CHEPTER-2

Pressure and Its measurement

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2.1 Introduction:

Pressure may be defined as the force exerted on a unit area.

Consider a small area dA in Large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA. Let dF is the force acting on the area dA in the normal direction. Then the ratio of $\frac{dF}{dA}$ is known as the intensity of pressure

$$P = \frac{dF}{dA}$$

The units of pressure:- 1) kgf/m^2 MKS unit

2) N/m^2 SI unit

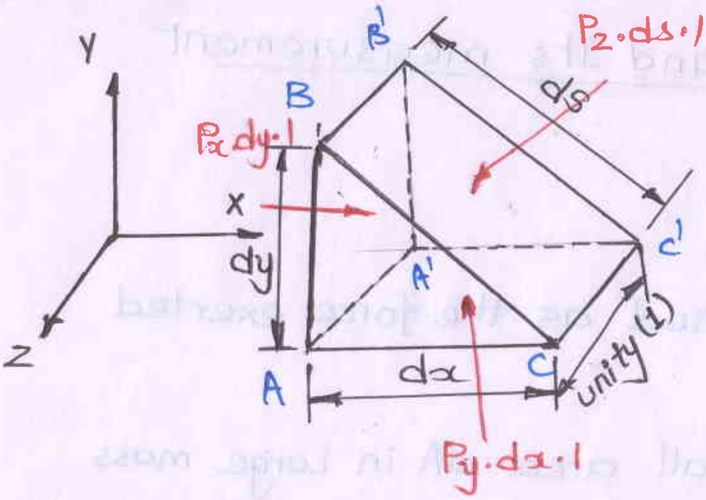
N/m^2 is known as Pascal (Pa)

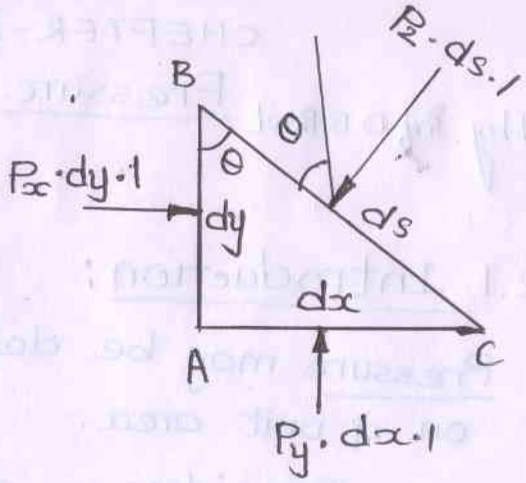
2.2 Pascal's Law:-

"It states that the pressure or intensity of pressure at a point in static fluid is equal in all directions."

This is proved as: The fluid element is of very small dimensions i.e., dx, dy and ds

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest





1) Pressure forces normal to the surface
2) Weight of element in the vertical direction

The forces on the faces are:

force on the face $ABB'A' = P_x \times$ Area of face $ABB'A'$

$$= P_x \cdot dy \cdot 1$$

Similarly force on the face $AA'C'C = P_y \cdot dx \cdot 1$

Force on the face $BB'C'C = P_2 \cdot ds \cdot 1$

Weight of element = Mass of element × g

$$= \rho g \forall = \rho g \times \frac{1}{2} AB \times AC \times 1 = \rho g \frac{dx \cdot dy}{2}$$

where ρ = density of fluid

Resolving the forces in x-direction

$$P_x \cdot dy \cdot 1 - P_2 \cdot ds \cdot 1 \cdot \sin(90^\circ - \theta) = 0$$

$$P_x \cdot dy - P_2 \cdot ds \cdot \cos\theta = 0 \qquad \left[\cos\theta = \frac{dy}{ds}\right]$$

$$P_x \cdot dy - P_2 \cdot ds \cdot \frac{dy}{ds} = 0$$

$$P_x = P_2 \quad \ldots\ldots\ldots (1)$$

similarly, resolving the forces in y-direction

$$P_y \cdot dx \cdot 1 - P_2 \cdot ds \cdot 1 \cdot \cos(90^\circ - \theta) - \frac{dx \cdot dy}{2} \cdot 1 \cdot \rho g = 0$$

$$P_y dx - P_2 ds \cdot \cos\theta - \frac{dx \cdot dy}{2} \cdot \rho g$$

$$P_y dx - P_z ds\cos\theta = 0$$

The wedge element is very small neglecting weight of fluid

$$P_y \cdot dx - P_z\, ds \cdot \frac{dx}{ds} = 0$$

$$P_y = P_z \quad \cdots (2)$$

From eq^n. (1) and (2)

