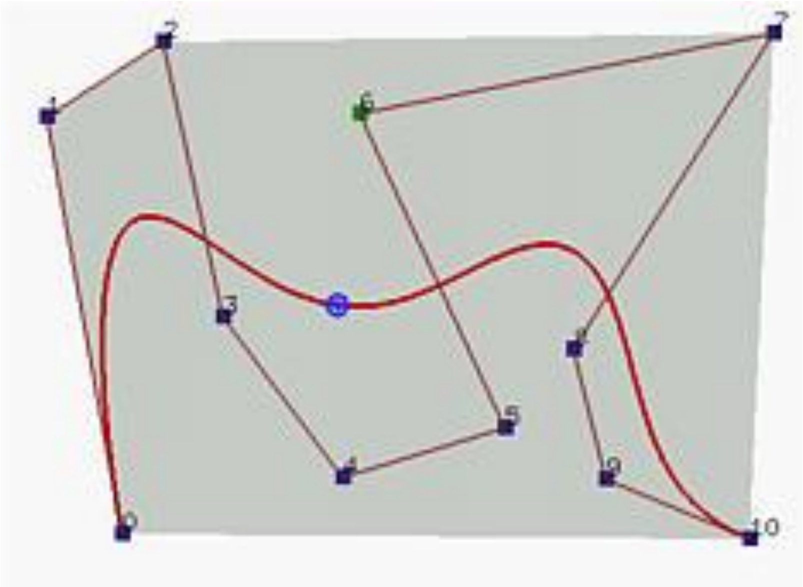
3. Bézier curves are tangent to their first and last edges of control polyline.





Properties of a Bézier Curve

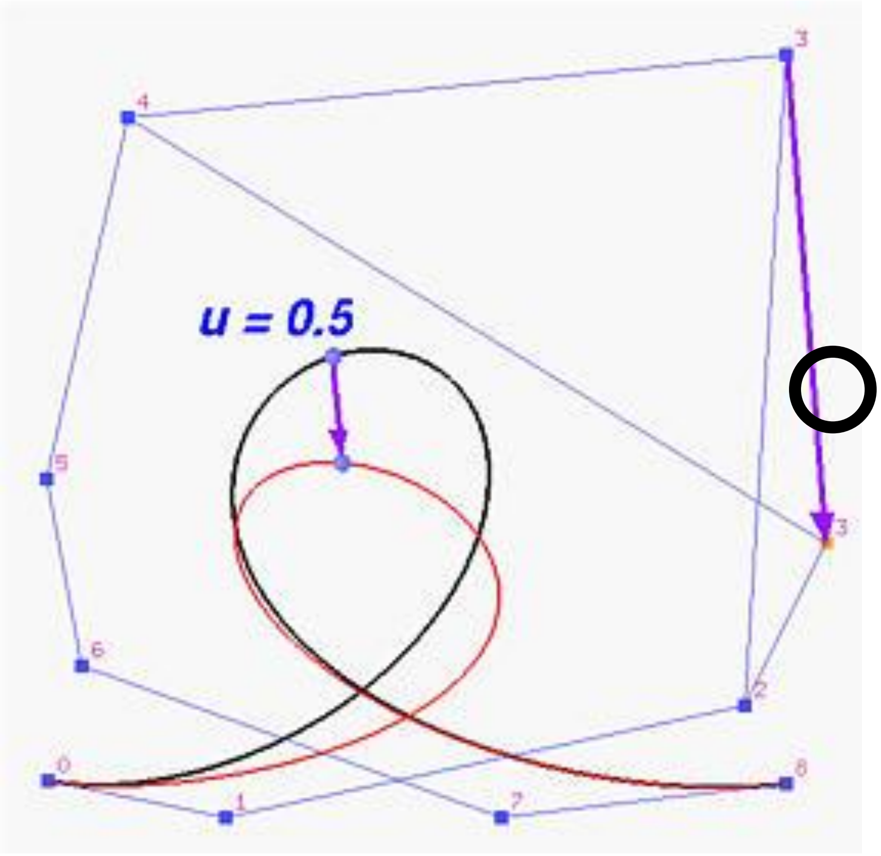
4. The Bézier curve lies completely in the **convex hull** of the given control points.



- Note that not all control points are on the boundary of the convex hull. For example, control points 3, 4, 5, 6, 8 and 9 are in the interior. The curve, except for the first two endpoints, lies completely in the convex hull.

