LUKHDHIRJI ENGINEERING COLLEGE, MORBI

General Department Sem-2 Subject: Mathematics-2 (3110015)

Assignment-1: Vector Calculus

Q-1: Find the unit vector to the surface $xy^3z^2 = 4$ at $(-1, -1, 2)$.
Ans. $\frac{1}{\sqrt{11}}(-\widehat{i} - 3\widehat{j} + \widehat{k})$

Q-2: Find the parametric representations of the following curves.
(a) $y^2 = 4x$, (b) $x^2 + y^2 = 16$, (c) $\frac{x^2}{4} + \frac{y^2}{9} = 1$.
Ans. (a) $\overrightarrow{r(t)} = \frac{t^2}{4}\widehat{i}+t\widehat{j}$, (b) $\overrightarrow{r(t)} = 4\cos t\widehat{i}+4\sin t\widehat{j}$, (c) $\overrightarrow{r(t)} = 2\cos t\widehat{i}+3\sin t\widehat{j}$.

Q-3: Find the angle between the surfaces $x^2+y^2+z^2 = 9$ and $x^2+y^2-z = 3$ at point $(2, -1, 2)$. **Ans.** $\frac{8}{3\sqrt{21}}$

Q-4: Find the directional derivative of $4xz^3 - 3x^2y^2z$ at the point $(2, -1, 2)$ in the direction $2\widehat{i} + 3\widehat{j} + 6\widehat{k}$. **Ans.** $\frac{648}{7}$

Q-5: Show that $\overrightarrow{F} = (y^2 - z^2 + 3yz - 2x)\widehat{i} + (3xz + 2xy)\widehat{j} + (3xy - 2xz + 2z)\widehat{k}$ is both solenoidal and irrotational.

Q-6: Show that the vector field is given by $\overrightarrow{F} = (y^2 \cos x + z^2)\widehat{i} + (2y \sin x)\widehat{j} + (2xz)\widehat{k}$ is irrotational. Find the scalar function ϕ such that $\overrightarrow{F} = \nabla\phi$.
Ans. $\phi = y^2 \sin x + xz^2 + c$.

Q-7: If $\overrightarrow{F} = (2x + y^2)\widehat{i} + (3y - 4x)\widehat{j}$ then evaluate $\int_c \overrightarrow{F}.dr$ around the parabolic arc $y^2 = x$ joining $(0, 0)$ to $(1, 1)$. **Ans.** $\frac{5}{3}$

Q-8: Find the work done by the force field $\overrightarrow{F} = (3x^2 - 3x)\widehat{i} + 3z\widehat{j} + \widehat{k}$ along the straight line $t\widehat{i} + twidehatj + t\widehat{k} : 0 \leq t \leq 1$. **Ans.** 2

Q-9: Apply Green's theorem to evaluate $\int_c (2x^2 - y^2)dx + (x^2 + y^2)dy$, where c is the boundary of the area enclosed by the axis and the upper half of the circle $x^2 + y^2 = 16$. **Ans.** $\frac{256}{3}$

Q-10: Verify Stoke's theorem for $\overrightarrow{F} = xy^2\widehat{i} + y\widehat{j} + z^2x\widehat{k}$ for the surface of rectangular lamina bounded by $x = 0, y = 0, x = 1, y = 2, z = 0$.

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Assignment-2: Laplace Transform

Q-1: Define Laplace transform and find laplace transform of the following functions
(a) e^{at}, (b) $\cos t$. **Ans.** (a) $\frac{1}{s-a}$, (b) $\frac{s}{s^2+a^2}$.

Q-2: Find the Laplace transform of $f(t) = \begin{cases} 1, & 0 < t < 1 \\ \sin t, & t > 1 \end{cases}$
Ans. $\frac{e^{-s}-1}{-s} + \frac{e^{-s}}{s^2+1}(s\sin 1 + \cos 1)$.

Q-3: Find the Laplace transform of $f(t) = \frac{\sin^2 t}{t^2}$.
Ans. $\frac{1}{4}\Big[-s\log\left(\frac{s^2+4}{s^2}\right) + 4\cos^{-1}\left(\frac{s}{2}\right)\Big]$.

Q-4: Find the Laplace transform of $\frac{d}{dt}\left(\frac{1-\cos t}{t}\right)$. **Ans**. $s\frac{1}{2}\log\left(\frac{s^2+1}{s^2}\right) - 1$.

Q-5: Prove that $\int_0^\infty e^{-2t} t\cos t dt = \frac{3}{25}$.

Q-6: Define inverse Laplace transform and their change of scale property.

Q-7: Find $L^{-1}\left[\frac{e^{-2s}}{(s^2+2)(s^2-3)}\right]$.
Ans. $\left[\frac{1}{5\sqrt{3}}\sin\sqrt{3}(t-2) - \frac{1}{5\sqrt{2}}\sin\sqrt{2}(t-2)\right]u(t-2)$.