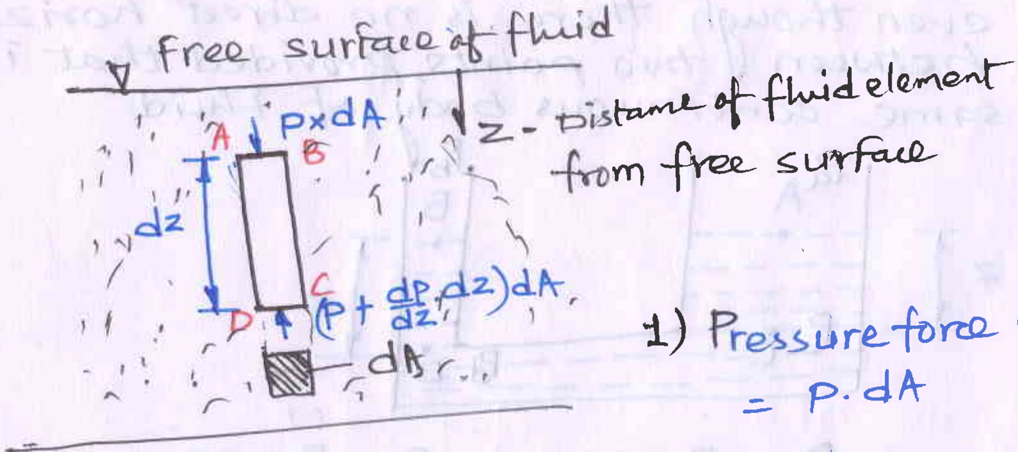
$$\boxed{P_x = P_y = P_z}$$

Application of Pascal's Law

1) In measurement system (menometer, pressure gauge etc)

2) Hydralic press, Hydralic ~~press~~ jack, Hydralic lift

2.3 Pressure variation in a fluid at Rest



1) Pressure force on AB $= p \cdot dA$

2) Pressure force on CD $= \left(P + \frac{dP}{dz} \cdot dz\right) \cdot dA$

3) weight of fluid element $= \rho g \times \text{volume} = \rho g \times dA \cdot dz$

For equilibrium of fluid element

$$p \cdot dA - \left(P + \frac{dP}{dz} \cdot dz\right) dA - \rho g \cdot dA \cdot dz = 0$$

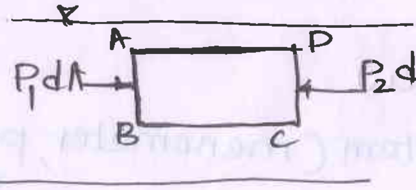
$$\frac{dP}{dz} = \rho g = w = \text{weight density}$$

Equation states that "rate of increase of pressure in vertical direction is equal to weight density of the fluid at that point" This is Hydrostatic Law

$$\int dp = \int \rho g\, dz$$

$$p = \rho g z \quad z \text{ is call pressure head}$$

→ **Equality of pressure at the same level in static fluid**

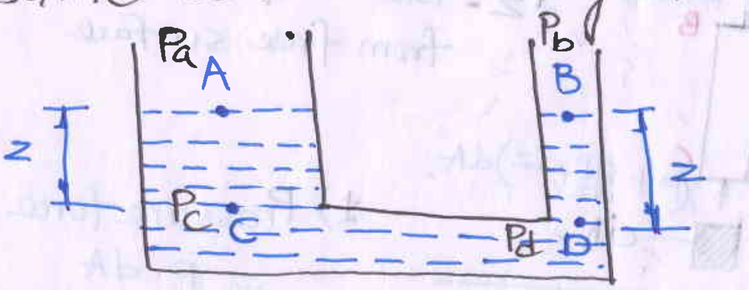


$$P_1 dA - P_2 dA = 0$$

$$P_1 = P_2$$

Pressure at any two points at the same level in body of fluid at rest will be the same