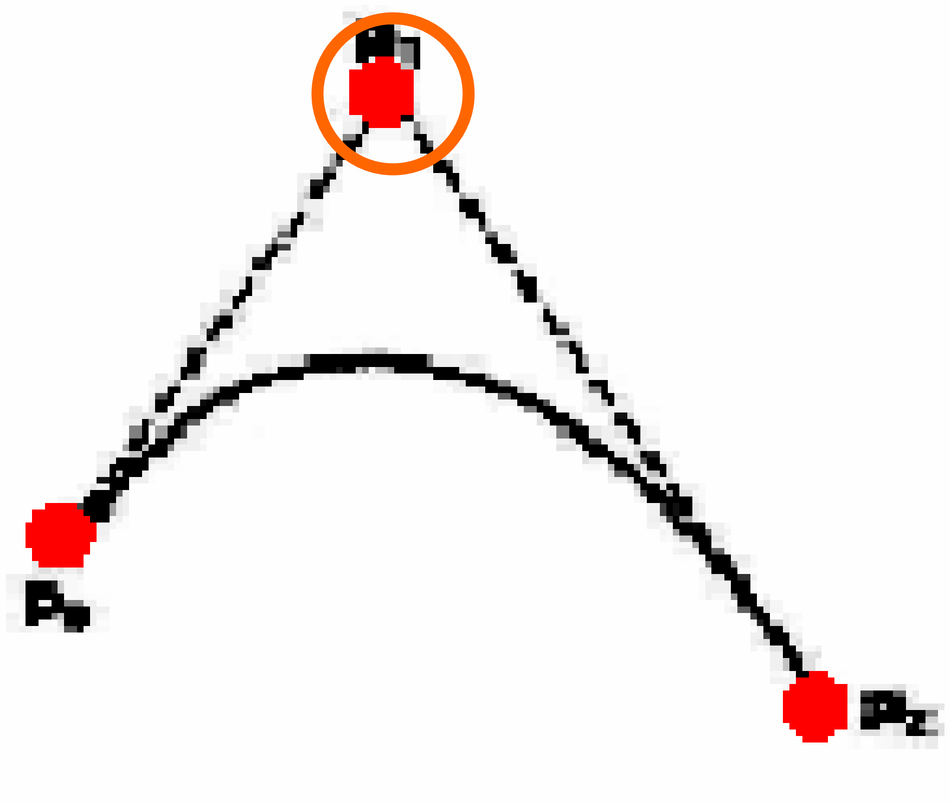
Properties of a Bézier Curve

5. Moving control points:



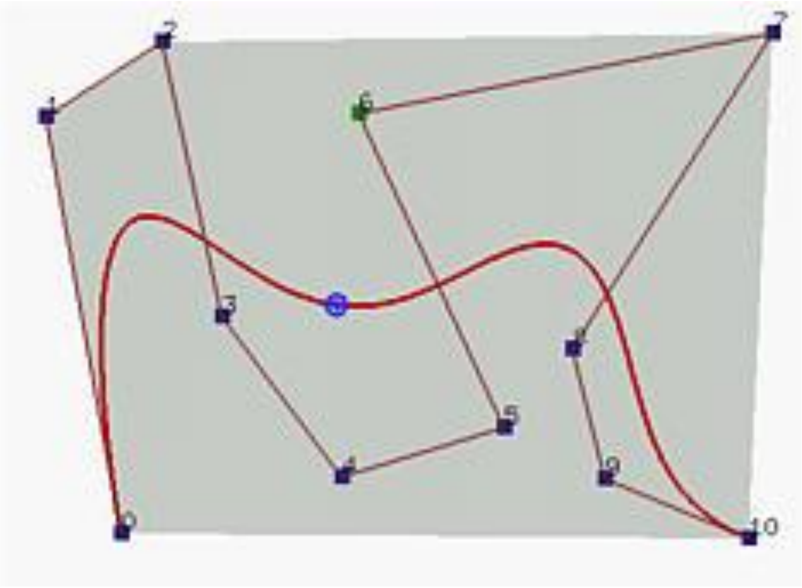
Properties of a Bézier Curve

5. **Moving control points:**



Bézier Curves

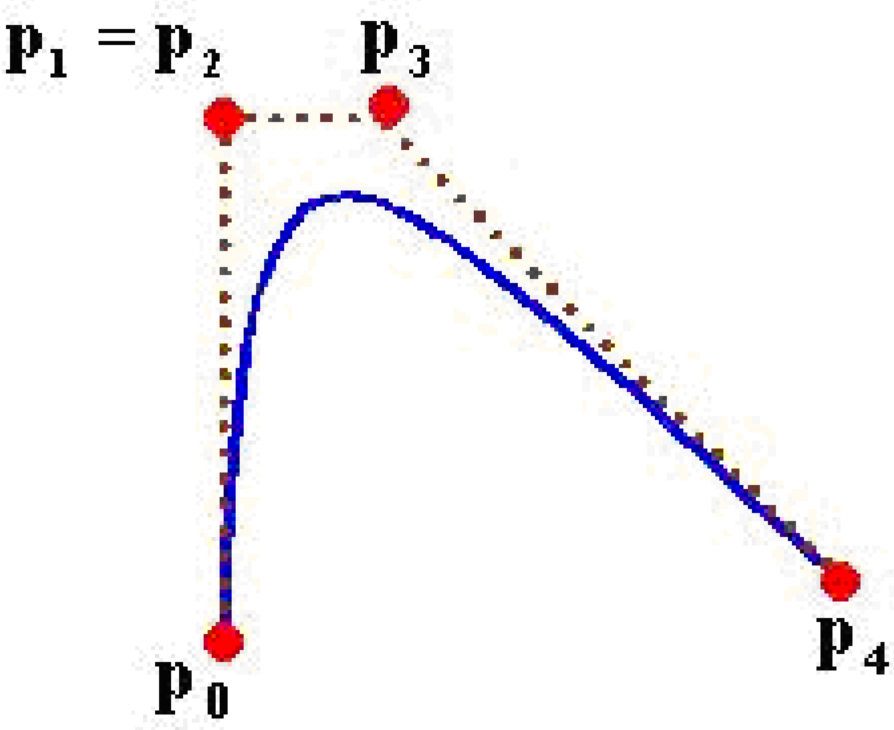
6. The point that corresponds to u on the Bézier curve is the **"weighted"** average of all control points, where the weights are the coefficients $B_{k,n}(u)$.



$$\mathbf{C}(u) = \sum_{k=0}^{n} \mathbf{p}_k B_{k,n}(u), \quad 0 \le u \le 1$$

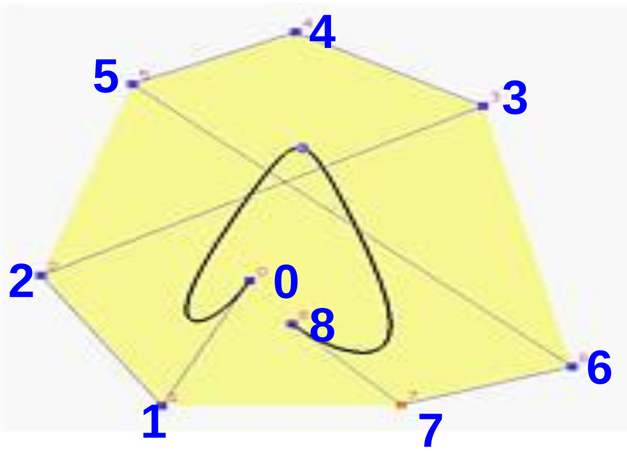
Design Techniques Using Bézier Curve (Weights)

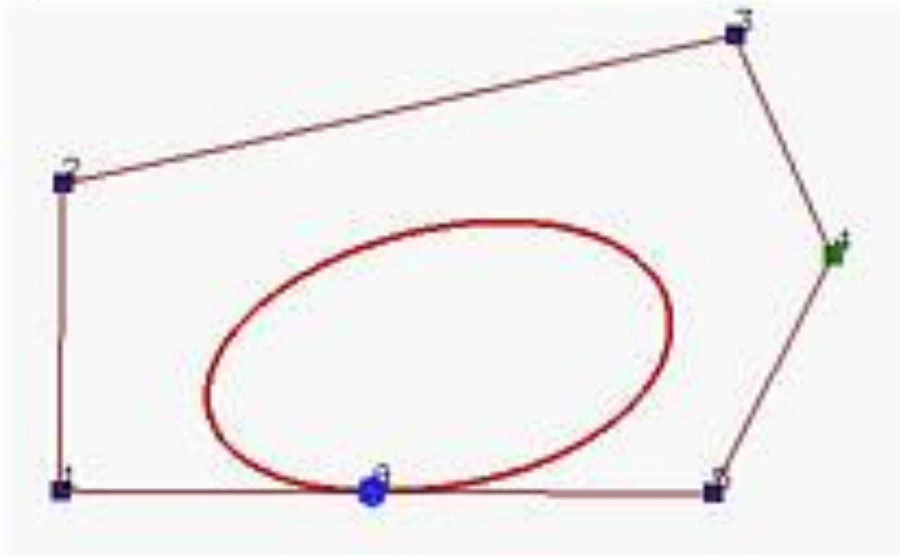
7. Multiple control points at a single coordinate position gives more weight to that position.



Design Techniques Using Bézier Curve (Closed Curves)

8. **Closed Bézier curves** are generated by specifying the first and the last control points at the same position.





Note: Bézier curves are polynomials which cannot represent circles and ellipses.

Properties of a Bézier Curve

9. If an affine transformation is applied to a Bézier curve, the result can be constructed from the affine images of its control points.

Construction

of

Bézier Curves

Bézier Curves

- Given n+1 control point positions:

$$\mathbf{C}(u) = \sum_{k=0}^{n} \mathbf{p}_k B_{k,n}(u), \quad 0 \le u \le 1$$

- The ***Bézier blending functions*** are the Bernstein polynomials:

$$B_{k,n}(u) = C(n,k)u^k(1-u)^{n-k}$$

- **The *C(n,k)* are the binomial coefficients:**

$$C(n,k) = \frac{n!}{k!(n-k)!}$$

Bézier Curves

- Given n+1 control point positions:

$$\boxed{\mathbf{C}(u) = \sum_{k=0}^{n} \mathbf{p}_k B_{k,n}(u), \quad 0 \le u \le 1}$$

$$x_i(u) = P_i(1-u)^3 + 3b_{i,1}u(1-u)^2 + 3b_{i,2}u^2(1-u) + P_{i+1}u^3$$

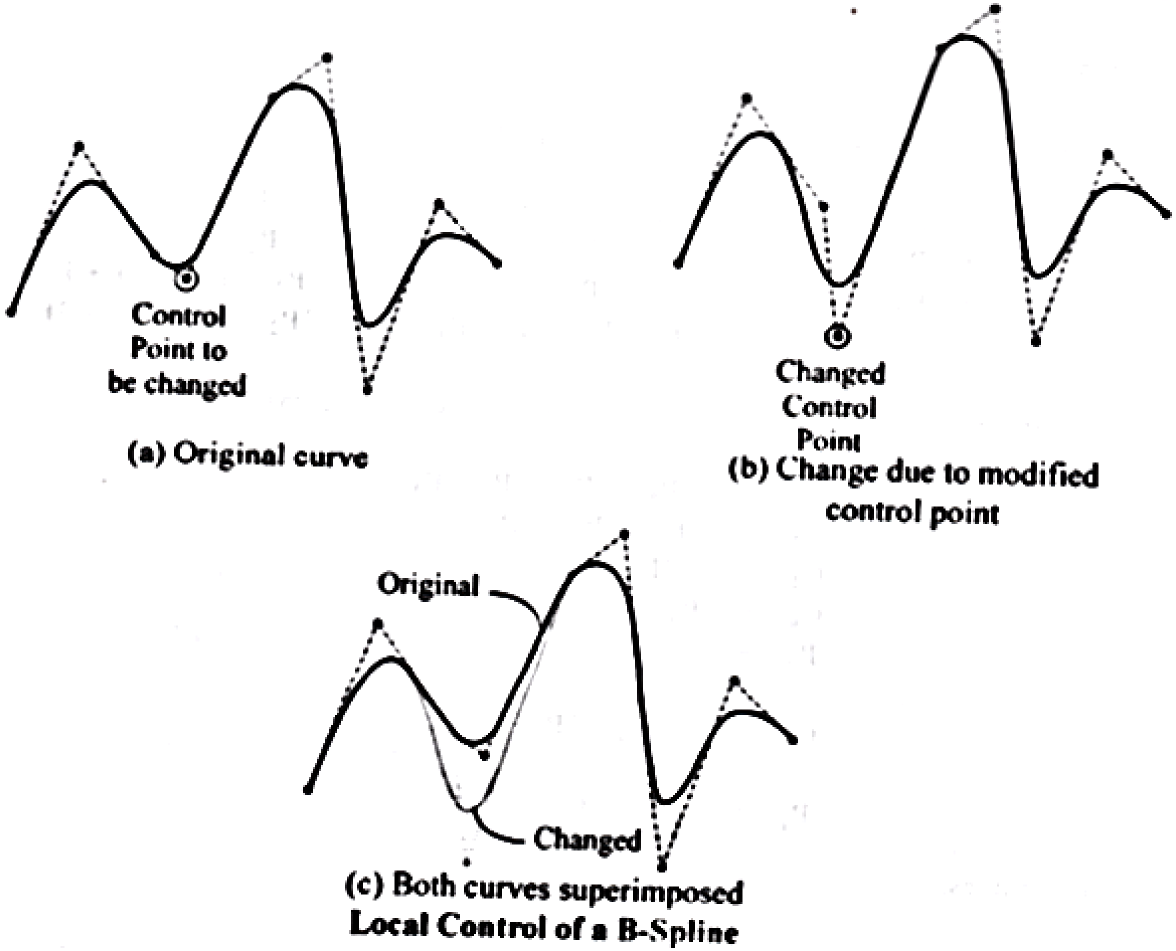
$$\boxed{\begin{aligned} B_{0,3}(u) &= (1-u)^3 \\ B_{1,3}(u) &= 3u(1-u)^2 \\ B_{2,3}(u) &= 3u^2(1-u) \\ B_{3,3}(u) &= u^3 \end{aligned}}$$

