

# To study Bernoulli's Principle (CO-3130502.2)

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## Objective

1. To verify Bernoulli's theorem.

## Chemicals

1. water

## Apparatus

Bernoulli's Apparatus, stop watch

## Theory

### Energy of a Liquid in Motion

The energy, in general, may be defined as the capacity to do work. Though the energy exists in many forms, yet the following are important from the subject point of view:

1. Potential Energy
2. Kinetic Energy, and
3. Pressure Energy.

### Potential Energy of a Liquid in Motion

It is the energy possessed by a liquid particle, by virtue of its position. If a liquid particle is  $Z$  meters above the horizontal datum (arbitrarily chosen), the potential energy of the particle will be  $Z$  meter-kilogram (briefly written as  $mkg$ ) per  $kg$  of liquid. Potential head of the liquid, at that point, will be  $Z$  meters of the liquid.

### Kinetic Energy of a Liquid Particle in Motion:

It is the energy, possessed by a liquid particle, by virtue of its motion or velocity. If a liquid particle is flowing with a mean velocity of  $v$  meter per second, then the kinetic energy of the particle will be  $v^2/2g$   $mkg$  per  $kg$  of liquid. Velocity head of the liquid, at that velocity, will be  $v^2/2g$  meter of the liquid.

### Pressure Energy of a Liquid Particle in Motion:

It is the energy, possessed by a liquid particle, by virtue of its existing pressure. If a liquid particle is under a pressure of  $p$   $kg/m^2$ , then the pressure energy of the particle will be  $p/w$   $mkg$  per  $kg$  of liquid, where  $w$  is the specific weight of the liquid. Pressure head of the liquid under that pressure will be  $p/w$  meter of the liquid.

### Total Energy of a Liquid Particle in Motion:

The total energy of a liquid particle, in motion, is the sum of its potential energy, kinetic energy and pressure energy. Mathematically,

Total Energy,

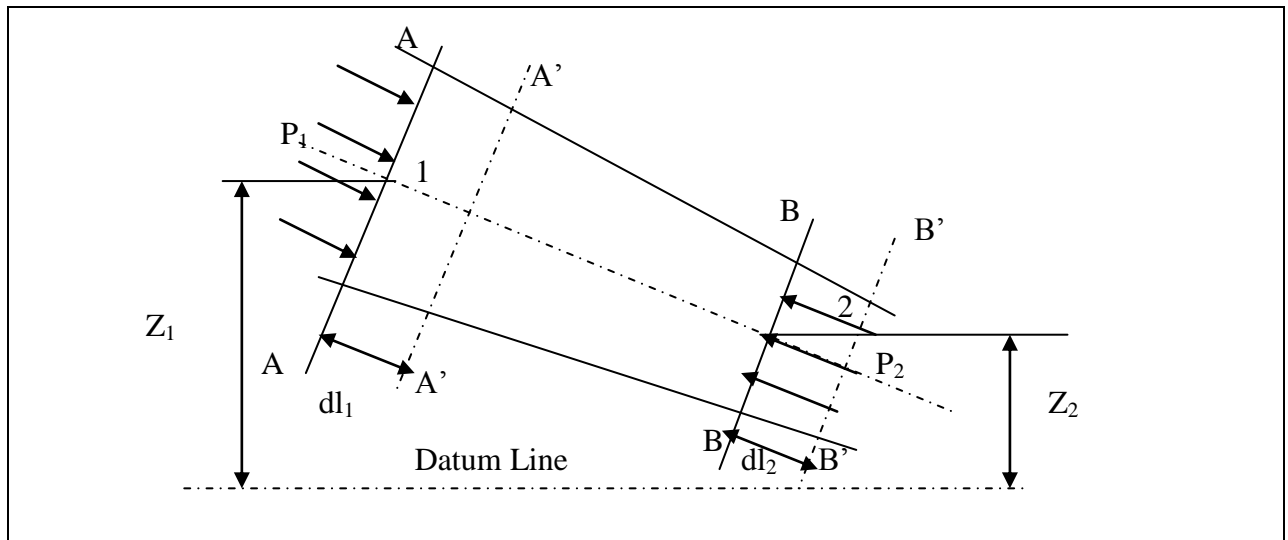
$$E = Z + \frac{v^2}{2g} + \frac{p}{w} \text{ mkg/ kg of liquid}$$

### BERNOULLI'S EQUATION:

It states, "For a perfect incompressible liquid, flowing in a continuous stream, the total energy of a particle remains the same; while the particle moves from one point to another." This statement is based on the assumption that there are no losses due to friction in pipe. Mathematically,

$$E = Z + \frac{v^2}{2g} + \frac{p}{w} = \text{Constant}$$

Consider a perfect incompressible liquid, flowing through a non-uniform pipe as shown in Figure.



**Figure: Bernoulli's Equation**

Let us consider two cross sections AA and BB of the pipe. Now let us assume that the pipe is running full and there is a continuity of flow between the two sections.

Let

|                      |   |                              |
|----------------------|---|------------------------------|
| $Z_1$                | = | Height of AA above the datum |
| $p_1$                | = | Pressure at AA               |
| $v_1$                | = | Velocity of liquid at AA     |
| $a_1$                | = | Area of pipe at AA           |
| $Z_2, p_2, v_2, a_2$ | = | corresponding values at BB   |

Let the liquid between the two sections AA and BB move to A'A' and B'B' through very small lengths  $dl_1$  and  $dl_2$  as shown in figure. This movement of the liquid between AA and A'A' to BB and B'B', the remaining liquid between A'A' and BB being unaffected.

Let W be the weight of the liquid between AA and A'A'. Since the flow is continuous, therefore

$$W = wa_1dl_1 = wa_2dl_2$$

Or

$$a_1 dl_1 = W/w$$

Similarly

$$a_2 dl_2 = W/w$$

Therefore,

$$a_1 dl_1 = a_2 dl_2$$

Work done by pressure at AA, in moving the liquid to A'A' = Force \* Distance =  $p_1 a_1 dl_1$

Similarly, work done by pressure at BB in moving the liquid to B'B' =  $- p_2 a_2 dl_2$   
(Minus sign is taken as the direction of  $p_2$  is opposite to that of  $p_1$ ).

$$\begin{aligned} \text{Total work done by the pressure} &= p_1 a_1 dl_1 - p_2 a_2 dl_2 \\ &= p_1 a_1 dl_1 - p_2 a_1 dl_1 \\ &= a_1 dl_1 (p_1 - p_2) \\ &= W/w (p_1 - p_2) \end{aligned}$$

$$\text{Loss of potential energy} = W (Z_1 - Z_2)$$

$$\text{And again in kinetic energy} = W \left( \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \right) = \frac{W}{2g} (v_2^2 - v_1^2)$$

We know that,

Loss of potential energy + work done by pressure = Gain in kinetic energy

$$W (Z_1 - Z_2) + W/w (p_1 - p_2) = \frac{W}{2g} (v_2^2 - v_1^2)$$

$$(Z_1 - Z_2) + \frac{p_1}{w} - \frac{p_2}{w} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

Or

$$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{w} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{w}$$

This proves the Bernoulli's equation.

### Bernoulli Equation:

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

--> 1

This is the basic form of *Bernoulli equation* for steady incompressible inviscid flows. It may be written for any two points 1 and 2 on the same streamline as

$$\frac{p_1}{\rho_1 g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho_2 g} + \frac{v_2^2}{2g} + z_2$$

--> 2

The constant of Bernoulli equation, can be named as *total head* ( $h_o$ ) has different values on different streamlines.

$$h_o = \frac{p}{\rho g} + \frac{v^2}{2g} + z$$

--> 3

The total head may be regarded as the sum of the *piezometric head*  $h^* = p/\rho g + z$  and the *kinetic head*  $v^2/2g$ .

**Bernoulli equation is arrived from the following assumptions:**

1. Steady flow - common assumption applicable to many flows.
2. Incompressible flow - acceptable if the flow Mach number is less than 0.3.
3. Frictionless flow - very restrictive; solid walls introduce friction effects.
4. Valid for flow along a single streamline; i.e., different streamlines may have different  $h_o$ .
5. No shaft work - no pump or turbines on the streamline.
6. No transfer of heat - either added or removed.

**Range of validity of the Bernoulli Equation:**

Bernoulli equation is valid along any streamline in any steady, inviscid, incompressible flow. There are no restrictions on the shape of the streamline or on the geometry of the overall flow. The equation is valid for flow in one, two or three dimensions.

**Modifications on Bernoulli equation:**

Bernoulli equation can be corrected and used in the following form for real cases.

$$\frac{p_1}{\rho_1 g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho_2 g} + \frac{v_2^2}{2g} + z_2 + h + w - q$$

Where 'q' is the work done by pump and 'w' is the work done by the fluid and h is the head loss by friction.

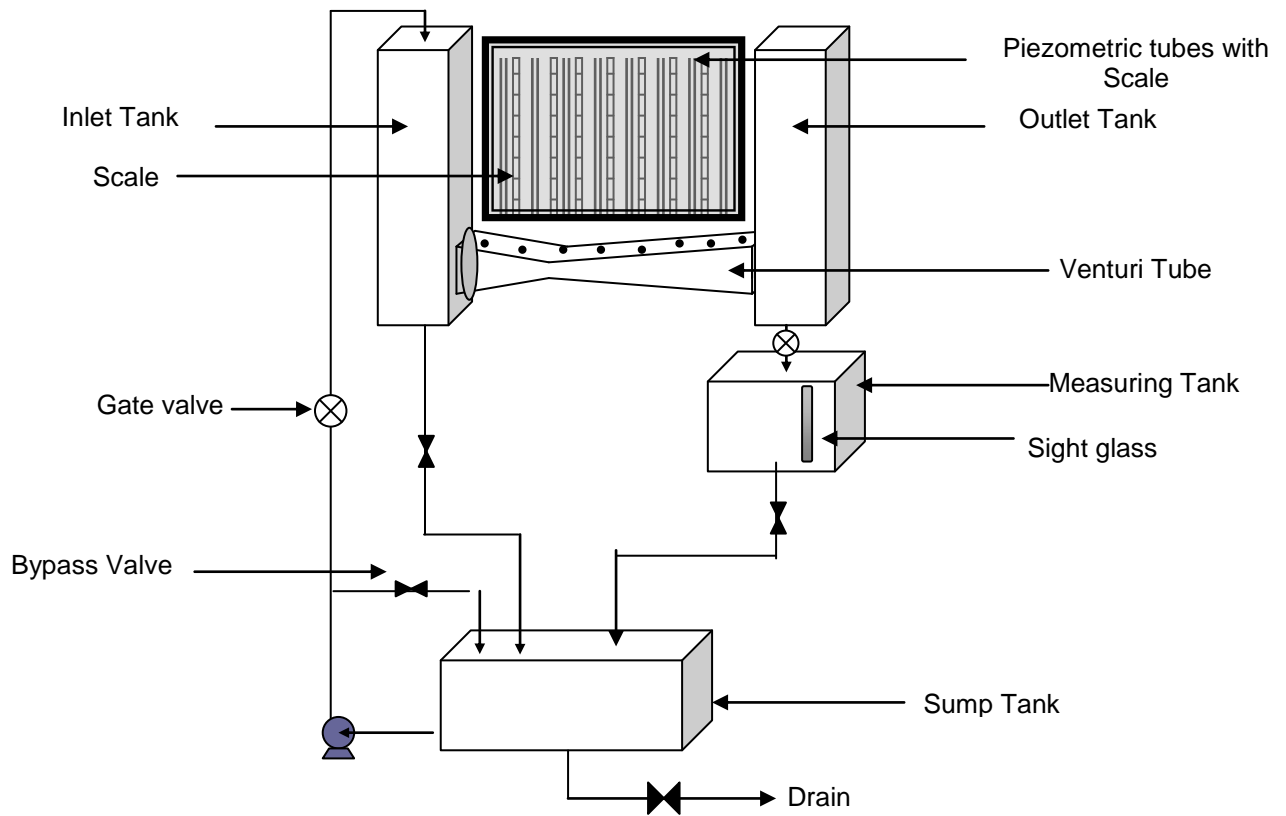


Fig. Bernoulli's Theorem Apparatus

### Description of Apparatus:

The apparatus is made from transparent acrylic and has both the convergent and divergent sections. Water is supplied from the constant head tank attached to the test section. Constant level is maintained in the supply tank. Piezometric tubes are attached at different distance on the test section. Water discharges to the discharge tank attached at the far end of the test section and from there it goes to the measuring tank through valve. The entire setup is mounted on a stand.