Q-8: Find inverse Laplace transform of $\frac{5s+3}{(s-1)(s^2+2s+5)}$.
Ans. $e^t - e^{-t}\cos 2t + \frac{3}{2}e^{-t}\sin 2t$.

Q-9: State the convolution theorem and find inverse Laplace transform of $\frac{1}{(s^2+4)^2}$ by convolution theorem.
Ans. $\frac{1}{16}[\sin 2t - 2t\cos 2t]$.

Q-10: Solve $ty'' + 2y' + ty = \cos t$ given that $y(0) = 1$.
Ans. $y(t) = \frac{\sin t}{t} + \frac{1}{2}\sin t$.

Q-11: Solve $\frac{dx}{dt} + y = \sin t$, $\frac{dy}{dt} + x = \cos t$ given that $x(0) = 0$ and $y(0) = 2$.
Ans. $x(t) = -2\sinh t$ and $y(t) = \sin t + 2\cosh t$.

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Assignment-3: Fourier Integral

Q-1: Define Fourier transform, Fourier sine transform and Fourier cosine transform.

Q-2: Using Fourier integral representation show that

$$\int_0^\infty \frac{\cos\lambda x + \lambda\sin\lambda x}{1+\lambda^2}d\lambda = \begin{cases} 0, & \text{if } x<0 \\ \frac{\pi}{2}, & \text{if } x=0 \\ \pi e^{-x}, & \text{if } x>0. \end{cases}$$

Q-3: Using Fourier integral representation show that

$$\int_0^\infty \frac{2}{1+\lambda^2}\cos\lambda x d\lambda = \begin{cases} 0, & \text{if } x<0 \\ \pi, & \text{if } x=0 \\ \pi e^{-x}, & \text{if } x>0. \end{cases}$$

Q-4: Using Fourier integral representation show that

$$\int_0^\infty \frac{\sin\lambda\pi}{1-\lambda^2}\sin\lambda x d\lambda = \begin{cases} \frac{\pi}{2}\sin x & \text{if } 0\le x\le\pi \\ 0, & \text{if } x>\pi. \end{cases}$$

Q-5: Find the Fourier cosine integral of $f(x)=e^{-kx}$, where $x>0$ and $k>0$.
Ans.

$$f(x)=\frac{2k}{\pi}\int_0^\infty \frac{1}{k^2+\lambda^2}\cos\lambda x d\lambda$$

Q-6: Find Fourier integral representation for the function

$$f(x)=\begin{cases} 0, & \text{if } x<0 \\ e^{-x}, & \text{if } x>0 \\ \frac{1}{2}, & \text{if } x=0. \end{cases}$$

Ans.

$$f(x)=\frac{1}{\pi}\int_0^\infty \frac{\cos\lambda x + \lambda\sin\lambda x}{1+\lambda^2}d\lambda.$$

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Assignment-4: First order ordinary differential equations

Q-1: Define order and degree of differential equations and determine the order and degree of the following differential equations

(a) $\left[1+\frac{dy}{dx}\right]^{\frac{3}{2}} = \frac{d^3y}{dx^3}$, (b) $\frac{dy}{dx} + \frac{5}{\frac{dx}{dy}} = 10.$

Ans. (a) 3, 2 (b) 1, 2

Q-2: Form differential equation whose general solution is $y = ae^{2x} + be^{3x}$.

Ans. $y'' - 5y' + 6y = 0.$

Q-3: Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0.$

Ans. $y\sin x + x\sin y + xy = C.$

Q-4: Solve $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0.$

Ans. $-\frac{1}{3xy} + \frac{2}{3}\log x - \frac{1}{3}\log y = C.$

Q-5: Solve $(x^2y^2 + 2)ydx + x(2 - x^2y^2)dy = 0$

Ans. $\log x - \frac{1}{x^2y^2} = C.$

Q-6: Solve $y\log y dx + (x - \log y)dy = 0.$

Ans. $x\log y - \frac{(\log y)^2}{2} = C.$

Q-7: Solve $(1 + \sin y)\frac{dx}{dy} = 2y\cos y - x(\sec y + \tan y).$

Ans. $x(\sec y + \tan y) - y^2 = C.$

Q-8: Solve $xy - \frac{dy}{dx} = y^3e^{-x^2}.$

Ans. $\frac{e^{x^2}}{y^2} = 2x + C.$

Q-9: Solve $xyP^2 + (3x^2 - 2y^2)P - 6xy = 0.$

Ans. $(y - Cx^2)(y^2 + 3x^2 - C) = 0.$

Q-10: Solve $y = x + a\tan^{-1}P.$

Ans. $x = \frac{a}{2}\left[\log\frac{P-1}{\sqrt{P^2+1}} - \tan^{-1}P\right] + C$ and $y = \frac{a}{2}\left[\log\frac{P-1}{\sqrt{P^2+1}} + \tan^{-1}P\right] + C.$

Q-11: Solve $y = 2Px + y^2P^3.$

Ans. $y^2 = 2Cx - C^3.$

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Assignment-5: Ordinary differential equations of Higher order

Q-1: Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 4y = 0$.
Ans. $y = e^{\frac{3}{2}x}[C_1 \cos \frac{\sqrt{7}}{2}x + C_2 \sin \frac{\sqrt{7}}{2}x$.