Prove that the pressure at same level will be equal even though there is no direct horizontal path between two points, provided that in the same continuous body of fluid



$$P_c = P_a + \rho g z, \quad P_d = P_b + \rho g z$$

$P_c = P_d$ at same level in static fluid

$$P_a + \rho g z = P_b + \rho g z$$

$$\boxed{P_a = P_b}$$

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2.4 Pressure and Head

1) Absolute pressure 2) Gauge pressure
3) Vacuum pressure 3) Atmospheric pressure

1) **Absolute pressure:-** It is defined as the pressure which is measured with reference to absolute vacuum pressure.

2) **Gauge pressure:-** It is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

3) **Vacuum pressure:-** It is defined as the pressure below the atmospheric pressure.

4) **Atmospheric pressure:-** It is pressure exerted by air on the surface of earth (due to weight of air). The air is compressible fluid the density of air vary from time to time due to change in its temperature, therefore atm. pressure is not constant.

The atmospheric pressure at sea level at 15°C is

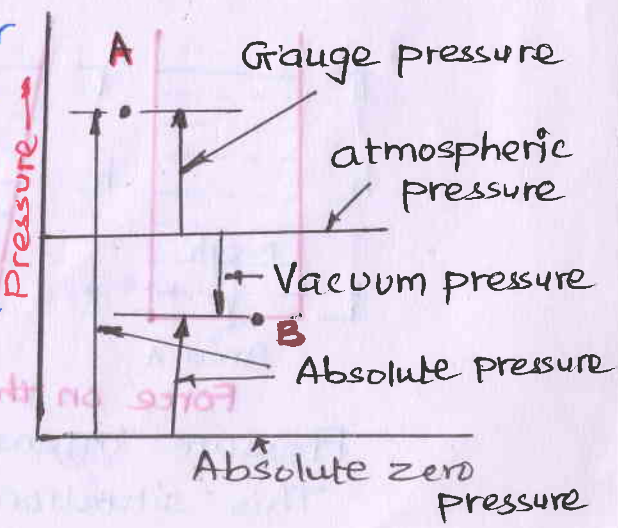
$101.325\ KN/m^2$ or 1.01325 bar } in SI unit

$1.033\ kgf/cm^2$ in MKS unit

We know $P = \rho g h$

$= 10.33$ m of water

$= 760$ mm of Hg



Mathematically

i) Absolute pressure (Pab) = Atm pressure (Patm) + Gauge pressure (Pg)

2) Vacuum pressure = Patm − Pab

2.5 Measurement of pressure :-

The pressure of a fluid is measured by following devices

1) Manometers 2) Mechanical Gauges

– Methods of pressure measurement

i) Measurement of elastic deformation of elastic elements like bourdon tube, diaphragm, bellows, capsule

Pressure → [Elastic element] → Displacement

(ii) Balancing the pressure exerted by fluid generally mercury column like in menometer Mcleod gauge, barometer etc.

Pressure → [Manometer] → Manometric liquid height

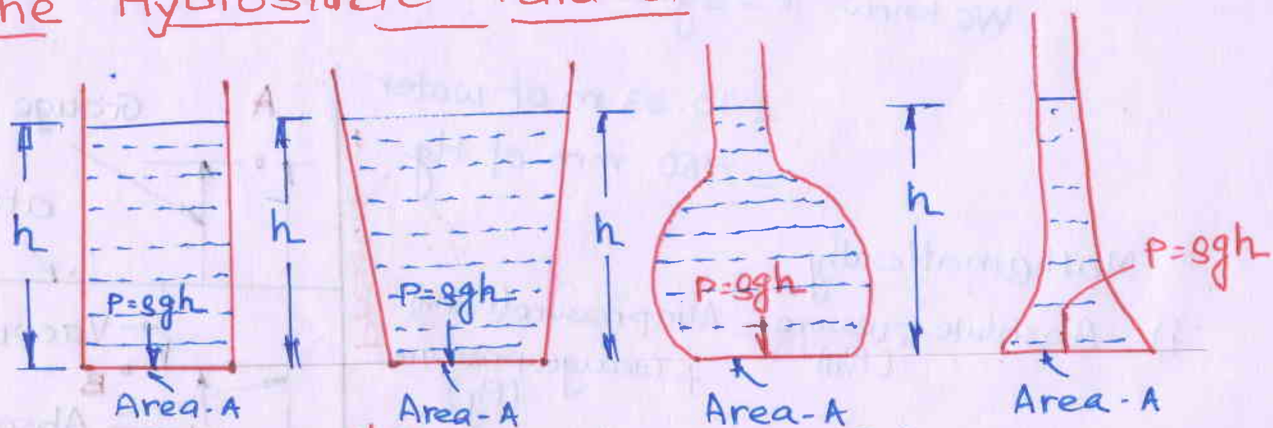
(iii) Balancing the force produced on a known area by measured force like in Dead weight tester.