### Inlet Tank:

|          |                      |
|----------|----------------------|
| Capacity | : 32 lit.            |
| Size     | : 200 * 200 * 800 mm |
| MOC      | : SS – 304           |

With provision for control valve to stabilize the flow and sight glass for Level Indication.

**Outlet Tank:**

Capacity : 32 lit.  
Size : 200 \* 200 \* 800 mm  
MOC : SS – 304

With provision for control valve and sight Glass for Level Indication.

**Sump Tank:**

Capacity : 150 liter  
Size : 750 \* 500 \* 400 mm  
MOC : SS – 304

With ½” drain valve

**Pump:**

Make : Aroma  
MOC : SS 304  
Type : Monoblock - Centrifugal  
Power : 0.5 HP, Single Phase

**Measuring Tank:**

Capacity : 51 liter  
Size : 400 \* 400 \* 320 mm  
MOC : SS - 304

With self graduated glass tube level indicator.

With 1” drain valve and piping leading the water to the sump tank.

**Testing Duct:**

MOC : Acrylic  
Type : Cylinder with divergent and convergent sections.

## Procedure

1. Connect the plug to the electric single phase 230 V AC, 50 Hz power supply.
2. Fill the sump tank with the help of water about 100 liters.
3. The water supply line is connected to the sump tank through pump. Make sure that the bypass valve is fully open.
4. The discharge line from the pump is connected to the supply tank with flow control valve. The supply tank is connected to the testing duct of acrylic. The outlet from the duct is connected to the outlet tank. From the outlet tank water flows to the measuring tank with flow control valve.
5. Switch ON the power supply.
6. Switch ON the Pump.
7. The pump bypass valve is then closed and flow control valve to the supply tank is opened. Adjusted until there is a steady level of liquid in the supply tank.
8. Keep drain valve of the discharge tank fully open.
9. After we get steady height of liquid in the supply tank adjust the drain valve on the discharge tank so as to get steady level there also.
10. Collect the predetermined quantity of water in the measuring tank and measure the time required for the same.
11. Also record the height of liquid in each of the piezometric tube.

## Observation Table

Distance between each piezometer = \_\_\_\_\_ cm

Density of water =  $0.001 \text{ kg/cm}^3$

Note down the sr. nos of Pitot tubes and their cross sectional areas.

Volume of water collected,  $V = \text{_____ cm}^3$

Time taken for collection of water,  $t = \text{_____ sec}$

Table-1

| SR. NO.                            |                |                                       |  |  |  |
|------------------------------------|----------------|---------------------------------------|--|--|--|
| Ht. Diff. In Measuring Tank (M)    | H <sub>1</sub> |                                       |  |  |  |
|                                    | H <sub>2</sub> |                                       |  |  |  |
|                                    | ΔH             |                                       |  |  |  |
| VOLUME (M <sup>3</sup> )           | V              |                                       |  |  |  |
| TIME (SEC.)                        | T              |                                       |  |  |  |
| VOL. FLOW RATE (M <sup>3</sup> /S) | Q              |                                       |  |  |  |
| PIEZOMETRIC TUBE POSITION          | 1              | H <sub>1</sub> =P <sub>1</sub> /w (m) |  |  |  |
|                                    |                | V <sub>1</sub> = Q/ A <sub>1</sub>    |  |  |  |
|                                    |                | v <sub>1</sub> <sup>2</sup> / 2g (m)  |  |  |  |
|                                    |                | TOTAL HEAD                            |  |  |  |
|                                    | 2              | H <sub>2</sub> =P <sub>2</sub> /w (m) |  |  |  |
|                                    |                | V <sub>2</sub> = Q/ A <sub>2</sub>    |  |  |  |
|                                    |                | v <sub>1</sub> <sup>2</sup> / 2g (m)  |  |  |  |
|                                    |                | TOTAL HEAD                            |  |  |  |
|                                    | 3              | H <sub>3</sub> =P <sub>3</sub> /w (m) |  |  |  |
|                                    |                | V <sub>3</sub> = Q/ A <sub>3</sub>    |  |  |  |
|                                    |                | v <sub>1</sub> <sup>2</sup> / 2g (m)  |  |  |  |
|                                    |                | TOTAL HEAD                            |  |  |  |
|                                    | 4              | H <sub>4</sub> =P <sub>4</sub> /w (m) |  |  |  |
|                                    |                | V <sub>4</sub> = Q/ A <sub>4</sub>    |  |  |  |
|                                    |                | v <sub>1</sub> <sup>2</sup> / 2g (m)  |  |  |  |
|                                    |                | TOTAL HEAD                            |  |  |  |
|                                    | 5              | H <sub>5</sub> =P <sub>5</sub> /w (m) |  |  |  |
|                                    |                | V <sub>5</sub> = Q/ A <sub>5</sub>    |  |  |  |
|                                    |                | v <sub>1</sub> <sup>2</sup> / 2g (m)  |  |  |  |
|                                    |                | TOTAL HEAD                            |  |  |  |
|                                    | 6              | H <sub>6</sub> =P <sub>6</sub> /w (m) |  |  |  |
|                                    |                | V <sub>6</sub> = Q/ A <sub>6</sub>    |  |  |  |
|                                    |                | v <sub>1</sub> <sup>2</sup> / 2g (m)  |  |  |  |
|                                    |                | TOTAL HEAD                            |  |  |  |
|                                    | 7              | H <sub>7</sub> =P <sub>7</sub> /w (m) |  |  |  |
|                                    |                | V <sub>7</sub> = Q/ A <sub>7</sub>    |  |  |  |
|                                    |                | v <sub>1</sub> <sup>2</sup> / 2g (m)  |  |  |  |
|                                    |                | TOTAL HEAD                            |  |  |  |



## Calculation

1. Discharge  $Q = V / t = \dots\dots\dots \text{cm}^3 / \text{sec}$
2. Velocity  $V = Q / A = \dots\dots\dots = \dots\dots\dots \text{cm/sec}$   
Where, A is the cross sectional area of the fluid flow
3. Velocity head  $V^2 / 2g = \dots\dots\dots \text{cm}$
4. Pressure head (actual measurement or piezometer tube reading)  
 $P/w = \dots\dots\dots \text{cm}$
5. Total Head  
 $H = \text{Pressure head} + \text{Velocity Head} = \dots\dots\dots \text{cm}$

## Results & Graphs

1. Plot the graph between  $P/w$  and  $x$ .
2. Plot the graph between  $V^2 / 2g$  and  $x$ .

## Conclusion

## Quiz

1. Bernoulli's equation holds good for non ideal fluids  
a) True b) False
2. The pressure head is given by  
a)  $P/\gamma$  b)  $V^2/2g$
3. Bernoulli's theorem deals with law conservation of momentum  
a) True b) false
4. What is piezometer tube?

# Reynolds apparatus (CO-3130502.1)

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## Objective

1. To visually observe laminar and turbulent flow patterns.
2. To determine the Reynolds Number for given flow.

## Chemicals

1. Water
2. Dye solution (tracer)

## Apparatus

Reynolds' apparatus, stop watch

## Theory

Pipe flow problems occur in the design of many types of engineering projects. A few of these include refineries (liquid flow), gasoline plants (gas or vapour flow), chemical plants etc. Fluid flow is an integral part of any chemical industry and understanding the phenomena that dictate the motion of any fluid through pipes is of utmost importance to any chemical engineer. There is always need of develop an efficient system. To reduce wastage of energy through pipe flow, it would be beneficial to first study the various flow regimes of fluids on a small scale.

Here are some formulae which we are using in this experiment:

$$N_{Re} = \frac{D V \rho}{\mu} = \frac{\text{inertia force}}{\text{viscous force}}$$

$N_{Re}$  is a dimensionless number called Reynolds number.  $\rho$  is density of fluid,  $V$  is velocity,  $D$  is internal diameter of pipe in which fluid flows and  $\mu$  is the viscosity of fluid. For calculation purpose all the parameters are taken in SI units.

The Reynolds number is important in analyzing any type of flow when there is substantial velocity gradient (i.e. shear.) It indicates the relative significance of the viscous effect compared to the inertia effect.

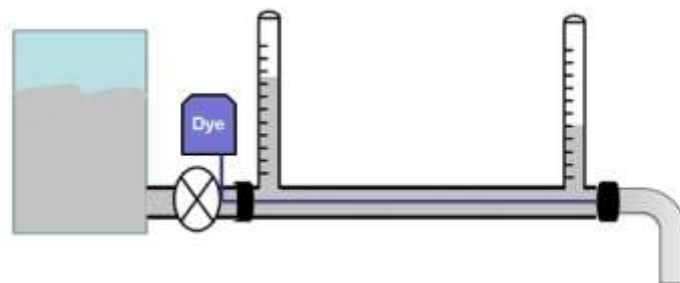
The flow is,

Laminar when  $N_{Re} < 2100$

Turbulent transient when  $2100 < N_{Re} < 4000$

Transient turbulent when  $4000 < N_{Re}$

At laminar region, viscous forces are dominant as compared to inertial forces. Under laminar flow condition the pressure drop per unit length is proportional to the velocity. At transition region, the experimental results are not reproducible. Finally, at turbulent region, inertial forces are dominant. For turbulent flow, the pressure drop becomes proportional to the velocity raised to a power of 2.



## Figure of Reynolds apparatus

The experimental setup consists of a glass tube (26 mm OD, 38 mm OD and 1320 mm long) with a side inlet for entry of water from a storage tank and with the discharge stop cock. The assembly is suitably supported on the stand.

A dye bottle (SS-304 Bottle 750 ml capacity) feeds potassium permanganate solution through capillary tube to the glass tube. The flow rate of the permanganate solution may be adjusted by a pinch arrangement in the connecting plastic tube.

The flow rate of water is controlled by a stopcock. The flow rate of water can be measured by collecting a definite quantity of water in a given time using a stopwatch and measuring cylinder.

### Procedure

1. Allow the water to fill the equipment and to flow at the lowest possible flow rate.
2. Adjust the flow of permanganate solution so that its velocity is about the same as the water in the tube.
3. Note that the color filament appears as a continuous thread without intermingling with water.
4. Determine the flow rate of water.
5. Repeat the experiment gradually by increasing the flow rate of water. Observe the flow rate at which the continuous thread just breaks up and the color gets diffused uniformly throughout the tube.