Bezier Curves: Eamples

- ..\bezier curve.xlsx

B-spline Curve

B-spline Curve



B-spline Curve

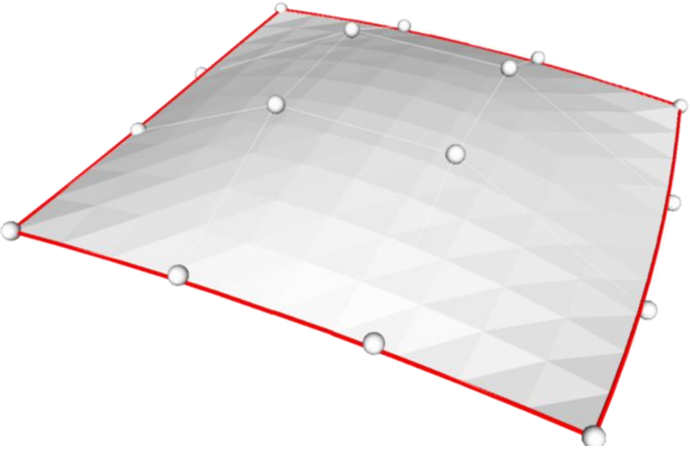
The major advantages of B-Spline Curves are :

1. The degree of the curve is independent of the number of control points. It was seen earlier that a four control point generated a cubic Bézier Curve. However, in case of B-splines, they can generate a linear, quadratic or cubic curve.
2. B-spline curve allows a local control over the shape of the spline curve. Consider the curve shown in Fig.
3. Besides the local control, B-spline curves allow us to vary the control points without changing the degree of the polynomial. Thus, any number of control points can be added or subtracted.
4. B-spline curves are widely used as they give better control. However, they are more complex as compared to Bézier Curves.

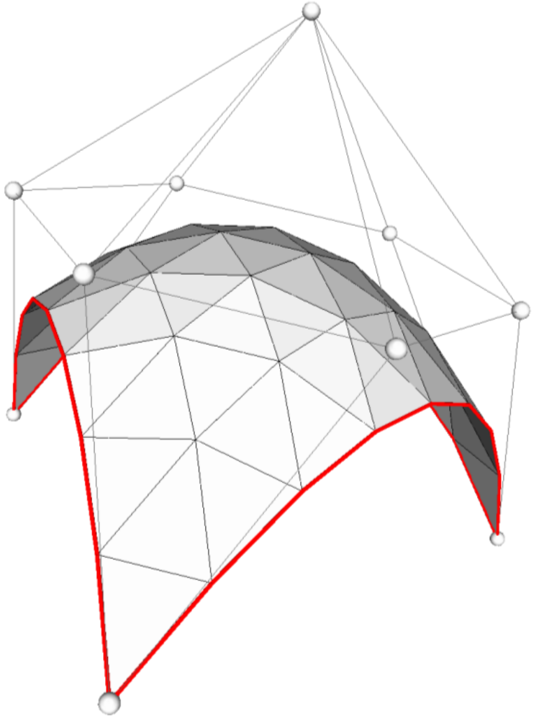
NURBS

- *NURBS ("Non-Uniform Rational B-Splines")* are a generalization of Beziers.
  - NU: *Non-Uniform*. The control points don't have to be weighted equally.
  - R: *Rational*. The spline may be defined by rational polynomials (homogeneous coordinates.)
  - BS: *B-Spline*. A chained series of Bezier splines with controllable degree.

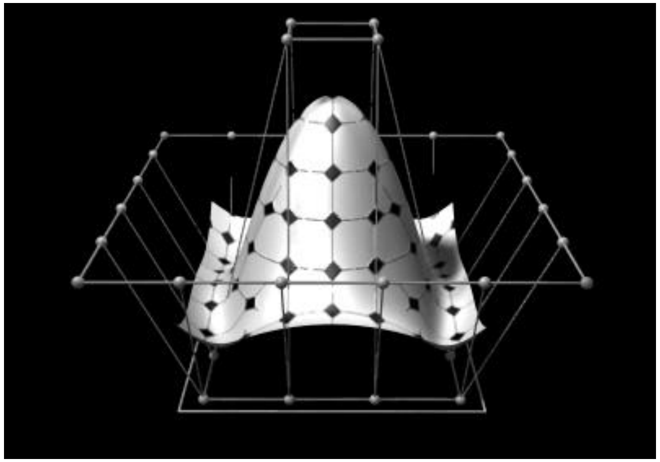
NURBS surfaces



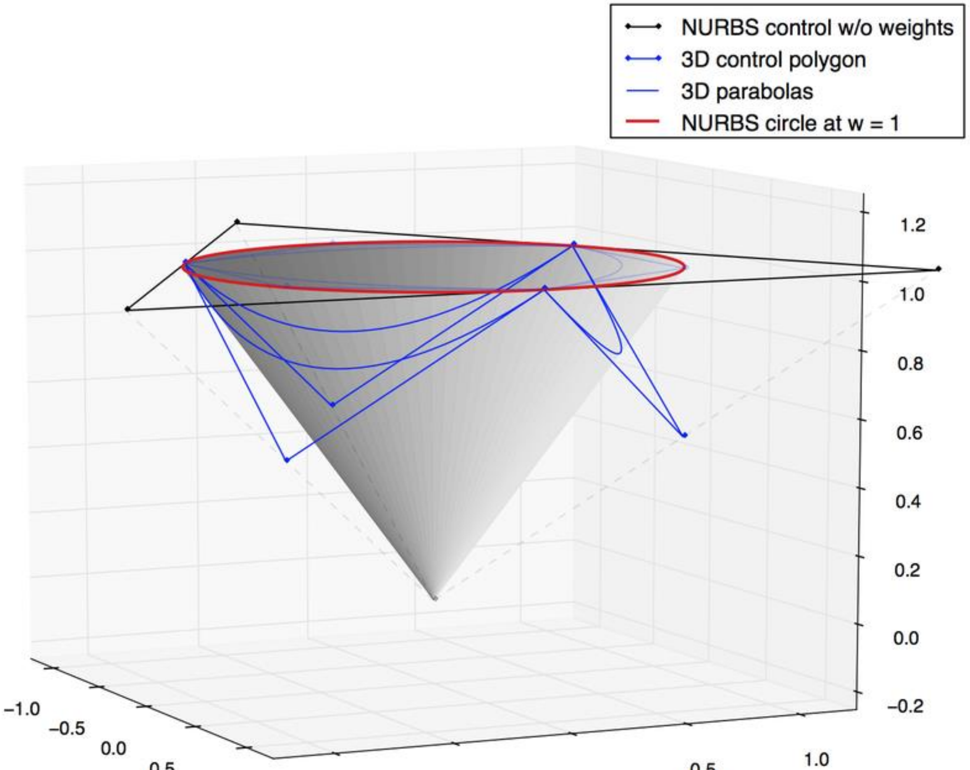
- ✓ The tensor product of the polynomial coefficients of two NURBS splines is a matrix of polynomial coefficients.
  - If curve A has degree *k* and *n+1* control points and curve B has degree *j* and *m+1* control points then AxB is an $(n+1)\times(m+1)$ matrix of polynomials of degree $max(j,k)$.
  - So all you need is $(n+1)(m+1)$ control points and you've got a rectangular surface patch!