Pressure → [Dead weight tester] → known force

iv) Measurement of electrical quantities.
strain gauge, LVDT

Pressure → [Bellows] → [LVDT] → Voltage

The Hydrostatic Paradox



Force on the base of vessel = P.A

Pressure intensity is independent on weight of fluid

This situation is called Hydrostatic paradox

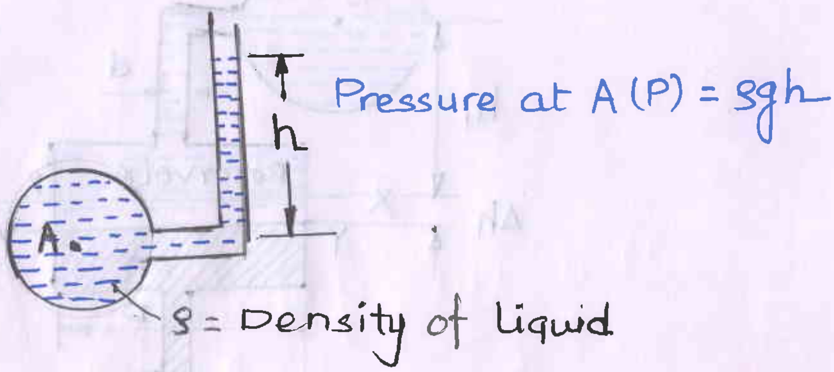
2.5.1 Manometers.

Manometers are defined as the devices used for measuring the pressure at point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified by

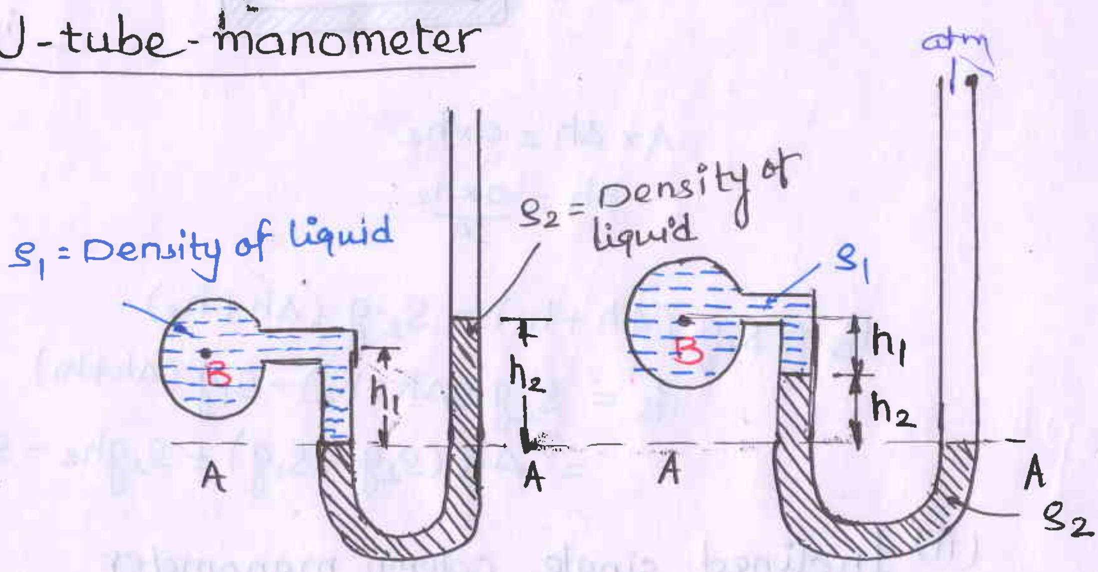
a) Simple manometers b) Differential manometers