### Observation Table

ID of the Glass Tube = 26mm

Table-1

|   | Value                | Unit              |
|---|----------------------|-------------------|
| Enter pipe diameter   |                      | Inch.             |
| Enter pipe length   |                      | Inch.             |
| Cross sectional Area of pipe, $A = (\pi D^2/4)$                               |                      |                   |
| Volumetric flow rate, Q<br>(volume collected / time required collecting vol.) |                      |                   |
| Velocity of fluid   |                      | m/s               |
| Density of Water, $\rho$  | 1000                 | kg/m <sup>3</sup> |
| Viscosity of water, $\mu$   | $8.9 \times 10^{-4}$ | kg/m.s            |

### Calculation

$$\text{Calculate Reynolds Number } N_{Re} = \frac{D V \rho}{\mu}$$

### Results

$$N_{Re} =$$

According to the classification of the regimes from Reynolds number what is the nature of flow?

1. Laminar
2. Transition
3. Turbulent

## Conclusion

## Quiz

1. What the value of critical Reynolds number for an incompressible fluid flowing through a closed conduit?

# To study Pressure drop through pipe (CO-3130502.3)

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## Objective

1. To study Pressure drop through pipe

## Chemicals

1. water

## Theory

In hydraulic engineering practice, it is frequently necessary to estimate the head loss incurred by a fluid as it flows along a pipeline. For example, it may be desired to predict the rate of flow along a proposed pipe connecting two reservoirs at different levels. Or it may be necessary to calculate what additional head would be required to double the rate of flow along an existing pipeline.

When a fluid with a uniform velocity enters a pipe, the layers of fluid adjacent to the walls are slowed down as on a plane surface and a boundary layer forms at the entrance. This builds up in thickness and as the fluid passes into the pipe. At some distance downstream from the mouth, the boundary layers reach a thickness equal to the pipe radius and join at the axis, after which conditions remain constant and fully developed flow exists. If the flow in the boundary layers is streamline when they meet, laminar flow exists in the pipe. If the transition has already taken place before they meet, turbulent flow will persist in the region of fully developed flow.

When a fluid flows in a steady state through a pipe, energy is dissipated in overcoming friction. The energy dissipated depends on the properties of flowing fluid and the confining pipe and their relative motion. The significant properties of the pipe are their internal diameter, length and roughness ratio  $\epsilon / D$  (where  $D$  is the inside diameter of the pipe and  $\epsilon$  is the average height of the projection of roughness inside the pipe) and of the fluid are its density and viscosity.

Loss of head is incurred by fluid mixing, which occurs at fittings such as bends or valves, and by frictional resistance at the pipe wall. Where there are numerous fittings and the pipe is short, the major part of the head loss will be due to the local mixing near the fittings. For a long pipeline, on the other hand, skin friction at the pipe wall will predominate. In the experiment described below, we investigate the frictional resistance to flow along a long straight pipe with smooth walls.

Losses due to friction for a fluid flowing through a pipe is given by

$$\Delta H = \Delta P / \rho g = 4 f (L / D) (V^2 / 2 g)$$

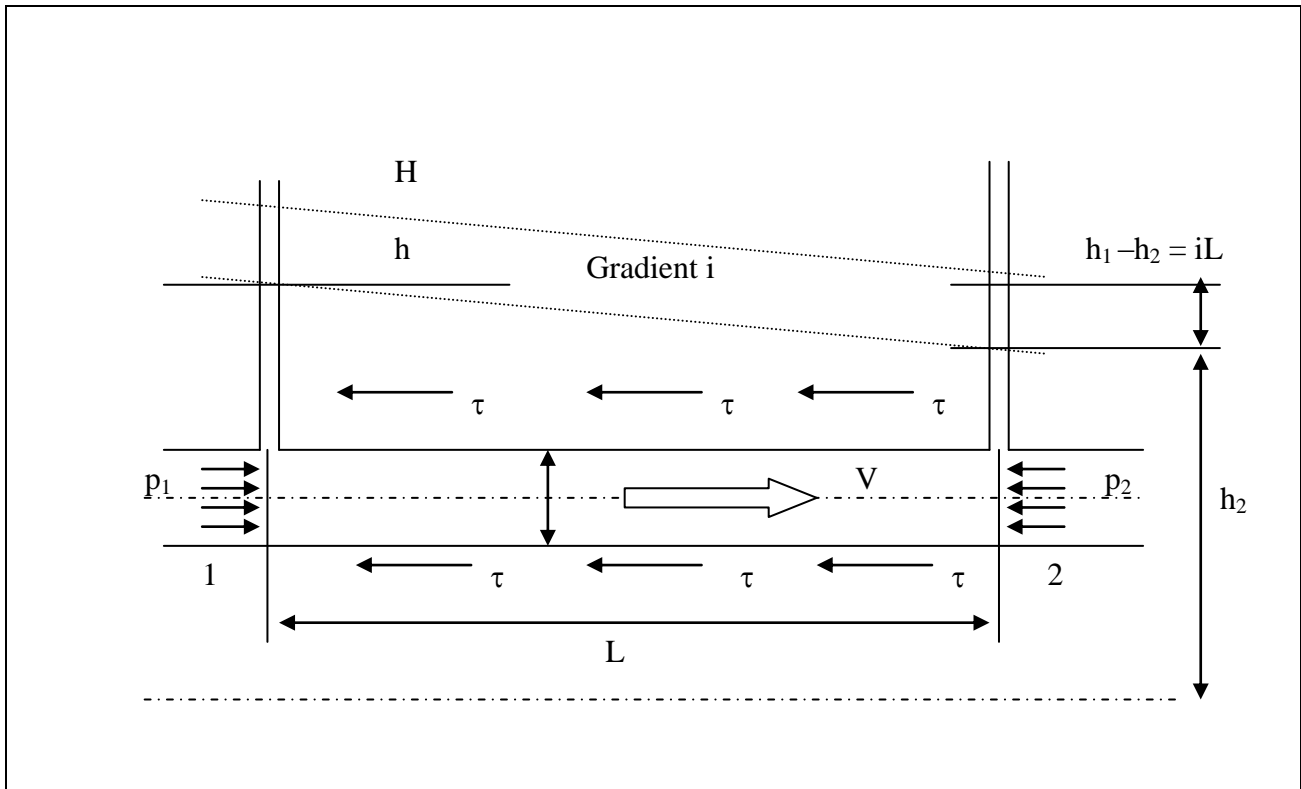
This equation is called Fanning's Equation.  $f$  is called the Fanning's friction factor and it is a function of Reynolds Number and the roughness factor.

$$f = \phi(N_{Re}) \Psi (\epsilon / D) = (\Delta P / 4 \rho) (D / L) (2 / V^2)$$

The friction factor may be determined by the following or other available equations (1)  $f = 16 / N_{Re}$ , for laminar flow ( $N_{Re} < 2100$ ) and  $f = 0.046 (N_{Re})^{-0.2}$  for turbulent flow, ( $N_{Re} > 4000$ ). For fluids flowing through smooth pipes, the roughness of which is zero.

**Friction Loss in Laminar and Turbulent Pipe Flow:**

Figure 1 illustrates flow along a length of straight uniform pipe of diameter D. All fittings such as valves or bends are sufficiently remote as to ensure that any disturbances due to them have died away so that the distribution of velocity across the pipe cross section does not change along the length of pipe under consideration.



**Fig. 1 Illustration of Fully Developed Flow along A Pipe**

Such a flow is said to be “fully developed”. The shear stress  $\tau$  at the wall, which is uniform around the perimeter and along the length, produces resistance to the flow. The pressure head  $h$  therefore falls at a uniform rate along the length, as shown in the figure 1. Since the velocity head is constant along the length of the pipe, the total head  $H$  also falls at the same rate.

The slope of pressure line is also frequently called the “hydraulic gradient”, and is denoted by symbol  $i$ :

$$i = -\frac{dh}{dl} = -\frac{dH}{dl} \quad (1)$$

(The minus sign are due to the fact that the head decreases in the direction of increasing  $l$ , which is measured positive in the same sense as the velocity  $V$ . The resulting value of  $i$  is then positive). Over the length  $L$  between sections 1 and 2, the fall in pressure head is

$$h_1 - h_2 = iL \quad (2)$$

Expressed in terms of pressures  $p_1$  and  $p_2$  at sections 1 and 2:

$$p_1 - p_2 = wiL = \rho giL \quad (3)$$

in which  $w$  is the specific weight and  $\rho$  is the density of water.

There is a simple relationship between wall shear stress  $\tau$  and hydraulic gradient  $i$ . The pressures  $p_1$  and  $p_2$  acting on the two ends of the length  $L$  of pipe produce a net force. This force, in the direction of flow, is

$$(p_1 - p_2) A$$

in which  $A$  is the cross sectional area of the pipe. This is opposed by an equal and opposite force generated by the shear stress  $\tau$  acting uniformly over the surface of the pipe wall. The area of pipe wall is  $P \cdot L$ , where  $P$  is the perimeter of the cross section, so the force due to shear stress is

$$\tau \cdot P \cdot L$$

Equating these forces:

$$(p_1 - p_2) A = \tau \cdot P \cdot L$$

This reduces, by the use of equation (3), to

$$\tau = \frac{A}{P} \rho gi \quad (4)$$

Now expressing  $A$  and  $P$  in terms of pipe diameter  $D$ , namely  $A = \pi D^2 / 4$  and  $P = \pi D$  so that  $(A/P) = D/4$ , we obtain the result:

$$\tau = \left( \frac{D}{4} \right) \rho gi \quad (5)$$

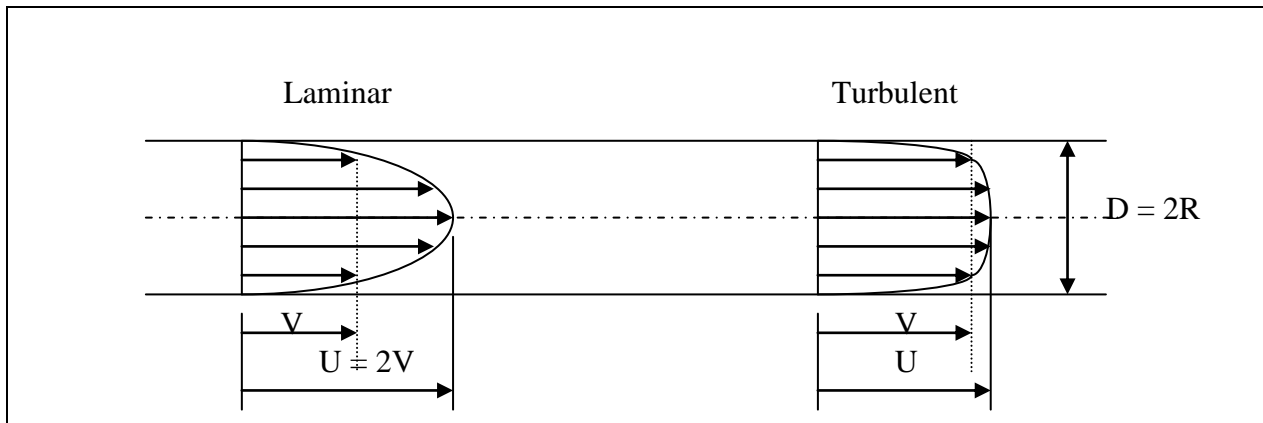
We may reasonably expect that  $\tau$  would increase in some way with increasing rate of flow. The relationship is not a simple one, and to understand it we must learn something about the nature of the motion, first described by Osborne Reynolds in 1883. By observing the behavior of a filament of dye introduced into the flow along a glass tube, he demonstrated the existence of two different types of motion. At low velocities, the filament appeared as a straight line passing down the whole length of the tube, indicating smooth or laminar flow. As the velocity was gradually increased in small steps, he observed that the filament, after passing a little way along the tube, mixed suddenly with the surrounding water, indicating a change to turbulent motion. Similarly, if the velocities were decreased in small steps, a transition from turbulent to laminar motion suddenly occurred. Experiments with pipes of different diameters and with water at various temperatures led Reynolds to conclude that the parameter, which determines whether the flow shall be laminar or turbulent in any particular case is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{V D}{\nu} \quad (6)$$

in which

|        |   |  |
|--------|---|--|
| Re     | = | Reynolds Number of the motion  |
| $\rho$ | = | Density of the fluid   |
| V      | = | Q/A denotes the mean velocity of flow, obtained by dividing the discharge rate Q by the cross sectional area A |
| $\mu$  | = | Coefficient of absolute viscosity of the fluid   |
| $\nu$  | = | $\mu / \rho$ Denotes the coefficient of kinematics viscosity of the fluid                                      |

The motion will be laminar or turbulent according as to whether the value of Re is less than or greater than a certain critical value. Experiments made with increasing flow rates show that the critical value of Re for transition to turbulent flow depends on the degree of care taken to eliminate disturbances in the supply and along the pipe. On the other hand, experiments with decreasing flow rates show that transition to laminar flow takes place at a value of Re which is much less sensitive to initial disturbance. This lower value of Re is found experimentally to be about 2000. Below this, the pipe flow becomes laminar sufficiently downstream of any disturbance, no matter how severe.



