Let $m = 320$ Kg $\Delta st = 0.002$ m $m_0 = 24$ Kg.

$$e = \frac{0.15}{2} = 0.075 \text{ m}.$$

$$\omega_n = \sqrt{\frac{g}{\Delta st}} = \sqrt{\frac{9.81}{0.002}} = 70 \text{ rad/s}.$$

resonance speed $= \frac{70}{2\pi} \times 60 = 670$ rpm

$$\omega = \frac{480 \times 2\pi}{60} = 50.4 \text{ rad/sec}.$$

$$\text{so } \left(\frac{\omega}{\omega_n}\right) = \frac{50.4}{70} = 0.72$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{490/0.3}{2 \times 320 \times 70} = 0.0364$$

$$\frac{m_0}{m} = \frac{24}{320} = 0.075$$

$$\text{Now } \frac{X}{\left(\frac{m_0 e}{m}\right)} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

$$\Rightarrow \frac{X}{0.075 \times .075} = \frac{(0.72)^2}{\sqrt{(1-.72)^2 + (2 \times .0364 \times .72)^2}}$$

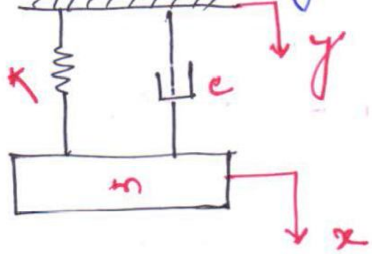
$$\Rightarrow \boxed{X = 0.006 \text{ m or } 6 \text{ mm}}$$

forced vibration due to base excitation:-

In most of the vibration related problems, a system is being excited by motion of the support. for example a vehicle is travelling on a wavy road, an engine mounted on a vibrating system etc.

- In this case the support is considered to be excited by a regular sinusoidal motion,

$$y = Y \sin \omega t \quad \text{---(1)}$$



Considering a spring-mass damper system the mass is attached with the support by means of a spring of stiffness K, a damper of damping coefficient c.

Absolute Amplitude :-

Let x = absolute motion of mass m.

Equation of motion for the system may be written as :

$$m\ddot{x} + c(\dot{x} - \dot{y}) + K(x - y) = 0$$

or $$m\ddot{x} + c\dot{x} + Kx = Ky + c\dot{y} \quad \text{--- (2)}$$

We have $y = Y \sin \omega t$

or $\dot{y} = \omega Y \cos \omega t$

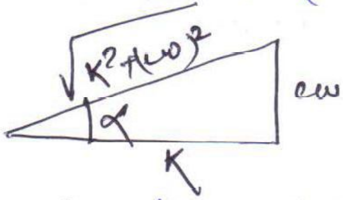
Substituting the value of y and $\dot{y}$ in eq. (2), we have

$$m\ddot{x} + c\dot{x} + Kx = KY \sin \omega t + c\omega Y \cos \omega t$$

or $$m\ddot{x} + c\dot{x} + Kx = Y\left[K \sin \omega t + c\omega \cos \omega t\right]$$

or $$m\ddot{x} + c\dot{x} + Kx = Y\sqrt{K^2 + (c\omega)^2}\left[\frac{K}{\sqrt{K^2 + (c\omega)^2}} \sin \omega t + \frac{c\omega}{\sqrt{K^2 + (c\omega)^2}} \cos \omega t\right]$$

or $$m\ddot{x} + c\dot{x} + Kx = Y\sqrt{K^2 + (c\omega)^2}\left[\cos\alpha \sin \omega t + \sin\alpha \cos \omega t\right]$$



or $$m\ddot{x} + c\dot{x} + Kx = Y\sqrt{K^2 + (c\omega)^2} \sin(\omega t + \alpha) \quad \text{--- (3)}$$

where $$\alpha = \tan^{-1}\left(\frac{c\omega}{K}\right) = \tan^{-1}\left(2\xi\frac{\omega}{\omega_n}\right) \quad \text{--- (4)}$$

Equation (3) is same as that of the equation of forced vibration with harmonic excitation

$$m\ddot{x} + c\dot{x} + Kx = F_0 \sin \omega t.$$

Therefore the steady-state solution of eq. (3) is

$$x = X \sin(\omega t - \phi) \quad \text{--- (5)}$$

where X = steady state amplitude

and $$X = \frac{Y\sqrt{K^2 + (c\omega)^2}}{\sqrt{(K - m\omega^2)^2 + (c\omega)^2}}$$

$\left[\because \text{same as forced vib. eq. } X = \dfrac{F_0}{\sqrt{(K - m\omega^2)^2 + (c\omega)^2}}\right]$

In a dimensionless form

$$\boxed{\frac{X}{Y} = \frac{\sqrt{1 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}} \quad \text{--- (6)}$$

and $$\boxed{\phi = \tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right]} \quad \text{--- (7)}$$

Comparing Eq.(1) and (5), it can be seen that the motion of mass 'm' lags that of the support through an angle $(\phi - \alpha)$.