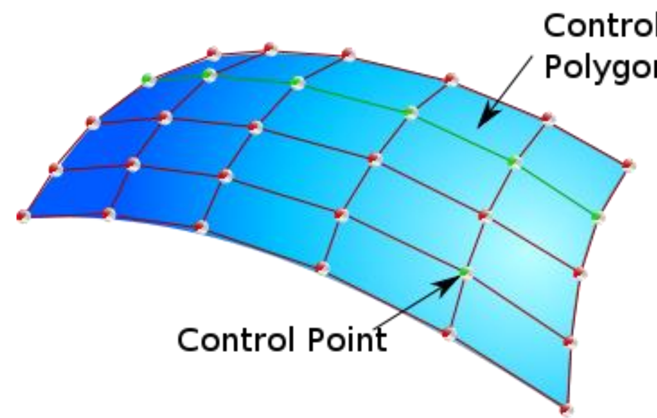


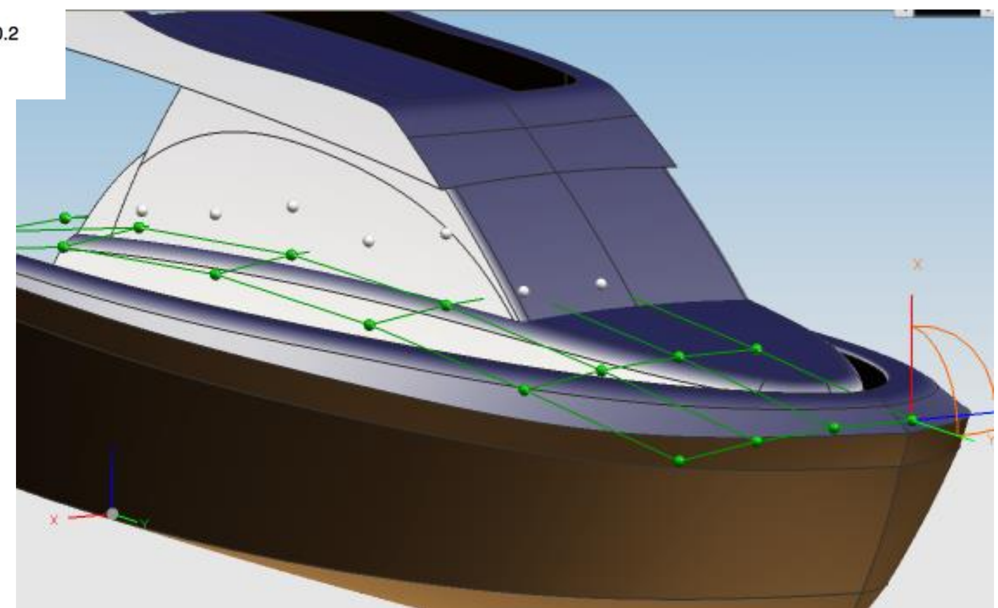
- ✓ This approach generalizes to triangles and arbitrary *n*-gons.



NURBS surfaces







Types of Surfaces

Geometric Modeling Approaches

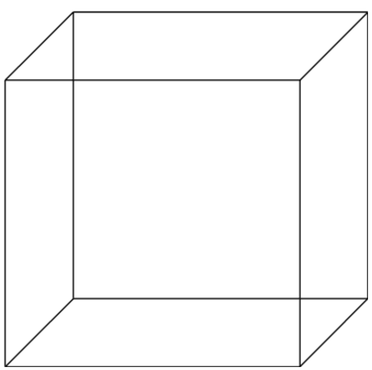
The basic geometric modelling approaches available to designers on CAD/CAM systems are:

1. Wire-frame modeling.
2. Surface modeling.
3. Solid modeling.

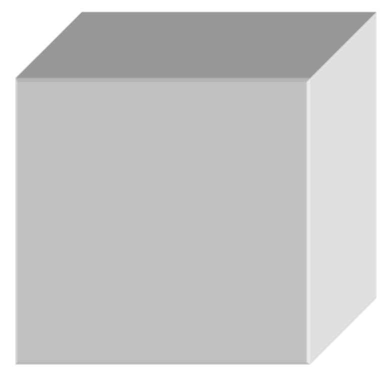
Wire-frame Modeling

Wire-frame modelling uses points and curves (i.e. lines, circles, arcs), and so forth to define objects.