a) Simple manometers

1) Piezometer,



2) U-tube-manometer



$\rho_1 < \rho_2$

a) For gauge pressure b) For vacuum pressure

a) Pressure above A-A in left column = Pressure above A-A in Right column

$$P_B + \rho_1 g h_1 = \rho_2 \cdot g h_2$$

$$P_B = \rho_2 g h_2 - \rho_1 g h_1$$

b) Pressure above AA in left column = Pressure above AA in Right column

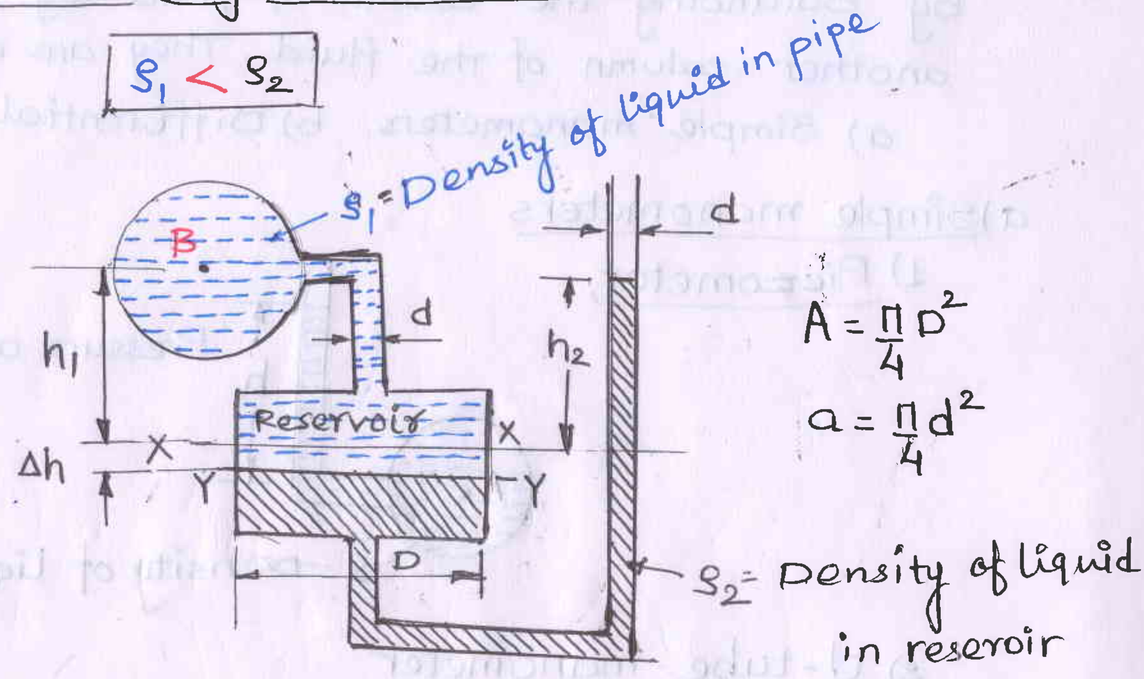
$$P_B + \rho_1 g h_1 + \rho_2 g h_2 = 0$$