**Fig. 2 Velocity distributions in laminar and turbulent pipe flows**

Figure 2 illustrates the difference between velocity profiles across the pipe cross sections in laminar and turbulent flow. In each case the velocity rises from zero at the wall to a maximum value U at the center of the pipe. The mean velocity V is of course less than U in both cases.

In the case of laminar flow, the velocity profile is parabolic. The ratio U/V of center line velocity to mean velocity is

$$\frac{U}{V} = 2 \quad (7)$$

And the velocity gradient at the wall is given by

$$\left( \frac{du}{dr} \right)_R = -\frac{4U}{D} = -\frac{8V}{D} \quad (8)$$

So that the wall shear stress  $\tau$  due to fluid viscosity is



$$\tau = \frac{8\mu V}{D} \quad (9)$$

Substituting for  $\tau$  in equation (5) from this equation leads to the result

$$i = \frac{32\nu W}{gD^2} \quad (10)$$

Which is the Poiseuille's equation.

In the case of turbulent flow, the velocity distribution is much flatter over most of the pipe cross section. As the Reynolds number increases, the profile becomes increasingly flat, the ratio of maximum to mean velocity reducing slightly. Typically,  $U/V$  falls from about 1.24 to about 1.12 as  $Re$  increases from  $10^4$  to  $10^7$ .

Because of the turbulent nature of the flow, it is not possible now to find a simple expression for the wall shear stress, so the value has to be found experimentally. When considering such experimental results, we might reasonably relate the wall shear stress  $\tau$  to the mean velocity pressure  $\frac{1}{2} \rho V^2$ . So a dimensionless **friction factor f** could be defined by

$$\tau = f \frac{1}{2} \rho V^2 \quad (11)$$

The hydraulic gradient  $i$  may now be expressed in terms of  $f$  by use of equation (5), and the following result is readily obtained:

$$i = \frac{4f}{D} \frac{V^2}{2g} \quad (12)$$

Therefore, the  $(h_1 - h_2)$  between sections 1 and 2 of a pipe of diameter  $D$ , along which the mean velocity is  $V$ , is seen from equation (2) to be given by

$$h_1 - h_2 = 4f \frac{L}{D} \frac{V^2}{2g} \quad (13)$$

where  $L$  is the length of pipe run between the sections. This is frequently referred to as Darcy's equation.

The results of many experiments on turbulent flow along pipes with smooth walls have shown  $f$  to be a slowly decreasing function of  $Re$ . Various correlations of the experimental data have been proposed, one of which is

$$\frac{1}{\sqrt{f}} = 4 \log(Re \sqrt{f}) - 0.4 \quad (14)$$

This expression, which is due to Prandtl, fits experimental results well in the range of  $Re$  from  $10^4$  to  $10^7$ , although it does have the slight disadvantage that  $f$  is not given explicitly.

Another correlation, due to Blasius, is:

$$f = 0.079 \text{ Re}^{-1/4} \quad (15)$$

This gives explicit values, which are in agreement with those from the more complicated equation (3.14) to within about 2% over the limited range of Re from  $10^4$  to  $10^5$ . Above  $10^5$ , however, the Blasius equation diverges substantially from experiment.

We have seen that when the flow is turbulent it is necessary to resort to experiment to find  $f$  as function of Re. However, in the case of laminar flow, the value of  $f$  may be found theoretically from Poiseuille's equation. Equating the expressions for  $I$  in equation (10) and (12):

$$\frac{32\mathcal{W}}{gD^2} = \frac{4f}{D} \frac{V^2}{2g}$$

After reduction this gives the result

$$f = \frac{16}{\text{Re}} \quad (16)$$

In summary, the hydraulic gradient  $i$  may conveniently be expressed in terms of a dimensionless wall friction factor  $f$ . This factor has the theoretical value  $f = 16/\text{Re}$  for laminar flow along a smooth walled pipe. There is no corresponding theoretical for turbulent flow, but good correlation of many experimental results on smooth walled pipes is given by equations such as (14) and (15).

For an incompressible flow in a mine airway the frictional pressure loss,  $\Delta P$ , over a certain length  $L$ , is given by:

$$\Delta P = \frac{f \rho v^2}{2D} = \frac{K S Q^2}{A^{3/5}} \quad (\text{Pa})$$

$f$  = Darcy-Weisbach resistance coefficient (dimensionless)

$L$  = length of airway (m)

$U$  = average velocity of airflow, m/s.

$\rho$  = Air density ( $\text{kg/m}^3$ )

$D$  = equivalent diameter of airway, m

$A$  = airway cross-sectional area,  $\text{m}^2$

$P$  = perimeter of the airway, m

$K$  = coefficient of friction =  $f\rho/8 \text{ N s}^2/\text{m}^4$

$S$  = rubbing surface =  $P * L$ ,  $\text{m}^2$

$Q$  = quantity flow rate,  $\text{m}^3/\text{s}$ .

The Darcy-Weisbach resistance coefficient is a function of the airway surface roughness, and the flow regime, given by the Reynolds number, (Re).

However, in practice, the density of air does not vary much in a mine and the flow in most cases can be considered turbulent enough for the value of ' $f$ ' to remain constant irrespective of Reynolds number of flow. Thus, the coefficient of friction  $K$  is essentially considered to vary with the surface roughness.

Based on the Reynolds number and the surface roughness, the value of 'f' is either read from the charts or computed empirically. For a smooth pipe flow (absence of surface roughness) two such empirical relationships exist.

Blasius Equation:

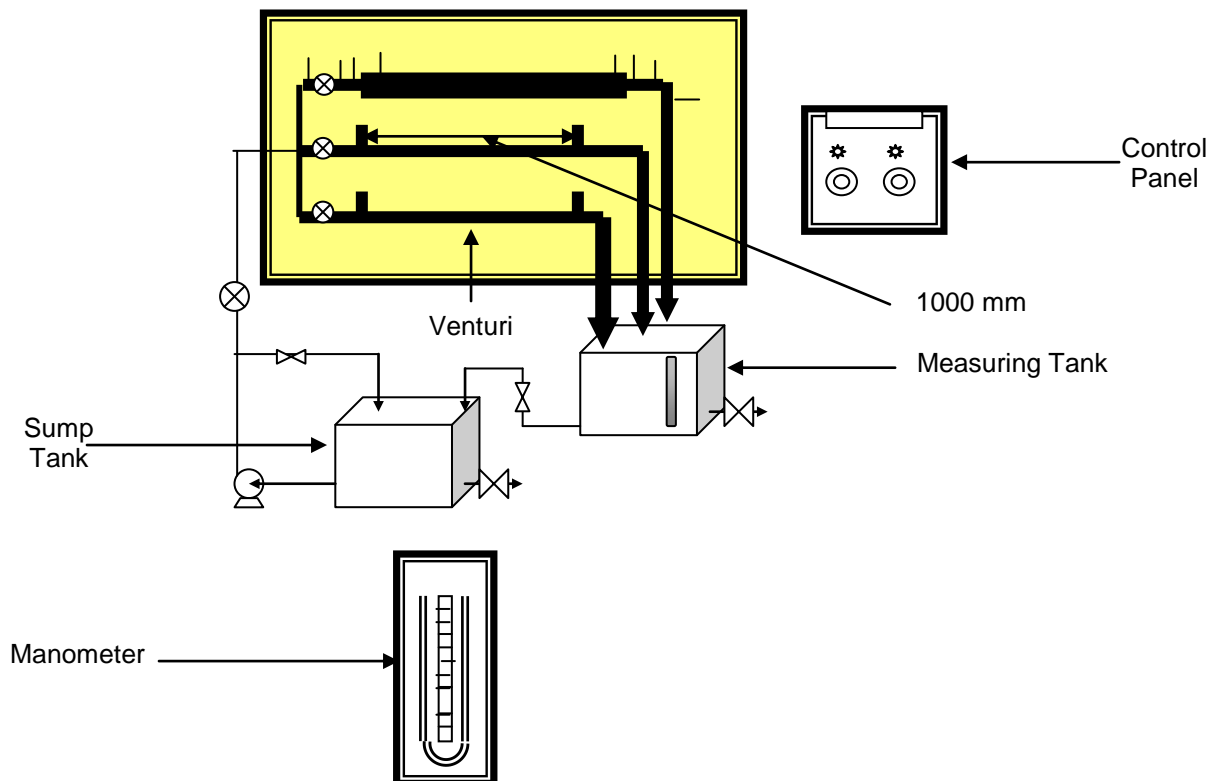
$$f = 0.316/(1/4) \text{-----} (2000 < Re < 100000)$$

Karman-Prandtl Equation:

$$f^{1/2} = [2 \log (Re * f^{1/2}) - 0.8] \text{-----} (\text{for } Re > 100,000)$$

The Reynolds number is given by  $Re = vD/\nu$ , where v is the average air velocity (m/s), D is the equivalent diameter (m), and  $\nu$  is the kinematic viscosity ( $m^2/s$ ). For the average ambient temperature  $\nu$  may be taken to be  $1.6 * 10^{-5} m^2/s$ .

## Apparatus



### Description of Apparatus:

The apparatus consists 25mm and 12 mm diameter pipes of GI having 1000 mm length for test section. Manometer tapping are made at an upstream and downstream of the test section which lie away from the pipe entry and exit. These clear lengths upstream and downstream of the test section are required to ensure that the results are not affected by the disturbances originating at the entrance or the exit of the pipe. The manometer tapping are connected to a manometer, which reads head loss in mm of mercury gauge.

**Test pipes:**

MOC : SS 304  
Size : 12 mm, 25 mm, mm Dia.  
Length : 1 meter

Pressure tapings are provided at suitable places. The flow through each pipe can be controlled using the valves provided.

**Sump Tank:**

Capacity : 50 liter  
Size : 400 \* 400 \* 300 mm  
MOC : SS – 304  
With ½” drain valve

**Pump:**

MOC : SS 304  
Type : Monoblock  
Power : 0.5 HP, Single Phase  
Delivery Size : 1/2”

**Measuring Tank:**

Capacity : 30 liter  
Size : 300 \* 300 \* 320 mm  
MOC : SS - 304,

with self graduated glass tube level indicator and With 1/2” drain valve and piping leading the water to the sump tank.

Necessary piping and valves (of SS – 304) is provided to supply water to flow meters. A 750 mm U-tube manometer filled with mercury (100 mm in each limb).