The user uses edges and vertices of the part to form a 3-D object

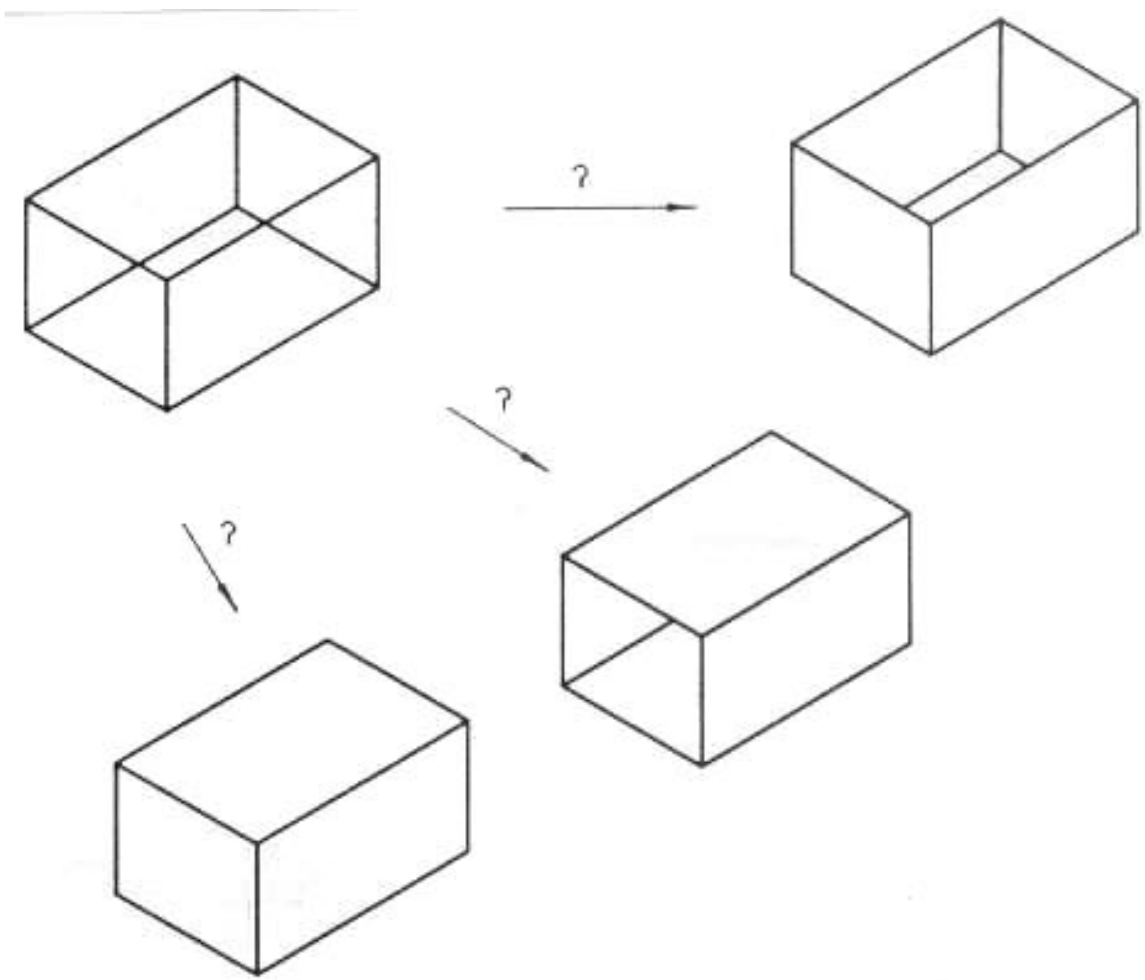


Wire-frame

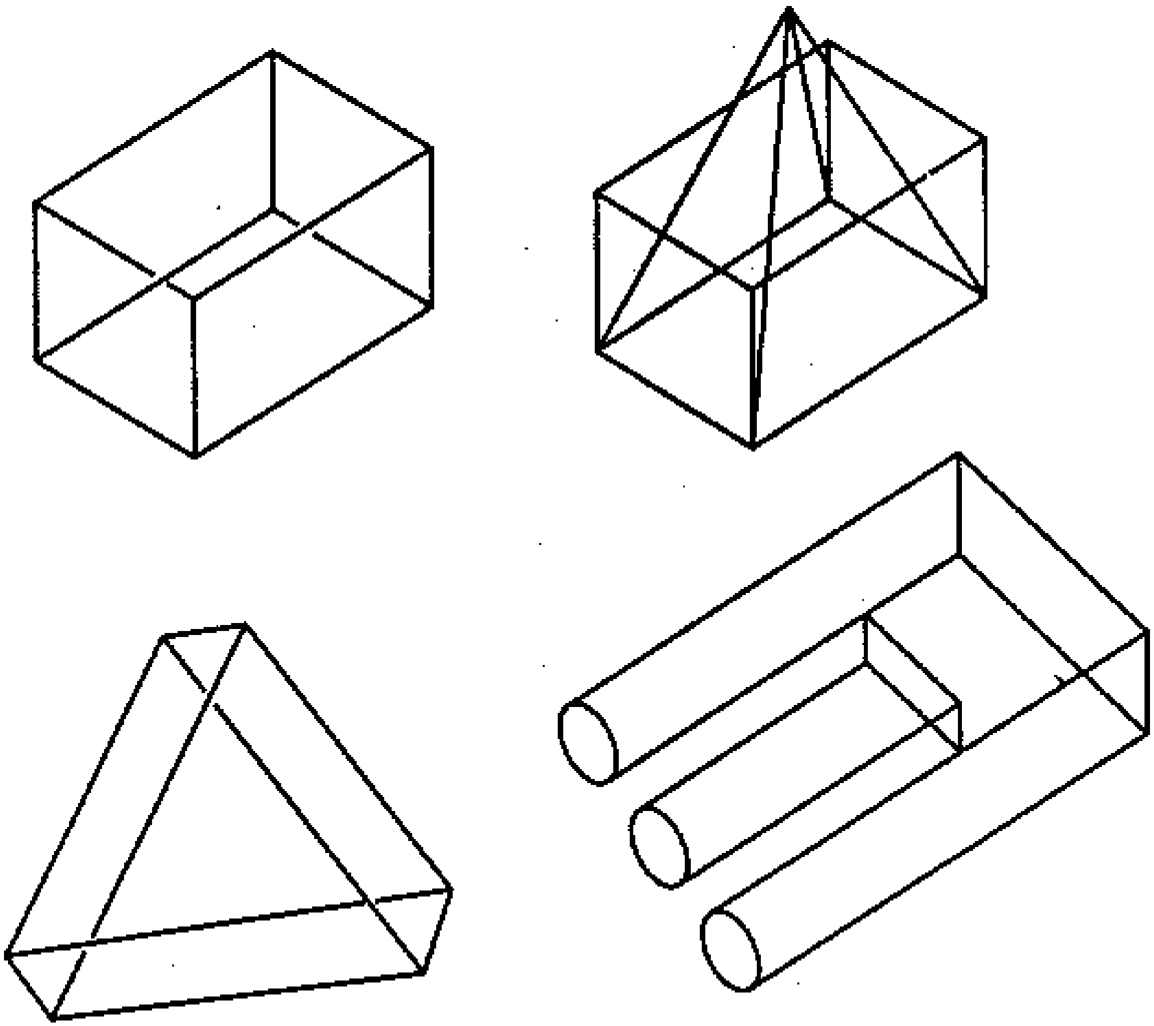


solid model

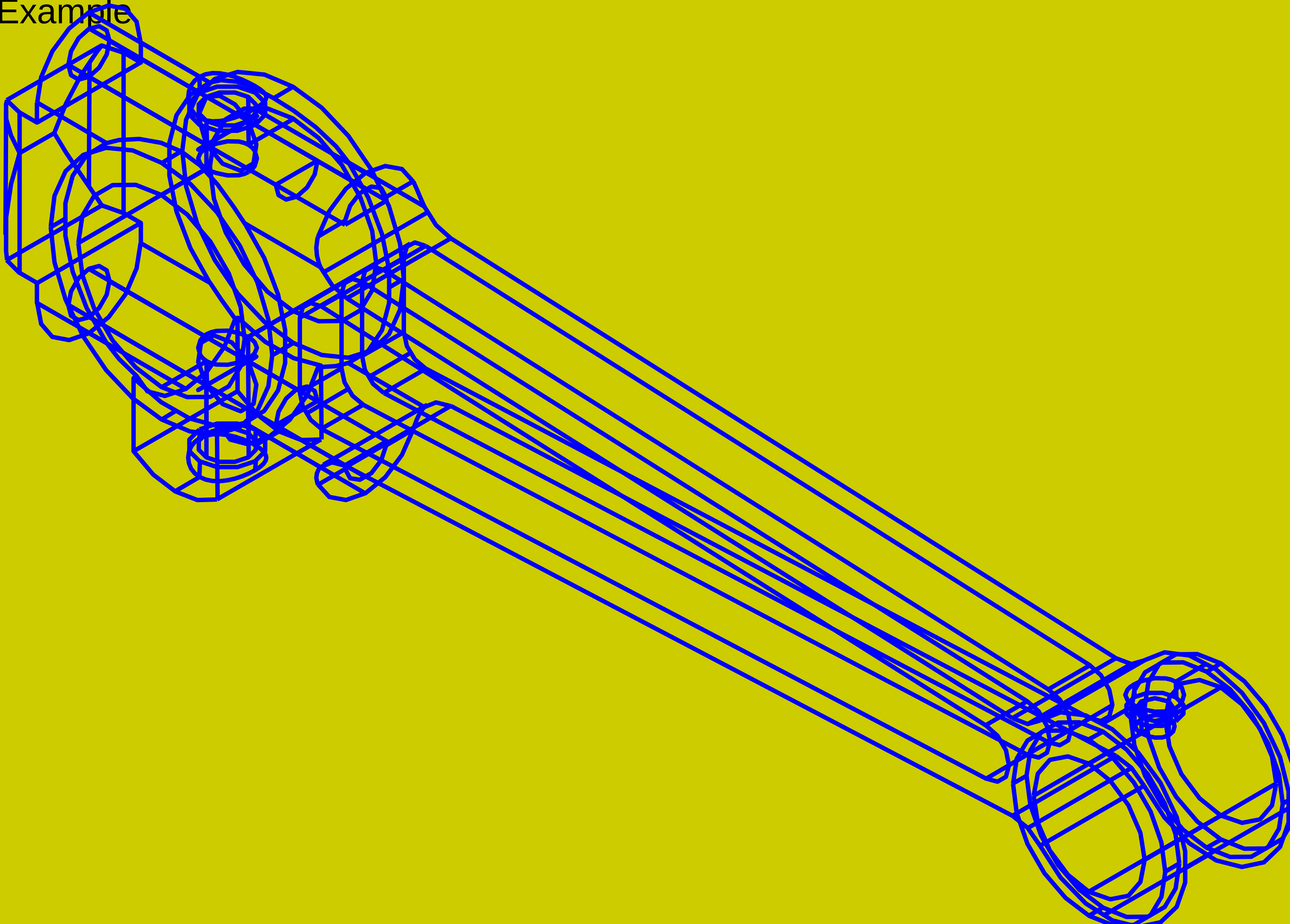
Wire-frame Models



Nonsense
Wire-frame Objects



Example



Surface Modeling

Surface modeling is more sophisticated than wireframe modeling in that it defines not only the edges of a 3D object, but also its surfaces.

In surface modeling, objects are defined by their bounding faces.

SURFACE ENTITIES

Similar to wireframe entities, existing CAD/CAM systems provide designers with both analytic and synthetic surface entities.

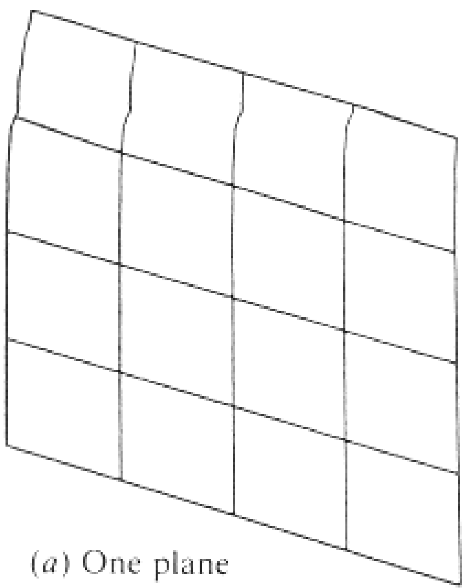
Analytic entities include :

- Plane surface,
- Ruled surface,
- Surface of revolution, and
- Tabulated cylinder.