Therefore the angle of lag $(\phi - \alpha)$.

$$\boxed{= \tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right] - \tan^{-1}\left[2\xi\frac{\omega}{\omega_n}\right]} \quad \text{— (8)}$$

Equation (5), (6) and (8) completely define the absolute motion of mass 'm' because of base excitation,

Relative amplitude :-

Let z = relative motion of mass m wrt the support

then $z = x - y$

or $x = y + z$

We have the eq. of motion of mass for an absolute amplitude case is

$$m\ddot{x} + c\dot{x} + Kx = Ky + c\dot{y}$$

or $m(\ddot{y}+\ddot{z}) + c(\dot{y}+\dot{z}) + K(y+z) = Ky + c\dot{y}$

or $m\ddot{z} + c\dot{z} + Kz = -m\ddot{y}$ — (9)

The base is excited by a regular sinusoidal equation

$y = Y\sin\omega t$

so $\dot{y} = \omega Y\cos\omega t$

$\ddot{y} = -\omega^2 Y\sin\omega t$

substituting the value of $\ddot{y}$ in eq. (9)

$$m\ddot{z} + c\dot{z} + Kz = m\omega^2 Y\sin\omega t \quad \text{— (10)}$$

Eq.(10) is same as that of equation of forced vibration with rotating unbalance

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + Kx = m_0 e\omega^2\sin\omega t$$

with a solution $X = \dfrac{(m_0 e\omega^2/K)}{\sqrt{\left(1-\frac{m\omega^2}{K}\right) + \left(\frac{c\omega}{K}\right)^2}}$

and therefore the solution in a dimensionless form

$$\frac{Y}{Z} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}} \qquad \text{--- (11)}$$

$$\text{and} \quad \phi = \tan^{-1}\left[\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right] \qquad \text{--- (12)}$$

Example-3

The support of a spring-mass system is vibrating with an amplitude of 5mm and a frequency of 1150 cycle/min. If the mass is 0.9 Kg and spring stiffness of 1960 N/m, determine the amplitude of vibration of mass. What amplitude will the amplitude of vibration of mass. What amplitude will result if a damping factor of 0.2 is included in the system?

Given data:-

Mass $m = 0.9$ Kg, $Y = 5$ mm, $K = 1960$ N/m

frequency = 1150 cycle/min = $1150 \times \frac{2\pi}{60} = 120.3$ rad/s

Now $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{1960}{0.9}} = 46.7$ rad/sec.

$$\frac{\omega}{\omega_n} = \frac{120.3}{46.7} = 2.58$$

The equation for base excitation for absolute amplitude

$$\frac{X}{Y} = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

for $\xi = 0$

$$\frac{X}{5} = \left|\frac{1}{[1-2.58^2]}\right| = \frac{1}{5.65}$$

$$\Rightarrow \boxed{X = 0.886 \text{ mm}}$$

for $\xi = 0.2$

$$\frac{X}{5} = \frac{\sqrt{1+(2\times0.2\times2.58)^2}}{\sqrt{(1-2.58^2)^2+(2\times0.2\times2.58)^2}} =$$

$$\Rightarrow \boxed{X = 1.25 \text{ mm}} \quad \text{(Ans)}$$

Observation from the $\frac{X}{\left(\frac{m_0 e}{m}\right)}$ vs $\left(\frac{\omega}{\omega_n}\right)$ plot :-

1. All the curves begin at zero amplitude,
2. At resonance, the amplitude of vibration is given by $\frac{X}{\left(\frac{m_0 e}{m}\right)} = \frac{1}{2\zeta}$, which indicates that the damping factor plays important role in controlling the vibration amplitude at resonance.
3. At very high speeds, $\frac{X}{\left(\frac{m_0 e}{m}\right)}$ tends to unity and damping has negligible effect.
4. for $0 < \zeta < \frac{1}{\sqrt{2}}$, the peak occur to the right of the resonance value of $\frac{\omega}{\omega_n} = 1$.

Vibration Isolation and Transmissibility :-

Most of the machines when mounted or installed on the foundations, cause undesirable vibrations because of unbalanced forces set up during their running. The vibration of large amplitude may damage the structure on which machines are mounted.

- Examples of these undesirable vibration cases are:
- inertia forces developed in reciprocating engine
- unbalanced force produced in any rotating m/c etc.