**Procedure:**

1. Fill the sump tank with water.
2. Attach the limbs of the manometer with the help of the flexible pipe to the pressure tapings of the pipe across which the pressure drop is to be determined.
3. Switch on the pump and adjust the flow through the pipe with the help of supply valve and bypass valve provided.
4. Remove any air from the system by opening the valve provided on the manometer.
5. After steady state is achieved record the difference in manometer limbs.
6. Collect about known quantity (height difference) of water in the collection tank with liquid level indicator and note down the time required for the same.
7. Repeat the above at least four - five times starting from the minimum flowrate and going to the maximum value.
8. Select another pipe and repeat the experiment.

**Observation:**

| PIPE                            |                |                   | 1                 | 2                 |
|---------------------------------|----------------|-------------------|-------------------|-------------------|
| DIAMETER OF PIPE                | D              | M                 | D <sub>1</sub> =  | D <sub>2</sub> =  |
| CROSS SECTIONAL AREA OF PIPE    | $\pi D^2/4$    | M <sup>2</sup>    | AP <sub>1</sub> = | AP <sub>2</sub> = |
| HT. DIFF. IN MEASURING TANK (M) | H <sub>1</sub> | M                 |                   |                   |
|                                 | H <sub>2</sub> | M                 |                   |                   |
|                                 | $\Delta H$     | M                 |                   |                   |
| VOLUME                          | V              | M <sup>3</sup>    |                   |                   |
| TIME                            | T              | SEC.              |                   |                   |
| VOL. FLOWRATE                   | Q              | M <sup>3</sup> /S |                   |                   |
| MANOMETER READING               | X <sub>1</sub> | M                 |                   |                   |
|                                 | X <sub>2</sub> | M                 |                   |                   |
|                                 | $\Delta X$     | M                 |                   |                   |
| I                               |                |                   |                   |                   |
| N <sub>RE</sub>                 |                |                   |                   |                   |
| F                               |                |                   |                   |                   |
| LOG I                           |                |                   |                   |                   |
| LOG N <sub>RE</sub>             |                |                   |                   |                   |
| LOG F                           |                |                   |                   |                   |

**CALCULATIONS:**

Height Difference in measuring tank =  $\Delta H$   
=  $H_1 - H_2$   
= \_\_\_\_\_ m

Area of Measuring Tank =  $A_m$   
=  
= \_\_\_\_\_  $m^2$

Volume Collected =  $V$   
=  $\Delta H * A_m$   
= \_\_\_\_\_  $m^3$

Volumetric Flowrate =  $Q$   
=  $V / t$   
= \_\_\_\_\_  $m^3/s$

Velocity of Fluid in pipe - 1 =  $v$   
=  $Q / A_p$   
= \_\_\_\_\_ m/s

Velocity Head in pipe - 1 =  $v^2 / (2 * g)$   
=  
= \_\_\_\_\_ m

Hydraulic Gradient of pipe - 1 =  $i$   
=  $\frac{12.6 * \Delta X}{L}$   
=  
= \_\_\_\_\_

$$\begin{aligned}
 \text{Reynolds Number for Pipe - 1} &= N_{Re} \\
 &= \frac{VD}{\nu} \\
 &= \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Pipe Friction Factor for pipe - 1} &= f \\
 &= i \frac{D}{4} \frac{1}{V^2 / 2g} \\
 &= \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Velocity of Fluid in pipe - 2} &= v \\
 &= Q / A_p \\
 &= \underline{\hspace{2cm}} \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 \text{Velocity Head in pipe - 2} &= v^2 / (2 * g) \\
 &= \\
 &= \underline{\hspace{2cm}} \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Hydraulic Gradient of pipe - 2} &= i \\
 &= \frac{12.6 * \Delta X}{L} \\
 &= \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Reynolds Number for Pipe - 2} &= N_{Re} \\
 &= \frac{VD}{\nu} \\
 &= \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

Pipe Friction Factor for pipe - 2

= f

$$= i \frac{D}{4} \frac{1}{V^2 / 2g}$$

=

= \_\_\_\_\_

### Graph:

Plot log i and log f as a function of log Re.

### Results:

### Conclusion:

### Quiz

1. Frictional loss in fittings is ----- that in straight pipes.

1. **Greater than**
2. Smaller than
3. Equal to

2. Equivalent length of a wide open gate valve is ----- the equivalent length of a half open gate valve of the same nominal size?

1. Lesser than
2. Greater than
3. **Same as**



# Flow through Orifice/Venturi meter (CO-3130502.4)

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## Objective

1. Determine the effect of Reynolds number on the coefficient of discharge (Cd).

## Chemicals

1. water

## Apparatus

1. Venturi meter setup, Manometer

## Theory

The most important class of flow meter is that in which the flow is either accelerated or retarded at the measuring sections by reducing the flow area, and the change in the kinetic energy is measured by recording the pressure difference produced.

This class includes:

1. **The Pitot tube:**

In which a small element of fluid is brought to rest at an orifice situated at right angles to the direction of flow. The flow rate is then obtained from the difference between the impact and the static pressure. With this instrument the velocity measured is that of a small filament of fluid.

2. **The Orifice meter:**

In which the fluid is accelerated at a sudden constriction (the orifice) and the pressure developed is then measured. This is a relatively cheap and reliable instrument though the overall pressure drop is high because most of the kinetic energy of the fluid at the orifice is wasted.

3. **The Venturi meter:**

In which the fluid is gradually accelerated to a throat and gradually retarded as the flow channel is expanded to the pipe size. A high proportion of kinetic energy is thus recovered but the instrument is expensive and bulky.

4. **The Nozzle:**

In which the fluid is gradually accelerated up to the throat of the instrument but expansion to pipe diameter is sudden as with an orifice. This instrument is again expensive because of the accuracy required over the inlet section.

5. **The Notch or Weir:**

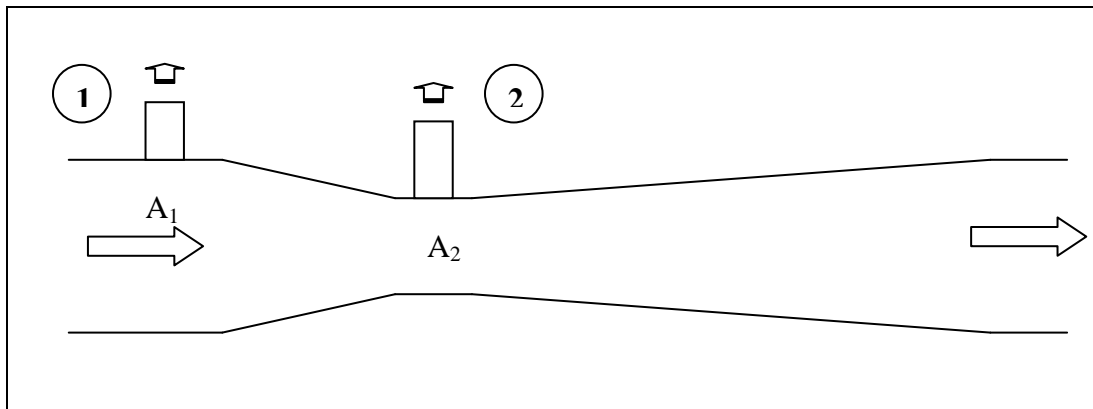
In which the fluid flows over the weir so that its kinetic energy is measured by determining the head of the fluid flowing over the

weir. This instrument is used in open channel flow and extensively in tray towers, where the height of the weir is adjusted to provide the necessary liquid depth for a given flow.

**6. The Variable Area meter or Rotameter:**

In the meters so far described the area of the constriction or orifice is constant and the drop in pressure is dependent on the rate of flow. In the variable area meter, the drop in pressure is constant and the Flowrate is a function of the constriction. A typical meter of this kind is commonly known as a Rotameter.

**VENTURIMETER**



**Fig. Venturimeter**

The Venturi tube (after Giovanni Battista Venturi (1746-1822), who performed experiments on flow in tapered tubes, including the convergent-divergent combination used in the Venturi meter of today) is a device, which has been used over many years for measuring the rate of flow along a pipe. As may be seen from figure, it consists essentially of a tapering contraction section, along which the fluid accelerates towards a short cylindrical throat, followed by a section which diverges gently back to the original diameter. (Such a slowly diverging section is frequently referred to as a diffuser). As the velocity increases from the inlet section to the throat, there is a fall in pressure, the magnitude of which depends on the rate of flow. The flow rate may therefore be inferred from the difference in pressure, as measured by manometers placed between the upstream end and the throat. Such a unit is referred to as a Venturi flow meter.

**Theory of the Venturi Meter:**

Consider flow of an incompressible, inviscid fluid through the convergent – divergent Venturi tube shown in Figure. The cross sectional area at the upstream section 1 is  $A_1$ , at the throat section 2 is  $A_2$ , and at any other arbitrary section  $n$  is  $A_n$ . Manometer tappings at these sections register  $h_1$  and  $h_2$ . Assume that both the velocity and the pressure head are constant over each of the sections considered. This amounts to assuming the flow to be one dimensional, so that the velocity and the pressure head vary only in the direction

of the tube length. We may then treat the convergent – divergent pipe as a stream tube, along which Bernoulli's theorem states.

$$\frac{u_1^2}{2g} + h_1 = \frac{u_2^2}{2g} + h_2 \quad (1.1)$$

in which  $u_1$  and  $u_2$  are the flow velocities at sections 1 and 2.

The equation of continuity is

$$u_1 A_1 = u_2 A_2 = Q \quad (1.2)$$

In which  $Q$  denotes the rate of volume flow or discharge. Substituting in Equation (1.1) for  $u_1$  from Equation (1.2), gives

$$\frac{u_2^2}{2g} \left[ \frac{A_2}{A_1} \right]^2 + h_1 = \frac{u_2^2}{2g} + h_2$$

and solving this for the velocity  $u_2$  in the throat leads to

$$u_2 = \sqrt{\frac{2g(h_1 - h_2)}{1 - (A_2 / A_1)^2}}$$

The rate of flow  $Q$  is found by multiplying the throat velocity  $u_2$  by the cross sectional area  $A_2$  at the throat, giving

$$Q = A_2 \sqrt{\frac{2g(h_1 - h_2)}{1 - (A_2 / A_1)^2}} \quad (1.3)$$

This is the **ideal** discharge rate, obtained by assuming inviscid, one-dimensional flow. In practice, there is some loss of head between sections 1 and 2. Also, the velocity is not absolutely constant across either of these sections. As a result, the actual values of  $Q$  fall a little short of those given by Equation (1.3). It is customary to allow for this by writing,

$$Q = CA_2 \sqrt{\frac{2g(h_1 - h_2)}{1 - (A_2 / A_1)^2}} \quad (1.4)$$

in which  $C$  is known as the discharge coefficient or simply the coefficient of the Venturi meter. Its value, which usually lies between 0.92 and 0.99, is established by experiment. It varies from one meter to another, and even for a given meter it may vary slightly with the flow rate.

Another way of metering the flow would be to insert a sharp edged orifice into the pipe; the differential pressure produced by flow through the orifice may similarly be used to infer the flow rate. Such a orifice meter has the advantage of simplicity and cheapness. In comparison with the Venturi tube, however, it causes a greater loss of total head than does a corresponding Venturi meter. This is because much of the velocity head at the throat is recovered as the fluid decelerates in the diffuser. Indeed, the differential Manometric head from inlet to the throat can be several times as great as the loss of total head across the whole device.