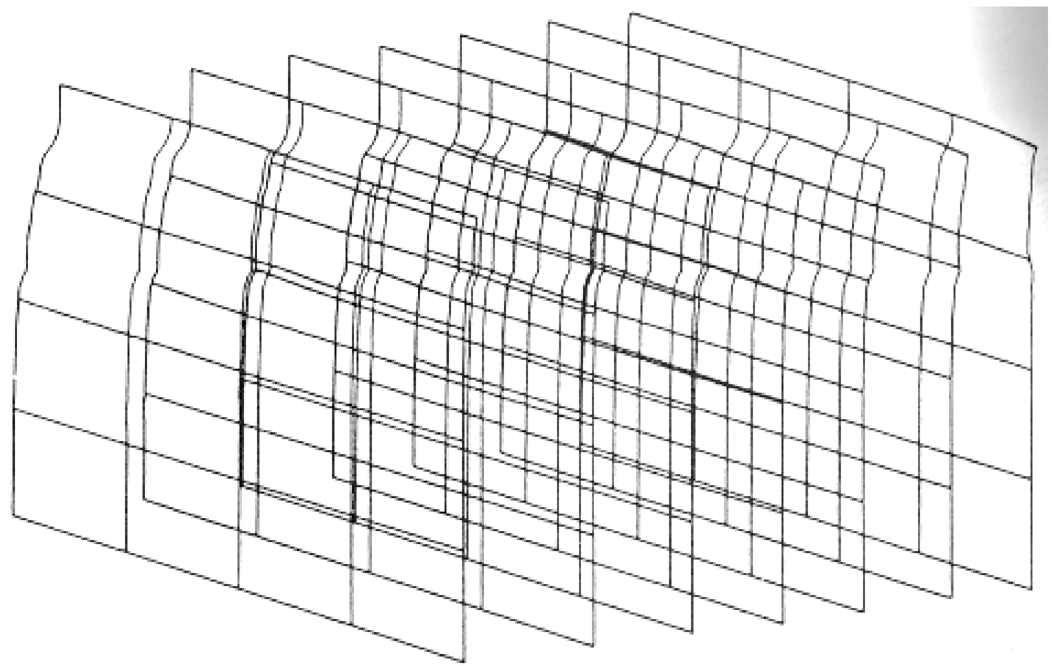
Synthetic entities include

- The bicubic Hermite spline surface,
- B-spline surface,
- Rectangular and triangular Bezier patches,
- Rectangular and triangular Coons patches, and
- Gordon surface.

Plane surface. This is the simplest surface. It requires three noncoincident points to define an infinite plane.

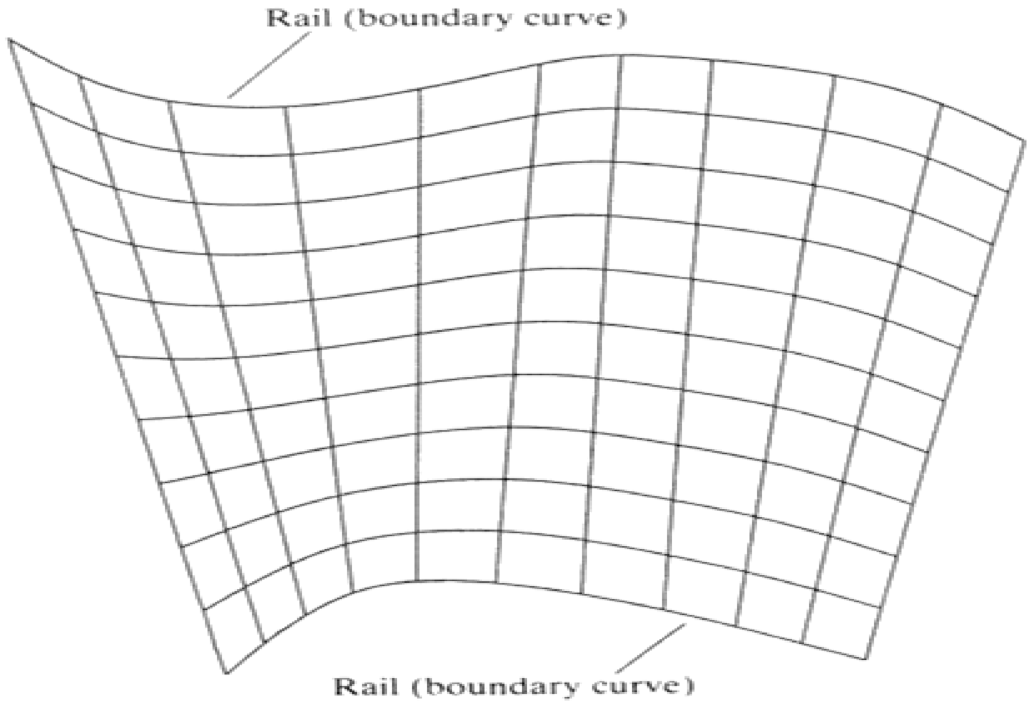


(*a*) One plane



(*b*) Multiple planes

Ruled (lofted) surface. This is a linear surface. It interpolates linearly between two boundary curves that define the surface (rails). Rails can be any wireframe entity. This entity is ideal to represent surfaces that do not have any twists or kinks.



Surface of revolution. This is an axisymmetric surface that can model axisymmetric objects. It is generated by rotating a planar wireframe entity in space about the axis of symmetry a certain angle.

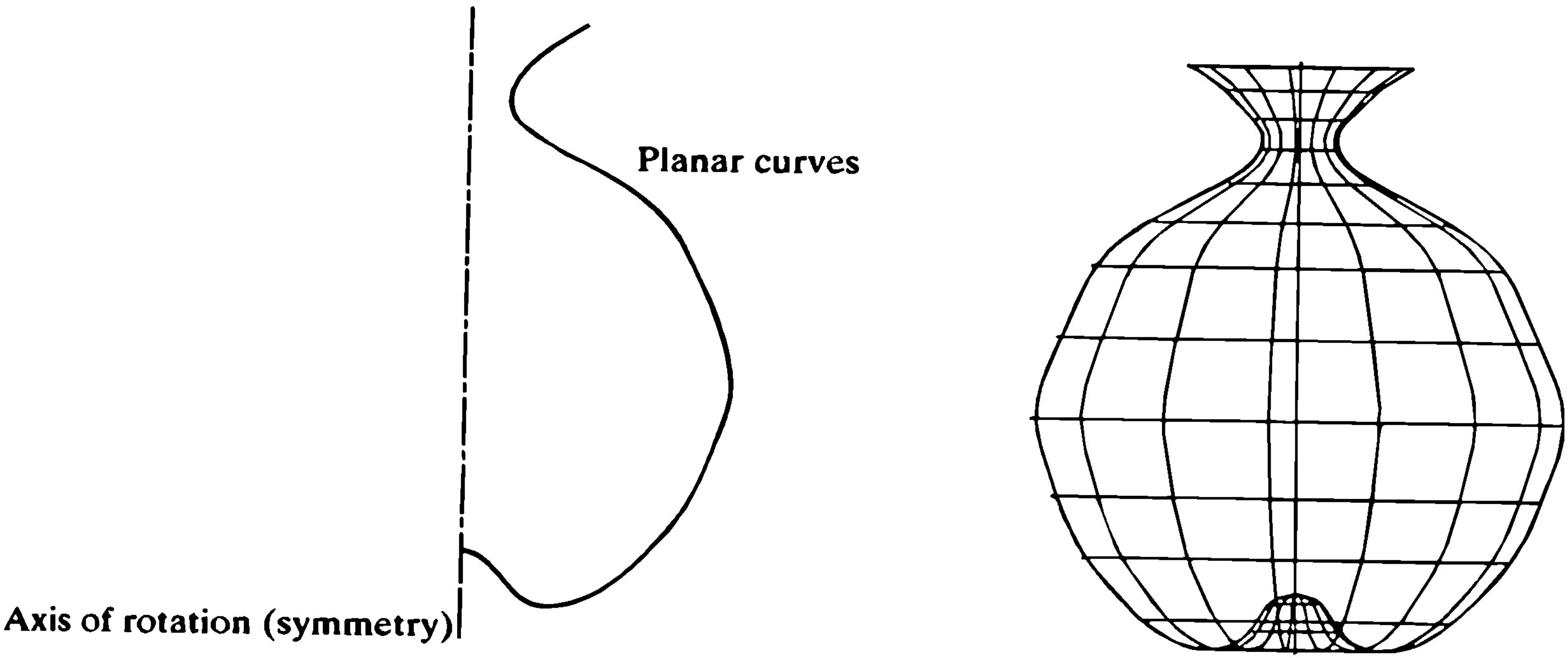


FIGURE 6-6
Surface of revolution.