The effectiveness of isolation may be measured in terms of the ratio of force or motion transmitted to that in existence. The first type is called
- force isolation and the second one is called
- motion isolation.

- The lesser the force or motion transmitted, the greater is said to be the isolation.
- for isolation different materials are used such as

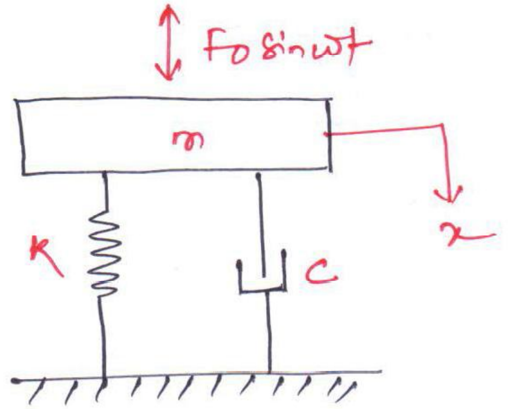
- pads of rubber
- felt or cork
- metallic spring etc.

- All these isolating materials are elastic and have damping properties.

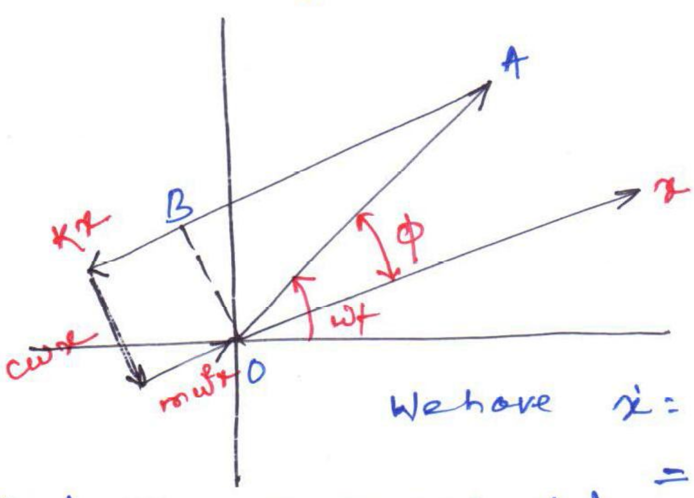
Force Transmissibility :-

Force transmissibility is defined as the ratio of

$$\boxed{\frac{\text{force transmitted to the foundation}}{\text{force Impressed on the system.}}}$$



considering a case, where a mass m is supported on the foundation by means of an isolator having equivalent stiffness and damping coefficient of K and C respectively. The system is excited by a force = $f_0 \sin\omega t$



The differential equation of motion is

$$m\ddot{x} + c\dot{x} + Kx = f_0 \sin\omega t \quad -(1)$$

Assuming a particular solution of eq.(1)

$$x = X \sin(\omega t - \phi) \quad -(2)$$

We have $\dot{x} = \omega X \cos(\omega t - \phi)$

$$= \omega X \sin(\omega t - \phi + \pi/2) \quad -(3)$$

And $\ddot{x} = -\omega^2 X \sin(\omega t - \phi)$

$$= \omega^2 X \sin(\omega t - \phi + \pi) \quad -(4)$$

substituting the value of x, $\dot{x}$ and $\ddot{x}$ in eq.(1)

$$m\omega^2 X \sin(\omega t - \phi + \pi) + c\omega X \sin(\omega t - \phi + \pi/2) + K X \sin(\omega t - \phi) = f_0 \sin \omega t$$

or $$f_0 \sin\omega t - KX \sin(\omega t - \phi) - c\omega X \sin(\omega t - \phi + \pi/2) - m\omega^2 X \sin(\omega t - \phi + \pi) = 0 \quad \text{— (5)}$$

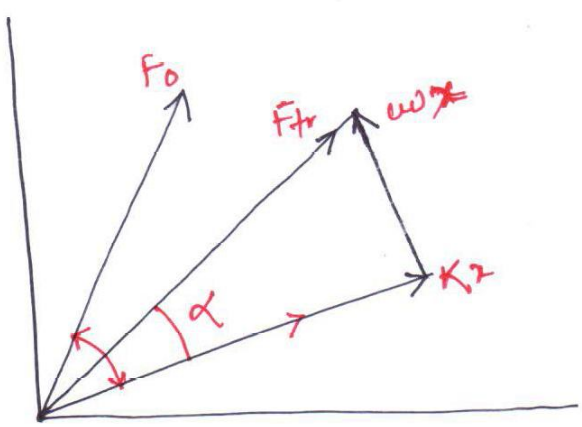
Total forces acting on the system are

1. External excitation force
2. Spring force
3. Dashpot force.
4. Inertial force.

Out of these four forces, the spring force KX and dashpot force $c\omega X$ are two common forces acting on the mass and on the foundation. Therefore the force transmitted to the foundation is the vector sum of these two forces.