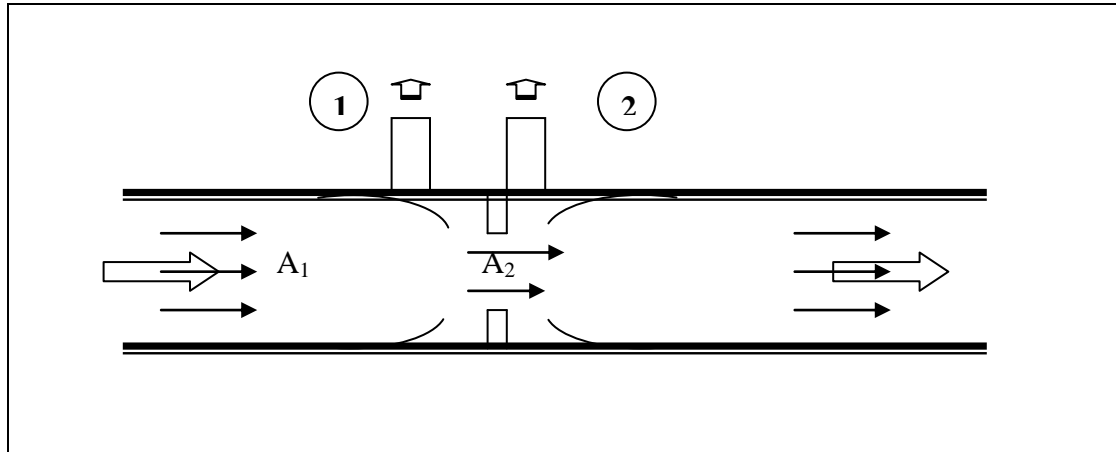
### **Orifice meter**

This is a measuring device where the fluid is accelerated by causing it to flow through a constriction, the kinetic energy is thereby increased and the pressure energy therefore decreases. The Flowrate is obtained by measuring the pressure difference between the inlet of the meter and a point of reduced pressure, as shown in figure. If the pressure difference is measured a short distance upstream where the flow is undisturbed and at the position where the area of the flow is a minimum.

The most important factors influencing the reading of an Orificemeter are the size of the orifice and the diameter of the pipe in which it is fitted, though a number of other factors do affect the reading to some extent. Thus the exact position and the method of fixing the pressure tappings are important because the area of flow, and hence the velocity, radially changes in the region of the orifice. The meter should not be located less than 50 pipe diameters from any pipefitting though the standard applies only for pipes of at least 150 mm diameter. The size of orifice is chosen to give convenient pressure drop. Although the Flowrate is proportional to the square root of the pressure drop, it is difficult to cover the wide range in flow with any size of orifice. Unlike the Pitot tube, the orifice meter gives the average Flowrate from a single reading.

The most serious disadvantage of the meter is that most of the pressures drop is not recoverable; that is it is inefficient. The velocity of the fluid is increased at the throat without much loss of total energy. The fluid is subsequently retarded as it mixes with the relatively slow-moving fluid downstream from the orifice. A high degree of turbulence is set up and most of the excess kinetic energy is decipitated as heat. Usually only about 5 or 10 % of the excess kinetic energy can be recovered as pressure energy. The pressure drop over the orifice meter is therefore high, and may prelude it from being used in a particular instance.

An orifice meter is used to measure the discharge in a pipe. An orifice meter, in its simplest form, consists of a plate having a sharp edged circular hole is known as an orifice. This plate is fixed inside a pipe as shown in Figure 2.1.



**Fig. Orificemeter**

**Theory of the Orifice Meter:**

Consider flow of an incompressible, inviscid fluid through the orifice plate shown in Figure. The cross sectional area at the upstream section 1 is  $A_1$  and at the plate section 2 is  $A_2$ . Manometer tappings at these sections register  $h_1$  and  $h_2$ . Assume that both the velocity and the pressure head are constant over each of the sections considered. This amounts to assuming the flow to be one dimensional, so that the velocity and the pressure head vary only in the direction of the flow. We may then apply Bernoulli's theorem across section 1 and section 2.

$$\frac{u_1^2}{2g} + h_1 = \frac{u_2^2}{2g} + h_2 \quad (2.1)$$

In which  $u_1$  and  $u_2$  are the flow velocities at sections 1 and 2.

The equation of continuity is

$$u_1 A_1 = u_2 A_2 = Q \quad (2.2)$$

In which  $Q$  denotes the rate of volume flow or discharge. Substituting in Equation (2.1) for  $u_1$  from Equation (2.2), gives

$$\frac{u_2^2}{2g} \left[ \frac{A_2}{A_1} \right]^2 + h_1 = \frac{u_2^2}{2g} + h_2$$

and solving this for the velocity  $u_2$  in the throat leads to

$$u_2 = \sqrt{\frac{2g(h_1 - h_2)}{1 - (A_2 / A_1)^2}}$$

The rate of flow  $Q$  is found by multiplying the throat velocity  $u_2$  by the cross sectional area  $A_2$  at the throat, giving

$$Q = A_2 \sqrt{\frac{2g(h_1 - h_2)}{1 - (A_2 / A_1)^2}} \quad (2.3)$$

This is the **ideal** discharge rate, obtained by assuming inviscid, one-dimensional flow. In practice, there is some loss of head between sections 1 and 2. Also, the velocity is not absolutely constant across either of these sections. As a result, the actual values of Q fall a little short of those given by Equation (2.3). It is customary to allow for this by writing,

$$Q = CA_2 \sqrt{\frac{2g(h_1 - h_2)}{1 - (A_2 / A_1)^2}} \quad (2.4)$$

In which C is known as the discharge coefficient or simply the coefficient of the Orifice meter. Its value, which usually lies between 0.6 and 0.65, is established by experiment. It varies from one meter to another, and even for a given meter it may vary slightly with the flow rate.

### Rotameter (Variable Area Meter):

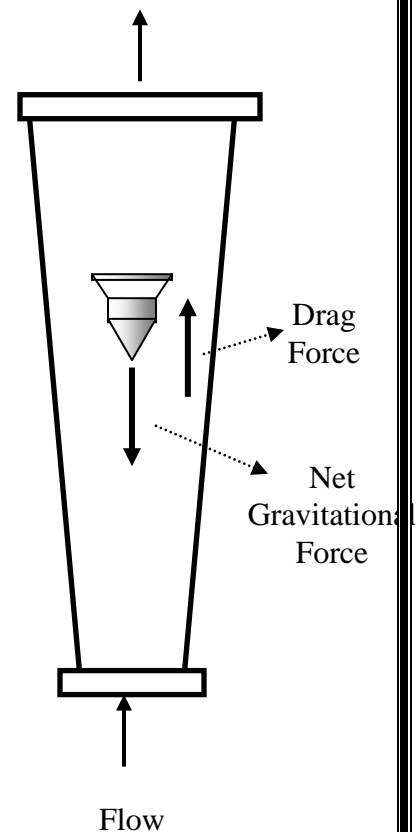
A typical meter of this kind, which is commonly as a Rotameter consists of a tapered tube with the smallest diameter at the bottom. The tube contains a freely moving float, which rests on a stop at the base of the tube. When the fluid is flowing the float rises until its weight is balanced by the upthrust of the fluid, its position then indicating the rate of flow. The pressure difference across the float is equal to its weight divided by its maximum cross sectional area in a horizontal plane. The area of annulus is the annulus formed between the float and the wall of the tube.

### Theory of Rotameter:

Variable area meter – Rotameter consist of a tapered tube with the smallest diameter at the bottom. The tube contains a freely moving float, which rests on a stop at the base of the tube. When fluid is flowing the float rises until its weight is balanced by the upthrust of the fluid, its position then indicating the rate of flow. The pressure difference across the float is equal to its weight divided by its maximum cross-sectional area in a horizontal plane. The area for flow is the annulus formed between the float and the wall of the tube.

For a given flow rate, the equilibrium position of the float in a Rotameter is established by a balance of three forces:

1. The weight of the float, (Acting Downwards)
2. The buoyant force of the fluid on the float, (Acting Upwards)



### 3. The drag force on the float (Acting Upwards).

For Equilibrium,

$$F_D g_c = v_f \rho_f g - v_f \rho g$$

Where,

- $F_D$  = Drag Force,
- $g$  = Acceleration of Gravity,
- $g_c$  = Newton's – law proportionality factor,
- $v_f$  = Volume of float,
- $\rho_f$  = Density of Float,
- $\rho$  = Density of Fluid.

The quantity  $v_f$  can be replaced by  $m_f / \rho_f$ , where  $m_f$  is the mass of the float, and equation becomes:

$$F_D g_c = m_f g [1 - (\rho / \rho_f)]$$

For a given meter operating on a certain fluid, the right-hand side of equation is constant and independent of the flow rate. Therefore  $F_D$  is also constant, and when the flow rate increases, the position of the float must change to keep the drag force constant. The drag force  $F_D$  can be expressed as a drag force co-efficient times the projected area of the float and the velocity head, but the velocity head is based on the maximum velocity past the float, which occurs at the largest diameter or metering edge of the float. Thus,

$$F_D = A_f C_D \rho \frac{u_{\max}^2}{2 g_c}$$

If the change in drag coefficient is small, which is usually the case for large Rotameters with low- or moderate- viscosity fluids, the maximum velocity stays the same with increasing flow rate, and total flow rate is proportional to the annular area between the float and the wall:

$$q = u_{\max} \frac{\pi}{4} (D_t^2 - D_f^2)$$

Where,

- $D_f$  = Float Diameter,
- $D_t$  = Tube Diameter

For a linearly tapered tube with a diameter at the bottom about equal to the float diameter, the area for flow is a quadratic function of the height of the float  $h$ ;

$$(D_t^2 - D_f^2) = (D_f + a h)^2 - D_f^2 = 2 D_f a h + a^2 h^2$$

When the clearance between float and tube wall is small, the term  $a^2 h^2$  is relatively unimportant and the flow is almost a linear function of the height  $h$ . therefore Rotameters tend to have a nearly linear relationship between flow and position of the float, compared with a calibration curve for an Orificemeter, for which the flow rate is proportional to the square root of the reading. The calibration of a Rotameter, unlike that of an orifice meter,

is not sensitive to the velocity distribution in the approaching stream, and neither long, straight, neither approaches nor straightening vanes are necessary.

Thus this meter may be considered as an orifice meter with a variable aperture, and the formulae already derived are therefore applicable with only minor changes. Both in the orifice – type meter and in the Rotameter the pressure drop arises from the conversion of pressure energy to kinetic energy and from frictional losses, which are accounted for in the coefficient of discharge. The pressure difference over the float  $-\Delta P$ , is given by:

$$-\Delta P = \frac{V_f (\rho_f - \rho) g}{A_f}$$

Where  $V_f$  is the volume of the float,  $\rho_f$  the density of the material of the float, and  $A_f$  is the maximum cross-sectional area of the float in a horizontal plane.

If the area of the annulus between the float and tube is  $A_2$  and the cross-sectional area of the tube is  $A_1$ , then

$$G = C_D A_2 \sqrt{\frac{2(-\Delta P)\rho}{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

Substituting for  $-\Delta P$  in above equation,, we have:

$$G = C_D A_2 \sqrt{\frac{2gV_f (\rho_f - \rho)\rho}{A_f [1 - \left(\frac{A_2}{A_1}\right)^2]}}$$

The coefficient  $C_D$  depends on the shape of the float and the Reynolds number (based on the velocity in the annulus and the hydraulic diameter of the annulus) for the flow through the annulus space of area  $A_2$ . In general, floats, which give the most nearly constant coefficient, are of such a shape that they set up eddy currents and give low values of  $C_D$ . The variation in the  $C_D$  largely arises from difference in viscous drag of fluid on the float, and if turbulence is artificially increased, the drag force rises quickly to a limiting but high value.

The constant coefficient for float  $C$  arises from turbulence promotion, and for this reason the coefficient is also substantially independent of the fluid viscosity. The meter can be relatively insensitive to changes in the density of the fluid by selection of the density of the float,  $\rho_f$ . thus the Flowrate for a given meter will be independent of  $\rho$  when  $dG/d\rho = 0$ .