$$P_B = -(\rho_1 g h_1 + \rho_2 g h_2)$$

3) Single column manometer:-

i) Vertical single column manometer

$\rho_1 < \rho_2$



$$A = \frac{\pi}{4} D^2$$

$$a = \frac{\pi}{4} d^2$$

$$A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a \times h_2}{A}$$

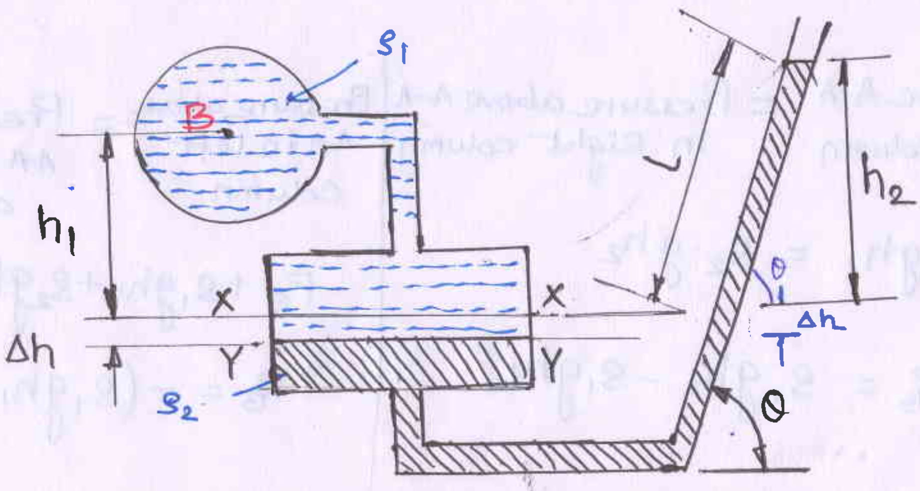
$$P_B + \rho_1 g (\Delta h + h_1) = \rho_2 g (\Delta h + h_2)$$

$$P_B = \rho_2 g (\Delta h + h_2) - \rho_1 g (\Delta h + h_1)$$

$$= \Delta h (\rho_2 g - \rho_1 g) + \rho_2 g h_2 - \rho_1 g h_1$$

(ii) Inclined single column manometer

$$A \times \Delta h = a \times L \sin\theta \Rightarrow \Delta h = \frac{a L \sin\theta}{A}$$



$$\sin\theta = \frac{h_2}{L}$$

$$h_2 = L \sin\theta$$

$$P_B + \rho_1 g (h_1 + \Delta h) = \rho_2 g (\Delta h + L\sin\theta)$$

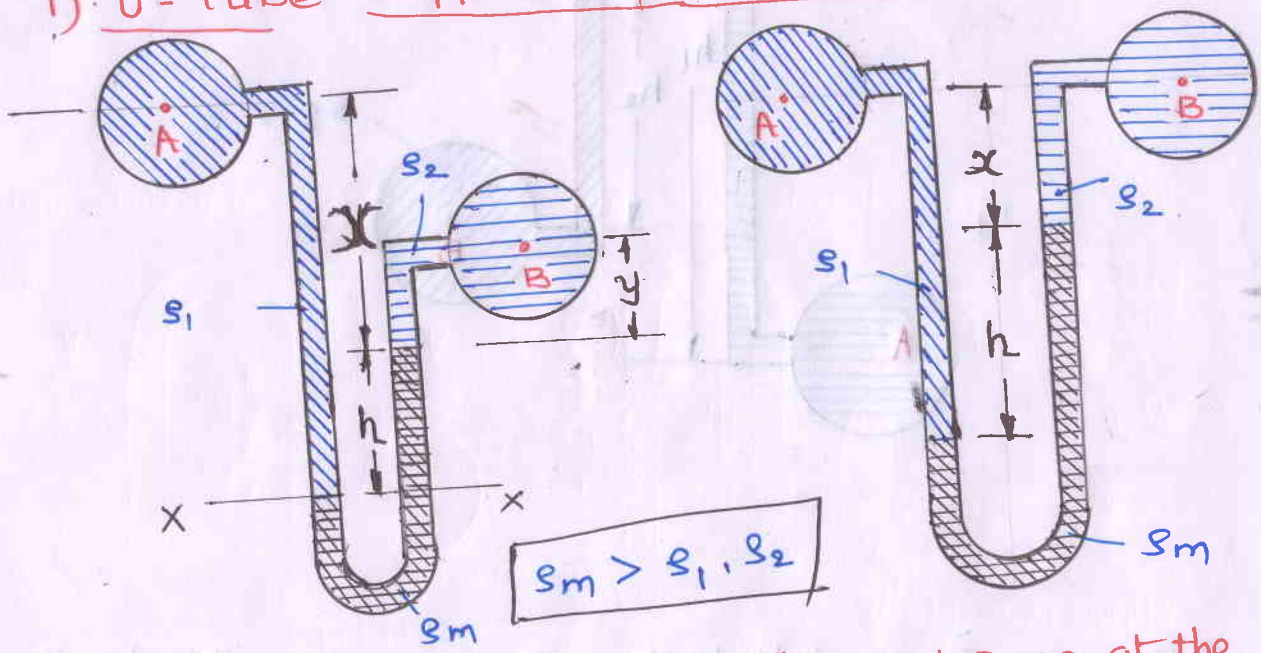
$$P_B = \Delta h (\rho_2 g - \rho_1 g) + \rho_2 g L \sin\theta - \rho_1 g h$$

b) Differential manometer

Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes.