Therefore $$F_{tr} = \sqrt{(KX)^2 + (c\omega X)^2}$$

$$\Rightarrow F_{tr} = X\sqrt{K^2 + (c\omega)^2} \quad \text{— (6)}$$



from the vector diagram, to find the value of X and ϕ in eq.(2) consider a ⊥r triangle OAB by dropping OB to AB

Now $$F_0 = \sqrt{(KX - m\omega^2 X)^2 + (c\omega X)^2}$$
$$= X\sqrt{(K - m\omega^2)^2 + (c\omega)^2}$$

$$\Rightarrow X = \frac{f_0}{\sqrt{(K - m\omega^2)^2 + (c\omega)^2}} \quad \text{— (7)}$$

and $$\phi = \tan^{-1}\left[\frac{c\omega}{K - m\omega^2}\right] \quad \text{— (8)}$$

substituting the value of X in eq. (6)

Force transmitted $F_{Tr} = \dfrac{F_0\sqrt{K^2+(c\omega)^2}}{\sqrt{(K-m\omega^2)^2+(c\omega)^2}}$ — (9)

Eq. (9) can be represented as a dimensionless form

transmissibility $\boxed{T_r = \dfrac{F_{Tr}}{F_0} = \dfrac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}}$ — (10)

The angle through which the transmitted force lags the impressed force is $(\phi-\alpha)$

Where $\alpha = \tan^{-1}\left(\dfrac{c\omega X}{KX}\right) = \tan^{-1}\left(\dfrac{c\omega}{K}\right)$

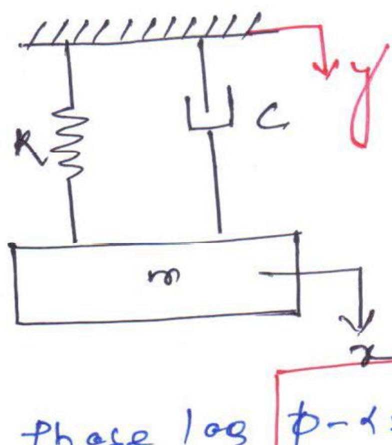
$\Rightarrow \alpha = \tan^{-1}\left(2\xi\dfrac{\omega}{\omega_n}\right)$

and angle $\phi = \tan^{-1}\left[\dfrac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$

so phase lag $\boxed{\phi-\alpha = \tan^{-1}\left[\dfrac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]-\tan^{-1}\left(2\xi\dfrac{\omega}{\omega_n}\right)}$

— (11)

Motion Transmissibility :-



Motion transmissibility

$\boxed{\dfrac{X}{Y} = \dfrac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}}$ — (12)

$\phi = \tan^{-1}\left[\dfrac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$ — (13)

Phase lag $\boxed{\phi-\alpha = \tan^{-1}\left[\dfrac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]-\tan^{-1}\left[2\xi\dfrac{\omega}{\omega_n}\right]}$ — (14)

Typical Isolators used:-

- Coil springs
- elastometers (rubber and neoprene)

Coil springs / steel springs:-

These are generally used for $f_n < 6$ Hz and $\Delta st > 7.5$ mm

Large coil diameter is chosen for larger deflection

Pad Mounts:-

Ribbed neoprene mounts are used for small static deflection. They can be used in a series for a total maximum static deflection of about 4 mm. They are generally used for printing machinery, saws, transformers, vacuum pumps wood working machinery etc.

General purpose Elastometric mounts:-

They are used in compression/shear, for static deflection from 2 mm to 16 mm corresponding to natural frequencies from 11 Hz to 4 Hz. They are used with great variety of machines including blowers, fans, pumps, bending m/cs diesel engine, motor generator sets etc.

Example-1

A 1000 kg machine is mounted on four identical springs of total spring constant K and having negligible damping. The machine is subjected to a harmonic external force of amplitude $F_0 = 490$ N and frequency 180 rpm. Determine (a) the amplitude of motion of the machine and maximum force transmitted to foundation because of the unbalanced force when $K = 1.96 \times 10^6$ N/m. (b) the same as in (a) for the case when $K = 9.8 \times 10^4$ N/m