The range of a meter can be increased by the use of floats of different densities, a given float covering a Flowrate range of about 10:1. for high pressure work the glass tube is



replaced by a metal tube. When a metal tube is used or when the liquid is very dark or dirty an external indicator is required.

**ADVANTAGES OF ROTAMETERS:**

- Direct visual index of flow rate,
- It can be used in wide range,
- It has nearly linear scale,
- It provides constant head loss,
- No extra pipelines as calming section are required for stabilizing the flow.

**DISADVANTAGE OF ROTAMETER:**

- It cannot be used in horizontal position.

**TECHNICAL SPECIFICATION:**

**Sump Tank:**

Capacity = 150 liter  
MOC = MS.  
Dimensions = 600 mm \*500 mm \*500mm

**Measuring tank:**

Capacity = 60 liter  
MOC = MS  
Dimensions = 400\*500\*300  
With self graduated glass level indicator.

**Pump:**

Type = Monoblock  
Power = 1 HP, Single Phase  
Delivery Size = 1”

**Venturimeter:**

Size = 26 mm  
Throat Size = 16 mm  
Dia Ratio = 0.6 – 0.64  
MOC = Brass

**Orificemeter:**

Size = 26 mm  
Orifice Size = 16 mm  
Dia Ratio = 0.6 – 0.64  
MOC = SS

**Rotameter:**

Range = 1-10 LPM  
MOC = Glass tube housed in MS Box and two perpex side

**Control Panel** Consisting of ON/OFF toggle switch & indicator lamp for pump and U-tube manometer filled with mercury (100 mm in each limb). The manometer is equipped with wooden scale (500 mm) and it is mounted on wooden board with necessary air venting system to vent system & drive out the air initially present in it.

**DESCRIPTION OF APPARATUS:**

The equipment consists of three flow measuring devices. i.e. Venturimeter, Orificemeter, and Rotameter. The supply tank / sump tank is connected with pump. The pumps draw the water to the different meters by opening the control valve of respective meters. The rate of flow of water is controlled by control valve and bypass valve. The manometer tappings in the wall of the flanges of the orifice meter & Venturimeter are connected to vertical manometer tubes, mounted in front of a scale marked in Centimeters. Rotameter is connected to the sections of pipe by threaded connection. A flow divider is given at the measuring tank to collect water in measuring tank or to direct it directly into the supply tank. The control panel comprises of ON/ OFF switch and Indicator Lamp for pump.

**PROCEDURE:**

1. Fill the storage tank / sump tank with the water.
2. Switch On the pump and keep the control valve fully open and close the bypass valve to have maximum flow rate through the meter.
3. To calibrate Venturimeter open control valve of the same & close other two valves.
4. Open the vent cocks provided at the top of the manometer to drive out the air from the manometer limbs and close both of them as soon as water start coming out.
5. Note down the difference of level in the manometer limbs.
6. Keep the drain valve of the collection tank closed till its time to start collecting the water.
7. Close the drain valve of the collection tank and note down the initial level of the water in the collection tank.
8. Collect known quantity of water in the collection tank and note down the time required for the same.
9. Change the flow rate of water through the meter with the help of control valve and repeat the above procedure.
10. To calibrate Orificemeter open control valve of the same & close other two valves. Repeat the same procedure indicated in step 4 – 9.
11. To calibrate Rotameter open control valve of the same & close other two valves. Repeat the same procedure indicated in step 4 – 9. At this time close both manometer tappings.
12. Take about 5-6 readings for different flow rates.

**OBSERVATION:****Venturimeter:**

|                                |  |   |
|--------------------------------|--|---|
| Diameter at inlet              | $D_1$  | = 26 mm                                   |
| Diameter at throat             | $D_2$  | = 16 mm                                   |
| Cross sectional area at throat | $A_2 = \pi \frac{D_2^2}{4}$                            | = $2.01062 \times 10^{-4} \text{ m}^2$    |
| Area ratio, throat to inlet    | $\frac{A_2}{A_1} = \left( \frac{D_2}{D_1} \right)^2 =$ | $\left( \frac{16}{29} \right)^2 = 0.3044$ |

| Sr. No.   |            | 1 | 2 | 3 | 4 |
|---|------------|---|---|---|---|
| Manometer Reading<br>m                            | X1         |   |   |   |   |
|   | X2         |   |   |   |   |
|   | $\Delta X$ |   |   |   |   |
| Diff. Pressure<br>$\Delta h$<br>m                 |            |   |   |   |   |
| Measuring Tank Height<br>m                        | H1         |   |   |   |   |
|   | H2         |   |   |   |   |
|   | $\Delta H$ |   |   |   |   |
| Time<br>T<br>sec                                  |            |   |   |   |   |
| Volumetric Flowrate<br>Q<br>$\text{m}^3/\text{s}$ |            |   |   |   |   |
| Co-efficient of Discharge<br>$C_D$                |            |   |   |   |   |
| Log h   |            |   |   |   |   |
| Log Q   |            |   |   |   |   |

**CALCULATION:**

Difference in Manometer Limbs =  $\Delta X = X1 - X2$  = \_\_\_\_\_ m

Pressure Difference in terms of water =  $h = h1 - h2 = \Delta X * \frac{(13.6-1.0)}{1.0}$  = \_\_\_\_\_ m

Height Difference in measuring tank =  $\Delta H = H1 - H2$  = \_\_\_\_\_ m

|                        |                    |            |         |
|------------------------|--------------------|------------|---------|
| Area of Measuring Tank | = $A_m$            | = 0.096211 | $m^2$   |
| Volume Collected V     | = $\Delta H * A_m$ | = _____    | $m^3$   |
| Volumetric Flowrate    | = $Q = V / T$      | = _____    | $m^3/s$ |

Values of C in the right hand column of the table are calculated using Equation (1.4), namely

$$Q = CA_2 \sqrt{\frac{2g(h_1 - h_2)}{1 - (A_2 / A_1)^2}}$$

Before substituting various values of Q and  $(h_1 - h_2)$  into this equation, it is useful first to establish the value of

$$A_2 \left( \frac{2g}{1 - (A_2 / A_1)^2} \right)^{1/2}$$

This has the value

$$2.01062 \times 10^{-4} \left( \frac{2 \times 9.81}{1 - 0.3044^2} \right)^{1/2} = 9.3496 \times 10^{-4} m - s \text{ units}$$

Using this value in Equation we obtain

$$Q = C \times 9.3496 \times 10^{-4} (h_1 - h_2)^{1/2}$$

or,

$$C = \frac{1069.56 * Q}{(h_1 - h_2)^{1/2}} = \underline{\hspace{2cm}}$$

**Result:**

**Conclusion:**

**Orificemeter:**

|                                 |  |   |
|---------------------------------|--|---|
| Diameter at inlet               | $D_1$  | = 26 mm                                 |
| Diameter at throat              | $D_2$  | = 16 mm                                 |
| Cross sectional area at Orifice | $A_2 = \pi \frac{D_2^2}{4}$                          | = $2.01062 \times 10^{-4} \text{ m}^2$  |
| Area ratio, throat to inlet     | $\frac{A_2}{A_1} = \left(\frac{D_2}{D_1}\right)^2 =$ | $\left(\frac{16}{29}\right)^2 = 0.3044$ |

| Sr. No.   |            | 1 | 2 | 3 | 4 |
|---|------------|---|---|---|---|
| Manometer Reading<br>m                            | X1         |   |   |   |   |
|   | X2         |   |   |   |   |
|   | $\Delta X$ |   |   |   |   |
| Diff. Pressure<br>$\Delta h$<br>m                 |            |   |   |   |   |
| Measuring Tank Height<br>m                        | H1         |   |   |   |   |
|   | H2         |   |   |   |   |
|   | $\Delta H$ |   |   |   |   |
| Time<br>T<br>sec                                  |            |   |   |   |   |
| Volumetric Flowrate<br>Q<br>$\text{m}^3/\text{s}$ |            |   |   |   |   |
| Co-efficient of Discharge<br>$C_D$                |            |   |   |   |   |
| Log h   |            |   |   |   |   |
| Log Q   |            |   |   |   |   |

**CALCULATION:**

Difference in Manometer Limbs =  $\Delta X = X1 - X2$  = \_\_\_\_\_ m

Pressure Difference in terms of water =  $h = h1 - h2 = \Delta X * \frac{(13.6-1.0)}{1.0}$  = \_\_\_\_\_ m

Height Difference in measuring tank =  $\Delta H = H1 - H2$  = \_\_\_\_\_ m

Area of Measuring Tank =  $A_m$  = 0.096211  $\text{m}^2$

|                     |                    |                           |
|---------------------|--------------------|---------------------------|
| Volume Collected V  | = $\Delta H * A_m$ | = _____ m <sup>3</sup>    |
| Volumetric Flowrate | = $Q = V / T$      | = _____ m <sup>3</sup> /s |

Values of C in the right hand column of the table are calculated using Equation (2.4), namely

$$Q = CA_2 \sqrt{\frac{2g(h_1 - h_2)}{1 - (A_2 / A_1)^2}}$$

Before substituting various values of Q and (h<sub>1</sub> - h<sub>2</sub>) into this equation, it is useful first to establish the value of

$$A_2 \left( \frac{2g}{1 - (A_2 / A_1)^2} \right)^{1/2}$$

This has the value

$$2.01062 \times 10^{-4} \left( \frac{2 \times 9.81}{1 - 0.3044^2} \right)^{1/2} = 9.3496 \times 10^{-4} m - s \text{ units}$$

Using this value in Equation we obtain

$$Q = C \times 9.3496 \times 10^{-4} (h_1 - h_2)^{1/2}$$

or,

$$C = \frac{1069.56 * Q}{(h_1 - h_2)^{1/2}} = \underline{\hspace{2cm}}$$

**Result:**

**Conclusion:**

**Rotameter:**

| Sr.No. | Rotameter Scale Reading in<br>R<br>LPM | Height Difference in Measuring Tank<br>H<br>(m) | Time required<br>t<br>(minute) | Volume collected<br>$V = \frac{(A * H)}{1000}$<br>Liter | Calculated Flowrate<br>$Q = V / t$<br>LPM |
|--------|--|---|--------------------------------|---|---|
|        |  |   |                                |   |   |
|        |  |   |                                |   |   |
|        |  |   |                                |   |   |
|        |  |   |                                |   |   |
|        |  |   |                                |   |   |
|        |  |   |                                |   |   |

**CALCULATION:**

Height Difference in measuring tank =  $\Delta H = H_1 - H_2$  = \_\_\_\_\_ m

Area of Measuring Tank =  $A_m$  = 0.096211 m<sup>2</sup>

Volume Collected V =  $\Delta H * A_m$  = \_\_\_\_\_ m<sup>3</sup>

Volumetric Flowrate =  $Q = V / T$  = \_\_\_\_\_ m<sup>3</sup>/s

Vol. Flowrate in LPM =  $Q (m^3/s) * 60000$  = \_\_\_\_\_ LPM

**GRAPH:**

Plot Rotameter Scale Reading (LPM) Vs Calculated flow rate (LPM) for both Rotameter separately.

**RESULT:**

**CONCLUSION:**

# Flow through v-notches (CO-3130502.4)

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## Objective

1. To know about notches and weirs, their applications and to calculate Cd of V-notch

## Chemicals

1. Water

## Theory

Different types of models are available to find discharge in an open channel as a notch, venture flume, weir etc. For calibration of rectangular notch, Trapezoidal notch or v-notch some flow is allowed in the flume. Once the flow becomes steady and uniform discharge coefficients can be determined for any model. In general, sharp crested notches are preferred where highly accurate discharge measurements are required, for example in hydraulic laboratories, industry and irrigation pilot schemes which do not carry derbies and sediments.