Tabulated cylinder. This is a surface generated by translating a planar curve a certain distance along a specified direction (axis of the cylinder).

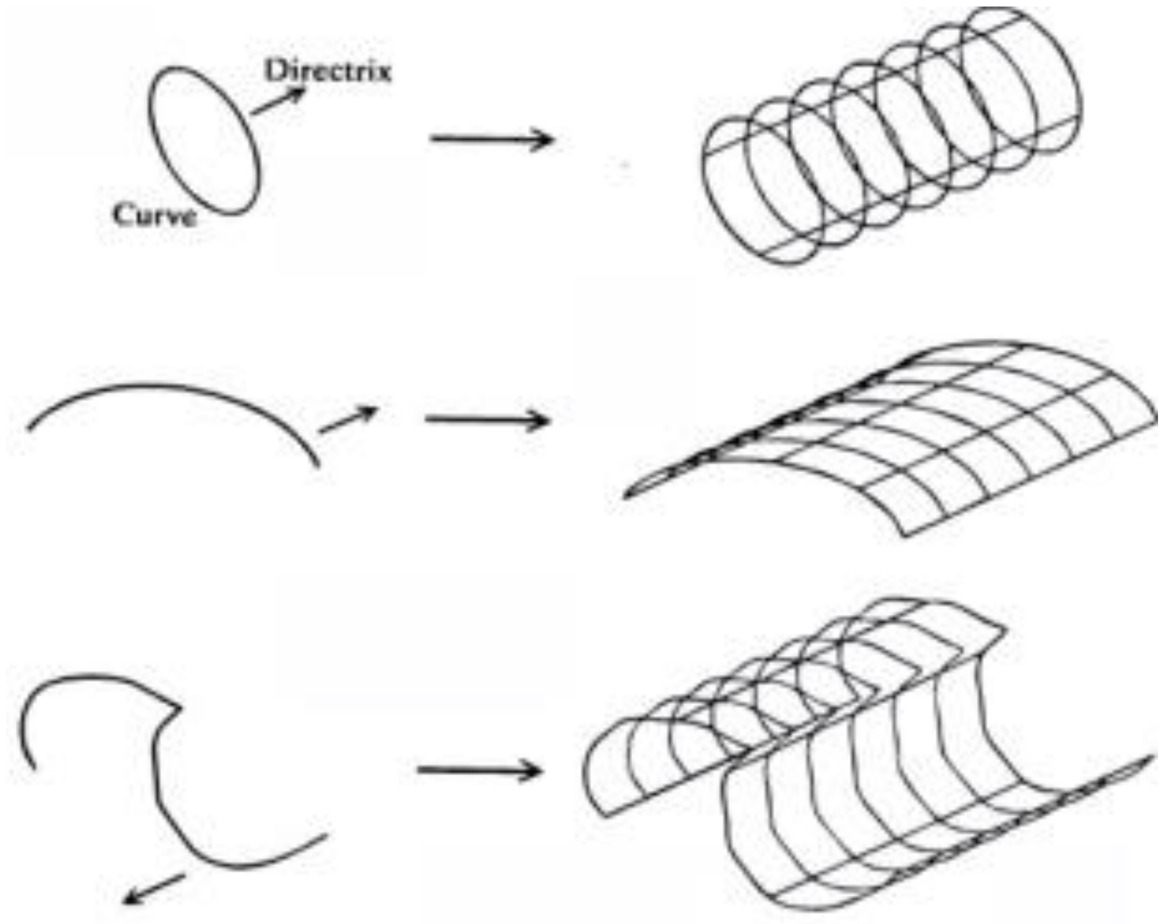
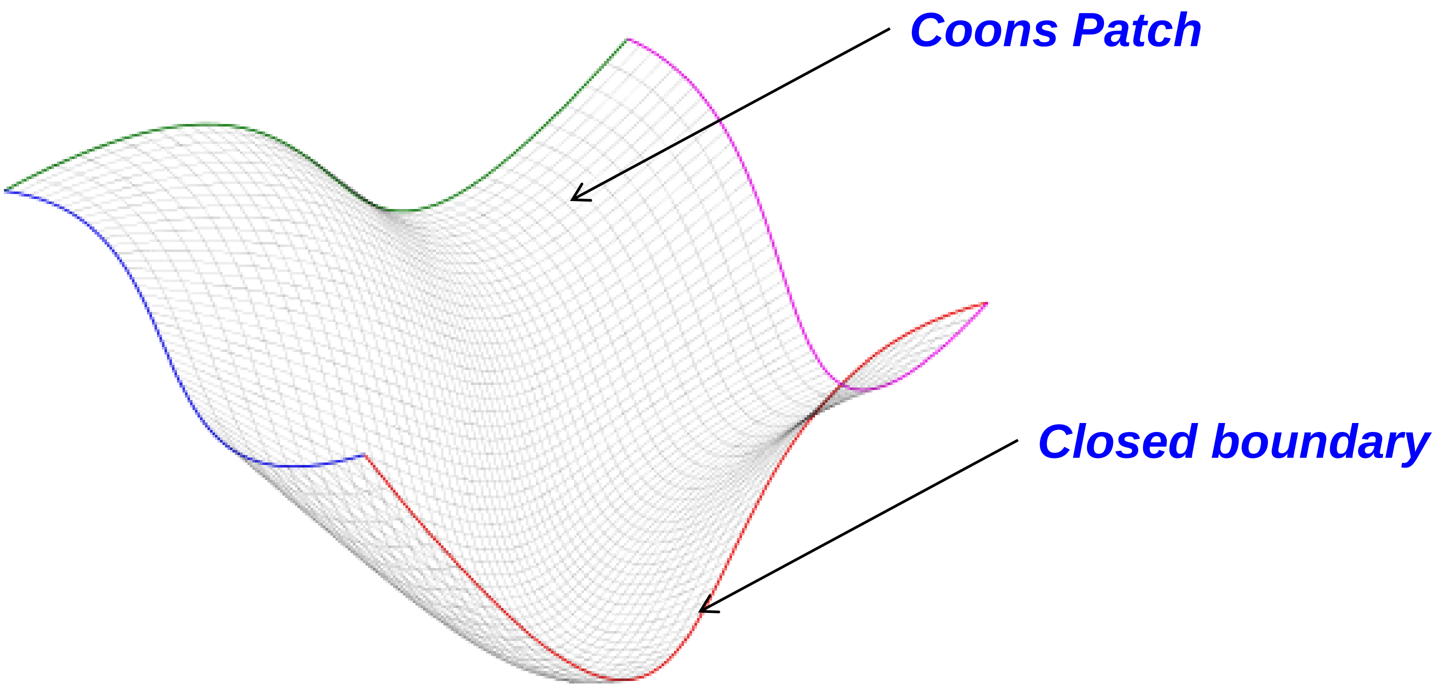
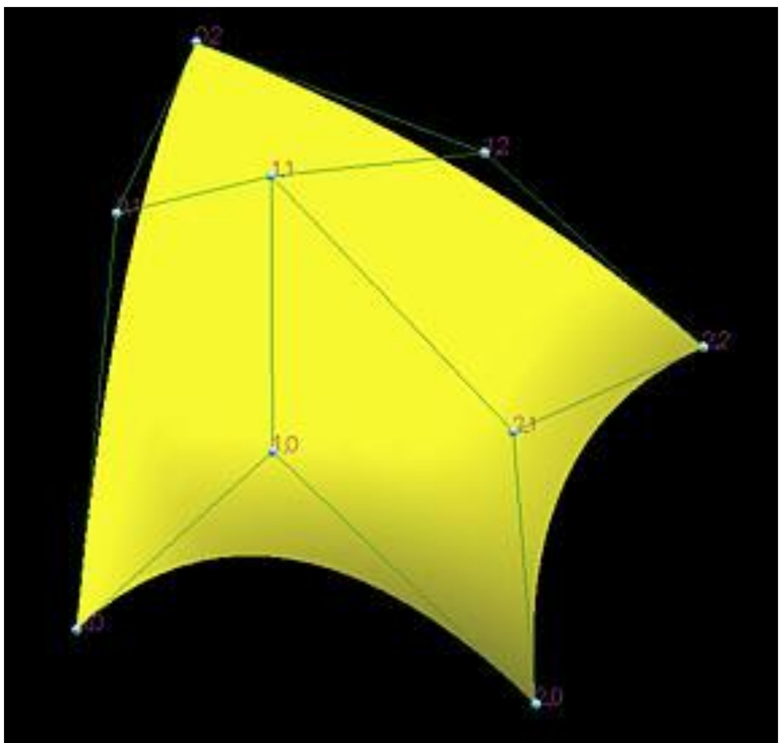


FIGURE 6-7
Tabulated cylinder.