(a) $K = 1.96\times10^6\ N/m \quad m = 1000\ Kg.$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{1.96\times10^6}{1000}} = 44.3\ rad/sec.$$

$$\frac{\omega}{\omega_n} = \frac{180\times2\pi}{60\times44.3} = 0.425$$

$F_0 = 490N \quad \xi = 0$

$$X_{st} = \frac{F_0}{K} = \frac{490}{1.96\times10^6} = 2.5\times10^{-4}\ m.$$

Amplitude

$$\frac{X}{(X_{st})} = \frac{1}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\} + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

As $\xi = 0$

$$\frac{X}{(X_{st})} = \left|\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right| = \frac{1}{\left|1-0.425^2\right|} = \boxed{0.305\ mm}$$

Transmitted force

$$\frac{F_{tr}}{490} = \frac{1}{(1-0.425^2)} \Rightarrow F_{tr} = \frac{490}{0.819} = \boxed{607\ N.}$$

(b) $K = 9.8\times10^4\ N/m. \quad m = 1000\ Kg.$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{9.8\times10^4}{1000}} = 9.9\ rad/sec.$$

$$\frac{\omega}{\omega_n} = \frac{180\times2\pi}{60\times9.9} = 1.90$$

$F_0 = 490\ N, \quad \xi = 0$

$$X_{st} = \frac{F_0}{K} = \frac{490}{9.8\times10^4} = 0.005\ m.$$

Now using the relation

$$\frac{X}{(X_{st})} = \frac{1}{\left|1-1.90^2\right|} = \frac{1}{2.61}$$

$\Rightarrow$ Amplitude $\boxed{X = 1.9\ mm}$

Transmitted force $\frac{F_{tr}}{490} = \frac{1}{\left|1-1.90^2\right|} \Rightarrow F_{tr} = \frac{490}{2.61} = \boxed{188N.}$

Example-2

A 75Kg machine is mounted on springs of stiffness K = 11.76×10^5 N/m with an assumed damping factor of $\xi = 0.20$. A 2Kg piston within the machine has a reciprocating motion with a stroke of 0.08 m and a speed of 3000 rpm. Assuming the motion of the piston to be harmonic, determine the amplitude of vibration of the machine and the vibrating force transmitted to the foundation.

Given data:-

mass of m/c $m = 75Kg$. spring stiffness $K = 11.76\times10^5 N/m$

damping factor $\xi = 0.2$ equivalent unbalanced mass $m_0 = 2 Kg$.

$e = \frac{0.08}{2} = 0.04 m$ speed = 3000 rpm

$\omega = \frac{3000\times2\pi}{60} = 100\pi$ rad/sec.

$\left(\frac{\omega}{\omega_n}\right) = \left(\frac{100\pi}{\omega_n}\right)$

Now $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{11.76\times10^5}{75}} = 125$ rad/sec.

so $\left(\frac{\omega}{\omega_n}\right) = \frac{100\pi}{125} = 2.51$

$\frac{m_0 e}{m} = \frac{2\times0.04}{75} = 10.67\times10^{-4}$ m.

$F_0 = m_0 e \omega^2 = 2\times0.04\times(100\pi)^2 = 7900 N.$

Now using the relation

$$\frac{X}{\left(\frac{m_0 e}{m}\right)} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\Rightarrow \frac{X}{10.67\times10^{-4}} = \frac{2.51^2}{\sqrt{(1-2.51^2)^2+(2\times0.2\times2.51)^2}}$$

$\Rightarrow \boxed{X = 1.25 mm}$

Transmitted force

We have

$$\frac{F_{Tr}}{F_0} = \frac{\sqrt{1+\{2\xi(\frac{\omega}{\omega_n})\}^2}}{\sqrt{\{1-(\frac{\omega}{\omega_n})^2\}^2+(2\xi\frac{\omega}{\omega_n})^2}}$$

$$\Rightarrow \frac{F_{Tr}}{7900} = \frac{\sqrt{1+(2\times0.2\times2.51)^2}}{\sqrt{(1-2.51^2)^2+(2\times0.2\times2.51)^2}}$$

$$\Rightarrow \boxed{F_{Tr} = 2078N.}$$ (Ans).

Example. 3

A radioset of 20kg mass must be isoted from a machine vibrating with an amplitude of 0.05 mm at 500 rpm. A set is mounted on four isolators, each having a spring scale of 31400 N/m and damping factor of 392 N-s/m

(a) What is the amplitude of vibration of the radio?

(b) What is the dynamic load on each isolator due to vibration?