Notches are those overflow structures whose length of crest in the direction of flow is accurately shaped. They may be rectangular, trapezoidal, v-notch etc. The v-notch is one of the most precise discharge-measuring devices suitable for a wide range of flow.

A triangular or v notch is having a triangular or 'v' shaped opening provided in its body so that water is discharged through this opening only. The line which bisects the angle of the notch should be vertical and at the same distance from both sides of the channel. The discharge coefficients Cd of a V notch may be determined by applying formula:

$$C_d = \frac{Q}{\frac{8}{15}(\sqrt{2g})H^{\frac{5}{2}} \tan \frac{\theta}{2}}$$

Where, Q =discharge over a triangular notch,

$\theta$  = the apex angle of notch,

H = head over the crest of the notch.

A rectangular notch, symmetrically located in a vertical thin plate, which is placed perpendicular to the sides and the bottom of a straight channel, is defined as a rectangular sharp-crested weir.

The discharge coefficient  $C_d$  of a rectangular notch may be determined by applying formula:



$$C_d = \frac{Q}{\frac{2}{3}(\sqrt{2g})BH^{\frac{3}{2}}}$$

Where,

Q = Discharge over a rectangular notch,

B = Width of notch,

H = Head over the crest of the notch, and g is acceleration due to gravity.

### **Apparatus for experiment:**

A constant steady water supply tank (Notch tank) with baffle wall, pointer gauge, collecting tank, models of

- (i) V-notch

### **Experimental setup:**

The experimental set up consists of a tank where inlet section is provided with 1 nos. of baffles for streamline flow. While at the upstream portion of the tank one can fix a notch of V-notch. A hook gauge is used to measure the head of water over the model. A collecting tank is used to find the actual discharge through the notch.

### **Procedures:**

1. Position the notch under test at the end of the tank, in a vertical plane, and with the sharp edge on the upstream side.
2. Fill the tank with water up to the crest level and subsequently note down the crest level of the notch by the help of the point gauge.
3. Adjust the flow regulating valve to give the maximum possible discharge without flooding the notch.
4. Allow the conditions to attain steady state.
5. Measure the head over the notch, i.e. 'H<sub>2</sub>' with the help of point gauge.
6. Reduce the flow rate in stages and the readings of discharge and H<sub>2</sub> were taken.
7. Repeat the procedure for other type of notch.

## Observation:

### 1. TRIANGULAR OR 'V' NOTCH:

Apex angle of notch  $\theta =$

Crest level of 'V' notch,  $H_1 =$  \_\_\_\_\_

Area of collecting tank,  $A =$  m<sup>2</sup>

| Sr. No. | Discharge Measurement |                  |              |                                    | Final reading of water level above the notch $H_2$ (cm) | Head over notch $H = H_2 - H_1$ | $C_d$ |
|---------|-----------------------|------------------|--------------|------------------------------------|---|---------------------------------|-------|
|         | Initial $h_1$ (cm)    | Final $h_2$ (cm) | Time T (sec) | Discharge Q (cm <sup>3</sup> /sec) |   |                                 |       |
|         |                       |                  |              |                                    |   |                                 |       |
|         |                       |                  |              |                                    |   |                                 |       |
|         |                       |                  |              |                                    |   |                                 |       |

## Calculation:

1) Discharge,  $Q = \frac{\text{Area of collecting tank, } a \times (\text{Final, } h_2 - \text{Initial, } h_1)}{\text{Time, } t}$

2) Head over notch,  $H = H_2 - H_1$

3) Co-efficient of discharge,  $C_d = \frac{Q}{\frac{8}{15} (\sqrt{2g}) H^{\frac{5}{2}} \tan \frac{\theta}{2}}$

## Graphical method

Equation for flow rate is:

$$Q = \frac{8}{15} C_d (\sqrt{2g}) H^{5/2} \tan \theta/2$$

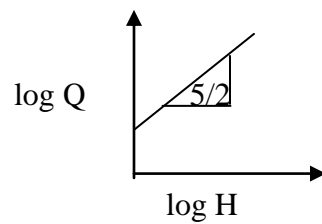
Taking log for both sides of above equation,

$$\log Q = \log \left[ \frac{8}{15} C_d (\sqrt{2g}) \tan \theta/2 \right] + 5/2 \log H.$$

Which is a straight-line equation similar to  $y = mx + C$ .

Where,  $y = \log Q$ ,  $x = \log H$ ,  $m = 5/2$ ,

$$c = \left[ \frac{8}{15} C_d (\sqrt{2g}) \tan \frac{\theta}{2} \right]$$



Plot the graph as explained above using  $\log Q$  &  $\log H$  data, read value of  $c$  from graph and find value of  $C_d$  graphically using following equation,

$$C_d = \frac{10^c}{\frac{8}{15} (\sqrt{2g}) \tan \frac{\theta}{2}}$$

**Result:**

|   | 'V' Notch |
|---|-----------|
| Co efficient of Discharge(Analytically) |           |
| Co efficient of Discharge(Graphically)  |           |

**Conclusion:****Questions:**

1. Define notch
2. Distinguish between notches and weirs.

# Fluidization apparatus

---

## Objective

1. To observe and study the behavior of bed during fluidization.

## Chemicals

1. water

## Apparatus

Fluidization setup, Manometer

## Theory

Fluidization is one of the methods available for contacting solids with fluids. The fluidization technique was first started by the petroleum refineries to find a better catalytic cracking process than the fixed bed. The main advantages of fluidized bed are greater interfacial surface area of contact, high rates of heat transfer, avoidance of hot spots and ease in solid handling. Fluidized beds also find applications in chemical reactions. Oxidation of naphthalene to phthalic anhydride using a fluidized bed is an example.

When a fluid passes upward through a bed of solids, there will be a certain pressure drop across the bed required to maintain the fluid flow. Depending upon the bed geometry, fluid velocity and particle characteristics the following phenomena occur with gradual increase in fluid velocity.

At low gas velocities, there is a pressure drop across the bed but the particles are stationary and the flow of fluid is through a fixed bed. As the bed velocity is gradually increased, a certain velocity is reached when the bed starts expanding. At this point the pressure drop across the bed equals the mass per unit area of the bed. This point is known as Fluidization Point; or Point of Onset of Fluidization or Incipient Fluidization. The movement of solids is at superficial velocities far below the terminal settling velocities of the solid particles and the process correspond to a situation which is approximately equivalent to hindered settling. The pressure drop is maximum at this point at this point the force exerted by fluid must not only act against the force of gravity on the particles but it must also overcome friction forces locking the particles together. Once the particles are separated, these forces drop off and the pressure required to maintain fluidization is less. As the velocities are still further increased the pressure drop continues to remain constant until the bed has assumed the loosest stable form of packing. With increase in fluid velocity the particles separate still further from one another, the bed continues to expand, the porosity increases, but the pressure difference does not change.

Particulate Fluidization, or Homogeneous Fluidization or Smooth Fluidization occurs when the difference in the density between the particle and fluid is small. This is typical of liquid fluidized bed. When the density difference between particle and fluid is very large, Aggregation Fluidization occurs. This is a typical of gas fluidized bed and is characterized by the gas rising through solid bed in a bubble form.

Froude's Number ( $V^2/gD_p$ ) is the criterion to determine the nature of fluidization. If  $N_{Fr}$  is less than unity, particulate fluidization occurs. Aggregative Fluidization occurs when  $N_{Fr}$  is greater than unity.

The pressure drop across a fixed bed, is given by Ergun's equation,

$$\frac{\Delta P}{\rho_f L} * \frac{e^3}{1 - e} * \frac{D_p}{V^2} = \frac{150 * (1 - e) * \mu_f}{D_p V \rho_f} \quad (1)$$

$$e, \text{ porosity at any time} = (L * A - W_s / \rho_p) / (L * A) = 1 - W_s / (LA \rho_p) \quad (2)$$

At the onset of fluidization, the pressure drop across the bed equals the weight of the bed per unit area of cross section.

$$\frac{\Delta P}{L} = g(\rho_p - \rho_f)(1 - e) \quad (3)$$

Substituting for  $\Delta P / L$  in equation (1) and solving for  $V_{mf}$ , the minimum fluidization velocity,

$$V_{mf} = D_p^2 g(\rho_p - \rho_f) e_{mf}^2 / (150 (1 - e_{mf}) \mu_f) \quad (4)$$

Leva has proposed a correlation to find minimum fluidizing velocity when gas is the fluidizing medium

$$V_{mf} = 0.0007 (N_{re})_{mf}^{-0.003} D_p^2 g(\rho_p - \rho_f) / \mu_f \quad (5)$$

### EXPERIMENTAL SETUP:

The experimental setup consists of a glass column of 48-50 mm diameter and 500 mm length. Pressure tappings are provided at the bottom and top of the column of the column to measure the pressure drop. Fluidizing Medium- Air is supplied with the help of 0.5 HP blower and through the supply valve and rotameter. Manometer is provided to provide the pressure drop across the column.

### PROCEDURE:

1. Add a weighed quantity of plastic balls in the column till the height of the bed equals the diameter of the column.
2. Switch on the air blower. Start with a minimum air flow rate by regulating globe valve. Allow steady state to be reached as indicated by the pressure and rotameter reading.
3. When the steady state is reached, note the rotameter reading and pressure drop as indicated by the water manometer.
4. Gradually increase the air flow rate and repeat the procedure. For every flow rate, observe the behavior of the bed.
5. Continue till the bed is fluidized and finally becomes turbulent (i.e. till there is no appreciable change in pressure drop indicated by manometer. Now gradually decrease the air flow back to zero, recording the same data as before.

### OBSERVATION TABLE:

Inside diameter of the column: m  
Cross sectional area of the column m<sup>2</sup>  
Length of the column (L) : m  
Fluidizing Medium : Air  
Density of the Fluid ( $\rho_f$ ) : kg/ m<sup>3</sup>  
Viscosity of the Fluid ( $\mu_f$ ) : kg/ m s  
Solids used : Plastic Balls  
Mass of Solids (m) : kg  
Density of Solids ( $\rho_s$ ) : kg/ m<sup>3</sup>  
Particle Diameter ( $D_p$ ) : m

| Sr. No. | Height of the Bed (H)<br>(m) | Flow Rate of Air (Q)<br>(m <sup>3</sup> / sec) | Superficial Velocity of Air (V)<br>(m/sec) | Pressure Drop across the column ( $\Delta P$ )<br>(N/m <sup>2</sup> ) | $\Delta P/ L$ | $N_{Re}$ |
|---------|------------------------------|--|--|---|---------------|----------|
|         |                              |  |  |   |               |          |
|         |                              |  |  |   |               |          |
|         |                              |  |  |   |               |          |

### **CALCULATIONS AND PLOTS:**

- Plot  $\Delta P/L$  ( $N/m^2$ )/ (height of the bed, m) as a function of  $V$ , m/s; on a log log plot. Show the ascending parts of the curve. Determine the incipient fluidization velocity corresponding to the intersection of the constant pressure line with descending velocity versus pressure line.
- Calculate  $\Delta P/L$  using equation (3) and  $V_{mf}$  using equation (4) and compare them with the values obtained from the graph.
- Plot  $e$  Vs.  $N_{Re}$ . Read from the graph the value of  $N_{Re}$  at  $e$  equal to unity. Calculate the velocity from the  $N_{Re}$  at  $e$  equal to unity.
- Calculate Froudes number at minimum fluidizing velocity. Check that it is less than unity.