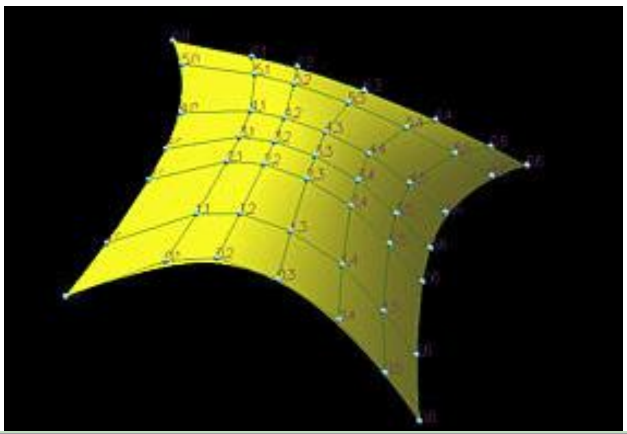
Coons Patch. This is a surface formed by curves which form a close boundary. Thus a set of curved can be used to create a surface.

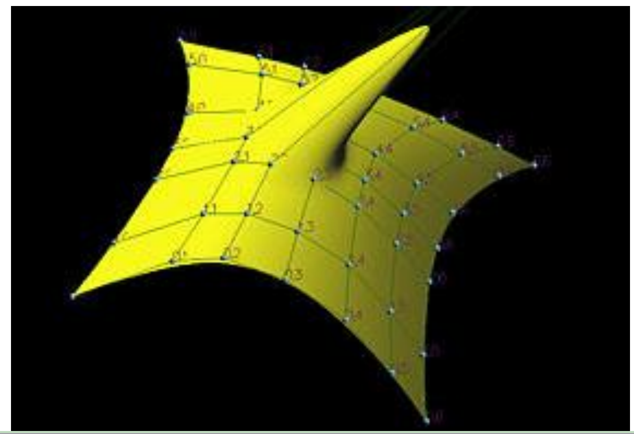


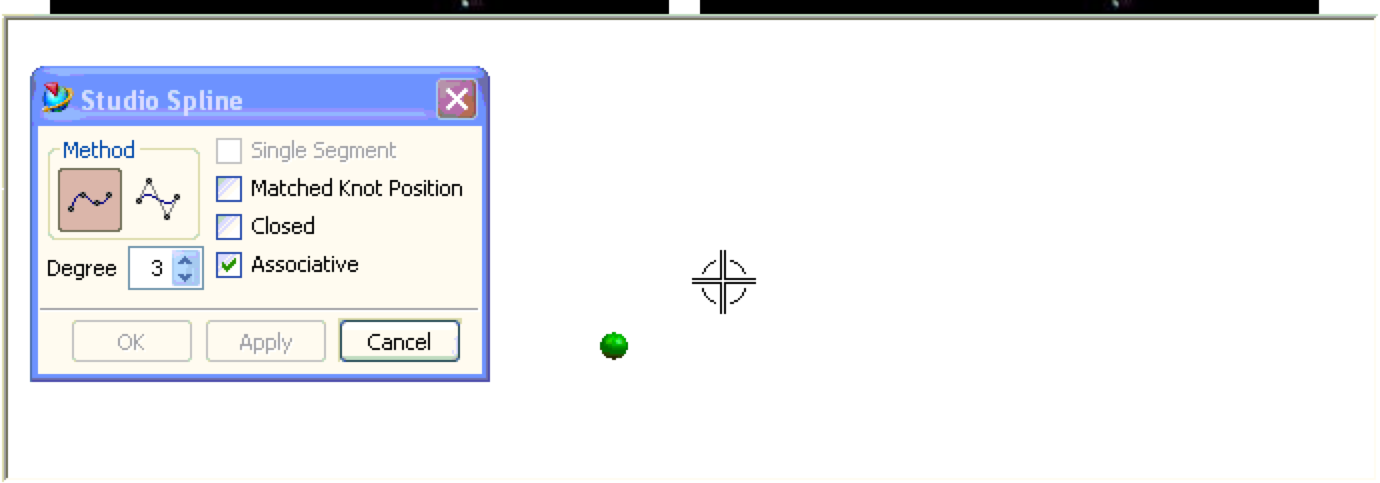
Bezier surface. This is a surface that approximates given input data. It is different from the previous surfaces in that it is a synthetic surface. Similarly to the Bezier curve, it does not pass through all given data points. It is a general surface that permits, twists, and kinks . The Bezier surface allows only global control of the surface.



B-spline surface. This is a surface that can approximate or interpolate given input data. It is a synthetic surface. It is a general surface like the Bezier surface but with the advantage of permitting local control of the surface.







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- G. Farin, J. Hoschek, M.-S. Kim, *Handbook of Computer Aided Geometric Design*, North-Holland (2002)
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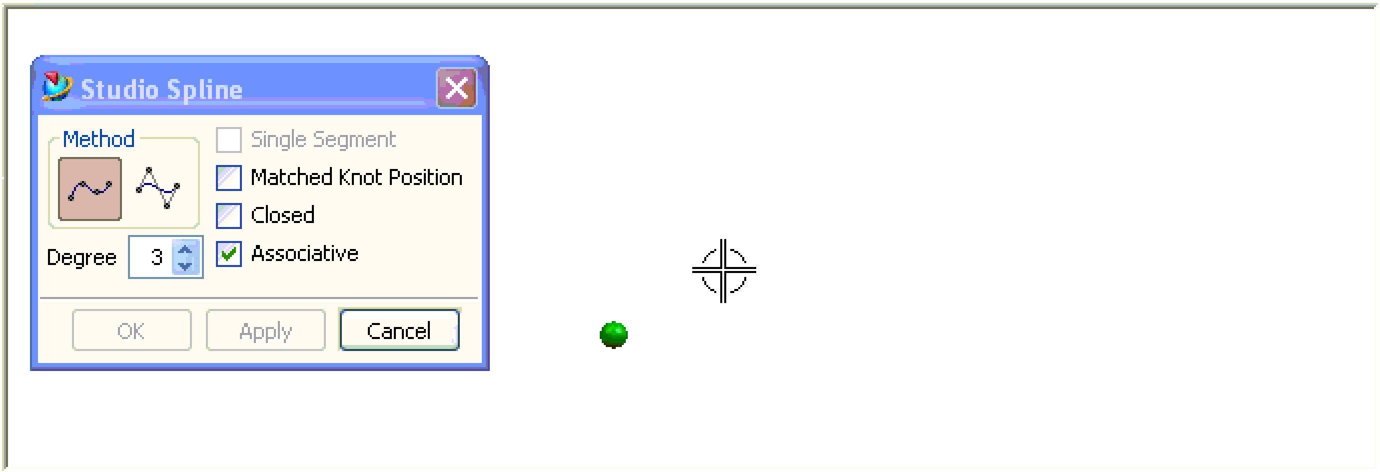
- https://en.wikipedia.org

Thanks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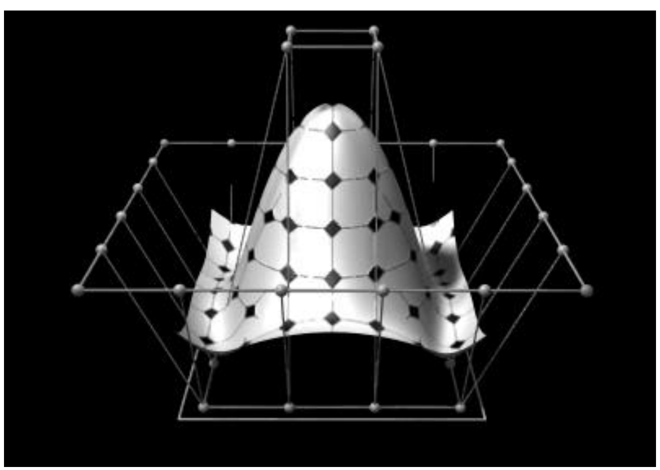


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