Let m ~~[illegible]~~ = mass of radio set

K = equivalent spring stiffness

c = damping coefficient of the four isolator

$m = 20kg \quad K = 4\times31400 = 125600\ N/m$

$C = 4\times392 = 1568\ NS/m$

$y = Y\sin\omega t$ and $Y = 0.05\ mm$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times500}{60} = 52.5\ rad/sec.$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{125600}{20}} = 79.2\ rad/sec$$

$$\left(\frac{\omega}{\omega_n}\right) = \frac{52.5}{79.2} = 0.662$$

$$\xi = \frac{c}{2\sqrt{Km}} = \frac{1568}{2\sqrt{125600\times20}} = \boxed{0.496}$$

(a) Amplitude of vibration of radio set

$$\frac{X}{Y} = \frac{\sqrt{1+(2\times\xi\frac{\omega}{\omega_n})^2}}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2+\{2\times\xi\times(\frac{\omega}{\omega_n})\}^2}}$$

$$\frac{X}{0.05} = \frac{\sqrt{1+(2\times0.496\times0.662)^2}}{\sqrt{(1-0.662)^2+(2\times0.496\times.662)^2}}$$

$$\Rightarrow \boxed{X = 0.069\,mm} \quad (Ans)$$

(b) The dynamic load on isolators due to vibration can be obtained by finding the relative Z amplitude and then

$$F_{Dyn} = Z\sqrt{K^2+(c\omega)^2}$$

Now using the relation

$$\frac{Z}{Y} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\times\zeta\times\frac{\omega}{\omega_n}\right)^2}}$$

$$\Rightarrow \frac{Z}{0.05} = \frac{(0.662)^2}{\sqrt{(1-0.662^2)^2+(2\times0.496\times0.662)^2}}$$

$$\Rightarrow Z = 0.025\,mm \text{ or } 2.5\times10^{-5}\,m$$

Now
$$F_{Dyn} = Z\sqrt{K^2+(c\omega)^2}$$
$$= 2.5\times10^{-5}\sqrt{125600^2+(1568\times52.5)^2}$$
$$= 3.77\,N.$$

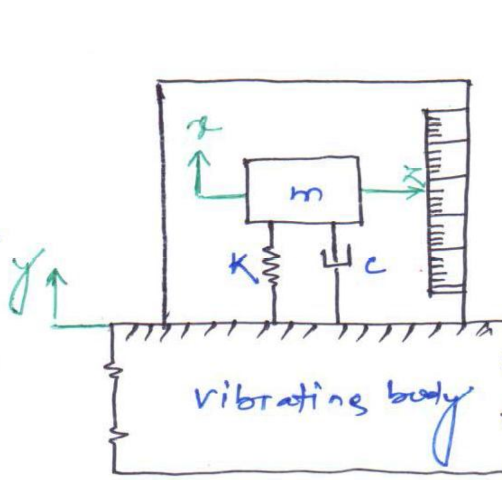
So the dynamic load on each isolator $= \frac{3.77}{4} = 0.94\,N.$

Vibration Measuring Instruments:-

The instrument used to measure any of the vibration related phenomenon i.e. displacement, velocity or acceleration of a vibrating system are referred to as vibration measuring instrument

The basic elements of most of the vibration measuring instrument is the seismic unit shown in the figure.

- It consists of a seismic mass m mounted on a spring K and dashpot c inside a box. The box is then placed on the vibrating machine or body.



The arrangement is similar to the spring-mass-dashpot system having support. The displacement of the mass relat to the box i.e. 'Z' can be measured by attaching a pointer to the mass and a scale to the box.

Vibrometer :- (Displacement Measuring Instrument) :-

Vibrometer is a used to measure the displacement of a vibrating body.

Considering the equation

$$\boxed{\frac{Z}{Y} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left\{2\xi\left(\frac{\omega}{\omega_n}\right)\right\}^2}}} \quad \text{--- (1)}$$

When the natural frequency of the instrument is low in comparision to vibrating frequency ω, the relative displacement approaches the amplitude of vibrating body irrespective of damping in the instrument

If $\frac{\omega}{\omega_n} \gg 1$, then eq. (1) may be written as

$$\boxed{\frac{Z}{Y} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2}} \simeq 1} \qquad \boxed{\because 1-\left(\frac{\omega}{\omega_n}\right)^2 \simeq \left(\frac{\omega}{\omega_n}\right)^2 \text{ for } \frac{\omega}{\omega_n} \gg 1}$$

$$\Rightarrow \boxed{Z \simeq Y} \quad \text{--- (2)}$$

- Thus, when $\frac{\omega}{\omega_n}$ is large, amplitude recorded is approximately equal to the amplitude of vibrating body. In most of the vibrometers, damping is kept as small as possible.
- Vibrometer are therefore known as low natural frequency instruments. The average value of natural frequency, ω_n for vibrometer is about 4 Hz.