1) U-tube differential manometer
2) Inverted U-tube differential manometer.

1) U-tube differential manometer



a) Two pipes at different levels

b) A and B are at the same level

a)

Pressure above x-x in the left limb = Pressure above x-x in the right limb

$$P_A + \rho_1 g(x+h) = \rho_2 g y + \rho_m g \cdot h + P_B$$

$$P_A - P_B = \rho_m \cdot g h + \rho_2 g y - \rho_1 g (h+x)$$

b)

Pressure above x-x in the left limb = Pressure above x-x in the right limb

$$P_A + \rho_1 g(x+h) = P_B + \rho_2 g x + \rho_m g h$$

$$P_A - P_B = \rho_m g h + \rho_2 g x - \rho_1 g (x+h)$$

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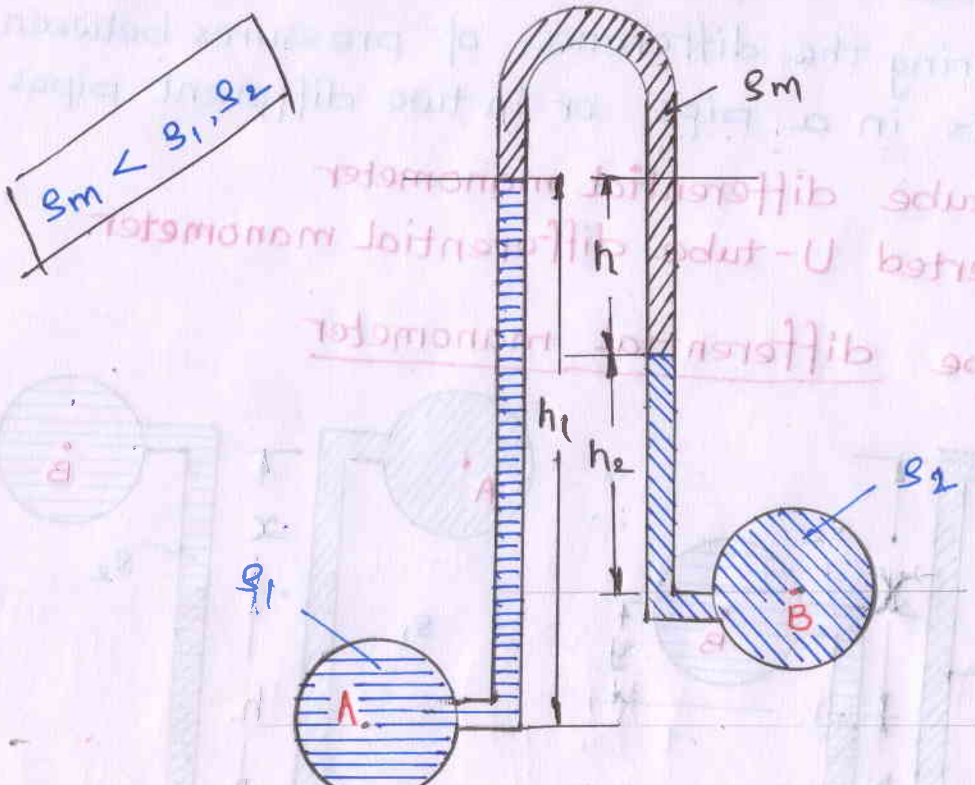
2) Inverted U-tube differential manometer



Pressure in the left limb below x-x = Pressure in right Limb below x-x

$$P_A - \rho_1 \cdot g \cdot h_1 = P_B - \rho_2 g h_2 - \rho_m \cdot g \cdot h$$

$$P_A - P_B = \rho_1 g h_1 - \rho_2 g h_2 - \rho_m \cdot g \cdot h$$