Q-2: Solve $y'' + 4y' + 4y = 0$, $y(0) = 0$, $y'(0) = 1$.
Ans. $y = (1 + 3x)e^{-2x}$.

Q-3: Solve $y''' - 6y'' + 11y' - 6y = 0$.
Ans. $y = C_1e^x + C_2e^{2x} + C_3e^{3x}$.

Q-4: Solve $y'' - 3y' + 2y = e^{3x}$. **Ans**. $y = C_1e^x + C_2e^{2x} + \frac{1}{2}e^{3x}$.

Q-5: Solve $y'' + 9y = 2 \sin 3x + \cos 3x$.
Ans. $y = C_1 \cos 3x + C_2 \sin 3x - \frac{x}{3} \cos 3x + \frac{x}{6} \sin 3x$.

Q-6: Solve $y'' + 16y = x^4 + e^{3x} + \cos 2x$.
Ans. $y = C_1 \cos 4x + C_2 \sin 4x + \frac{1}{16}\left[x^4 - \frac{3x^2}{4} + \frac{3}{32}\right] + \frac{1}{25}e^{3x} + \frac{1}{25} \cos 2x$.

Q-7: Solve $y''' - 2y' + 4y = e^{-4x} \cos x$.
Ans. $y = C_1e^{-2x} + e^x(C_2 \cos x + C_3 \sin x) - \frac{1}{20}xe^x(-3 \sin x + \cos x)$.

Q-8: Solve $y'' - 2y' + y = xe^x \cos x$.
Ans. $y = (C_1 + C_2x)e^x + e^x[-x \cos x + 2 \sin x]$.

Q-9: Solve $y'' + 3y' + 2y = e^{x^x}$.
Ans. $y = C_1e^{-x} + (C_2e^{-2x} + e^{x^x}e^{-2x}$.

Q-10: Solve $(3x + 2)^2y'' - (3x + 2)y' - 12y = 6x$.
Ans. $y = C_1(3x + 2)^2 + C_2(3x + 2)^{-\frac{2}{3}} + \frac{1}{3} - \frac{2}{15}(3x + 2)$.

Q-11: Use method of variation of parameters to solve $y'' + a^2y = \sec ax$.
Ans. $y = C_1 \cos ax + C_2 \sin ax - \frac{\cos ax}{a^2} \log \sec ax + \frac{x}{a} \sin ax$.

Q-12: Which of the following are linearly independent.
(a) $1 + x$, $1 + 2x$, x^2 (b) 3^x, 3^{x+2}.
Ans. (a) Linearly independent, (b) Linearly independent.

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Assignment-6: Series solutions of ODE and special functions

Q-1: Solve by power series method $\frac{d^2y}{dx^2} + x^2y = 0$.
Ans. $y = a_0\left(1 - \frac{x^4}{3.4} + \frac{x^8}{3.4.7.8} + ...\right) + a_1\left(x - \frac{x^5}{4.5} + \frac{x^9}{4.5.8.9} + ...\right)$.

Q-2:Solve by Frobenius method $4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$.
Ans. $y = A\left(1 - \frac{x}{2} + \frac{x^2}{24} + ...\right) + B\sqrt{x}\left(1 - \frac{x}{6} + \frac{x^2}{120} - .....\right)$.

Q-3: Write and find series (Frobenius) solution of Bessel's differential equation.

Q-4: Prove the following recurrence formulae
$(a)\ xJ_n' = nJ_n - xJ_{n+1}$, $(b)\ xJ_n' = -nJ_n + xJ_{n-1}$.

Q-5: Prove the following relations
$(a)\ J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$, $(b)\ J_{-\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{-\cos x}{x} - \sin x\right)$.

Q-6: Write and find series (Frobenius) solution of Legendre's differential equation.

Q-7: Prove that $P_n(x)$ is the coefficient of z^n in the expansion of $(1 - 2xz + z^2)^{-\frac{1}{2}}$ in ascending powers of z, where $|x| \leq 1$, $|z| < 1$.

Q-8: Prove the following recurrence formulae
$(a)\ (2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$, $(b)\ nP_n = (2n-1)xP_{n-1} - (n-1)P_{n-2}$.

Q-9: State and prove Rodrigue formula.

Q-10: Prove that $\int_{-1}^{1} P_m(x)P_n(x) = \begin{cases} 0, & \text{if} \quad m \neq n \\ \frac{2}{2n+1}, & \text{if} \quad m = n \end{cases}$.

Q-11: Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials .
Ans. $f(x) = \frac{8}{35}P_4(x) + \frac{6}{5}P_3(x) - \frac{2}{21}P_2(x) + \frac{34}{5}P_1(x) - \frac{434}{105}P_0(x)$.