Example-1

A vibrometer has a period of free vibration of 2 seconds. It is attached to a machine with a vertical harmonic frequency of 1 Hz. If the vibrometer mass has an amplitude of 2.5mm relative to the vibrometer frame what is the amplitude of vibration of machine ?

time period $\tau = 2$ sec $\quad Z = 2.5$ mm

$\omega = 1 \times 2\pi = 2\pi$ rad/sec.

Natural frequency $\omega_n = \frac{2\pi}{\tau} = \frac{2\pi}{2} = \pi$ rad/sec.

$\xi = 0$, for vibrometers.

Now using the relation

$$\frac{Z}{Y} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \cancel{\left(2\xi\frac{\omega}{\omega_n}\right)^2}}}$$

$$\Rightarrow \frac{Z}{Y} = \frac{(\omega/\omega_n)^2}{\sqrt{[1-(\omega/\omega_n)^2]^2}}$$

$$\Rightarrow \frac{2.5}{Y} = \frac{2^2}{\sqrt{(1-2^2)^2}}$$

$$\Rightarrow Y = \frac{2.5 \times \sqrt{(1-2^2)^2}}{2^2} = \boxed{1.875 \text{ mm}}$$

which is the amplitude of vibration of support of m/c in this case.

Example-2

A seismic instrument having natural frequency of 5Hz is used to measure the vibration of a machine operating at 110 rpm. The relative displacement of seismic mass as read from the instrument is 0.02m. Determine the amplitude of vibration of the machine. Neglect damping.

Data given:-

$f_n = 5$Hz, $N = 110$ rpm, $Z = 0.02$ m $\quad \xi = 0$.

Now $\omega_n = 2\pi f_n = 10\pi$ rad/sec. , $\omega = \frac{2\pi N}{60} = 11.52$ rad/s.

For a vibrometer, the governing equation is

$$\frac{Z}{Y} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

Neglecting damping, we have

$$\frac{Z}{Y} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2}}$$

$$\Rightarrow \frac{0.02}{Y} = \frac{\left(\frac{11.52}{31.416}\right)^2}{\left[1-\left(\frac{11.52}{31.416}\right)^2\right]}$$

$$\Rightarrow \boxed{Y = 0.129\,m}$$

Example - 3

A commericial vibrometer having amplitude of vibration of the m/c part as 5mm and damping factor $\xi = 0.2$, performs harmonic motion. If the difference between the max and minimum recorded value is 12 mm and the frequency of vibrating part is 15 rad/sec, find out the natural frequency of vibrometer.

Given data:-

$Y = 5\,mm$ $\quad \xi = 0.2$ $\quad Z = \frac{12}{2} = 6\,mm$ $\quad \omega = 15\,rad/sec$

Using the relation

$$\frac{Z}{Y} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\Rightarrow \left(\frac{6}{5}\right)^2 = \frac{(\omega/\omega_n)^4}{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(2\times 0.2\times\frac{\omega}{\omega_n}\right)^2}$$

$$\Rightarrow 1.44 = \frac{(\omega/\omega_n)^4}{(1-\omega^2/\omega_n^2)^2 + (0.4\,\omega/\omega_n)^2}$$

$$\Rightarrow 1.44 + 1.44\frac{\omega^4}{\omega_n^4} - 1.84\frac{\omega^2}{\omega_n^2} = \frac{\omega^4}{\omega_n^4}$$

$$\Rightarrow 0.44\frac{\omega^4}{\omega_n^4} - 1.84\frac{\omega^2}{\omega_n^2} + 1.44 = 0$$

Solving the above equation we have $\frac{\omega}{\omega_n} = 1.772$

$$\Rightarrow \omega_n = \frac{\omega}{1.772} = \frac{15}{1.772} = \boxed{8.465\,rad/sec}.$$

So $f_n = \frac{\omega_n}{2\pi} = \frac{8.465}{2\pi} = \boxed{1.35\ Hz.}$

Example-4

A Vibrometer indicates 1% error in measurement and its natural frequency is 4 Hz. If the lowest frequency that can be measured is 36 Hz, find the value of damping factor.

Since the reading recorded by vibrometer is Z

So $Z = 1.01\ Y$

$$\text{Now} \quad \frac{Z}{Y} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

$$\Rightarrow 1.01 = \frac{(36/4)^2}{\sqrt{\left\{1-(36/4)^2\right\}^2 + (2\zeta \cdot 36/4)^2}}$$

$$\Rightarrow 1.01 = \frac{81}{\sqrt{6400 + 324\zeta^2}}$$

$$\Rightarrow (1.01)^2 = \frac{81^2}{6400 + 324\zeta^2}$$

$$\Rightarrow \boxed{\zeta = 0.313}$$

~~Velocity Pick-ups (Vibrometer):-~~

~~Vibrometer is used to measure the displacement of a vibrating body.~~

~~Considering the equation~~

$$\cancel{\frac{Z}{Y} = \frac{(\omega/\omega_n)^2}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}}$$

Velocity Pick-ups / velometers :-

velocity of the vibrating system can be expressed from the equation

$$y = Y \sin\omega t \quad \text{---(1)}$$

so velocity $\dot{y} = \omega Y \cos\omega t$ ---(2)

Now we have the equation

$$\frac{Z}{Y} = \frac{(\omega/\omega_n)^2}{\sqrt{\{1-(\frac{\omega}{\omega_n})^2\}^2 + (2\xi\frac{\omega}{\omega_n})^2}} \quad \text{---(3)}$$

$$\Rightarrow Z = \frac{Y.(\omega/\omega_n)^2}{\sqrt{\{1-(\frac{\omega}{\omega_n})^2\}^2 + (2\xi\frac{\omega}{\omega_n})^2}}$$

The relative velocity

$$\dot{z} = \frac{\omega.Y(\frac{\omega}{\omega_n})^2}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2 + (2\xi\frac{\omega}{\omega_n})^2}} \cos(\omega t - \phi) \quad \text{---(4)}$$

for $\omega/\omega_n \gtrsim 1$

$$\frac{(\frac{\omega}{\omega_n})^2}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2 + (2\xi\frac{\omega}{\omega_n})^2}} \approx 1 \quad \text{---(5)}$$

so from eq. (4)

$$\boxed{\dot{z} = \omega Y \cos(\omega t - \phi)} \quad \text{---(6)}$$

comparing eq. (2) and (6) it can be obserbed that for a phase difference ϕ $\dot{z}$ gives velocity of base as long as eq. (5) is satisfied and this is possible for large value of $(\frac{\omega}{\omega_n})$, else velocity of system can be computed from eq. (4).

Acceleration measuring instruments or Accelerometer

Accelerometer is used to measure the acceleration of a vibrating body.

If $\left(\frac{\omega}{\omega_n}\right) \leq 1$, the equation for relative amplitude reduces to

$$\frac{Z}{Y} = \left(\frac{\omega}{\omega_n}\right)^2$$

or

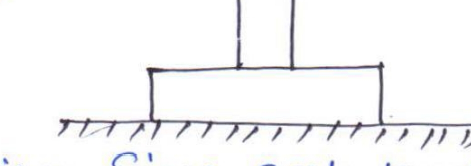
$$\boxed{Z = \frac{\omega^2 Y}{\omega_n^2} = \frac{\text{Acceleration}}{\omega_n^2}} \quad \text{--- (1)}$$

The expression $\omega^2 Y$ in the above equation is equal to the acceleration amplitude of the body vibrating with frequency ω and having a displacement amplitude Y. So the amplitude recorded Z, under these conditions is proportional to the acceleration of the vibrating body, as ω_n is a constant for the instrument.

Frequency Measuring Instruments:-

The frequency measuring devices are based on resonance principle. For the frequency less than about 100Hz. reed tachometers are quite useful. Two types of reed tachometers are generally used.

(a) Single-Reed Instrument:-

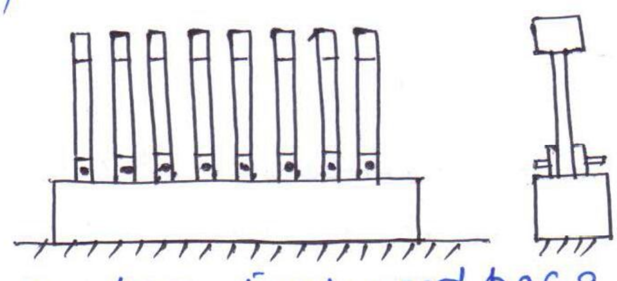


The instrument consists of a cantilever strip, held in a clamp at one end while a mass is attached at the other end. The free length of the strip can be adjusted by means of a screw mechanism. Since each length of strip correspond to a different natural frequency so the value of natural frequency are marked along length of the reed. The instrument is held firmly against the vibrating member and free length of the strip is altered untill at one particular length, resonance occur. The frequency is then directly read from the strip.

- The instrument is also known as fullarton Tachometer

(b) Multi Reed Instruments:-

The instrument is also called Frahm Tachometer. It essentially consists of a series of cantilevered reeds carrying small



concentrated mass at their tips. Each reed has a different natural frequency so it is possible to cover a wide frequency range. In practise the instrument is mounted on the vibrating body. The reed whose natural frequency matches with the unknown frequency of the body will undergo resonance and vibrate with larger amplitude. The frequency of the vibrating body can be then found from the known natural frequency of that reed