Lect_15

Logical Agents

- Knowledge: Representation/store Knowledge
- Inference/Derive new facts/actions
- Learning

Ex: A doctor — acquired knowledge during studies in college

Expert system for medical diagnosis (MYSIN)

- Interacts with patient, takes symptoms as i/p, asks questions and predicts desease ~~a~~ as O/p.
- Learn based on experience

Ex: Driverless car – Traffic rules, basic driving skills
map

– percept or i/p is taken through camera.
Action is taken – e.g. apply brake or
reduce speed.

– Learning

Knowledge Representation
Inference
Learning

1. Knowledge-based Agent (KB-Agent)

t - time, starts at 0.

function KB-Agent (percept) returns action

TELL (KB, MAKE-PERCEPT-SENTENCE(percept, t))

action ← ASK (KB, MAKE_ACTION-QUERY(t))

TELL (KB, MAKE-ACTION-SENTENCE(action, t))

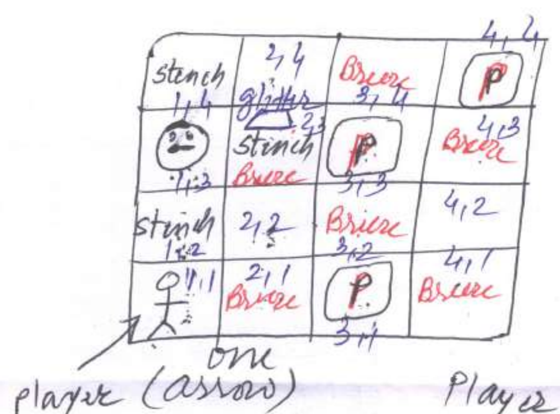
t = t + 1

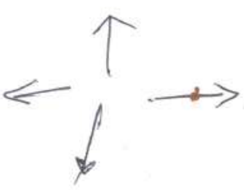
return action

KB - collection of sentences

Sentences - are written in some specific knowledge representation language

2. WUMPUS World





Player has only one arrow.

PEAS Description:

Performance, Environment, Actuators, Sensors

Performance : +1000 points - get gold
- 1000 points - caught by WUMPUS / fall in a pit
- 1 - for each action
- 10 - when an arrow is used.

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Environment : 4x4 grid

A Wumpus is present in any one cell
one or ~~p~~ more pits are also present.

Actuators : Actions taken by the agent.

- Move (L, R, U, D)
- shoot (shoot an arrow)
- Grab (Grab the gold)

Sensors : things which are sensed/perceived by the agent.

Percepts
- stench
- breeze
- glitter
- scream

$[\underset{S}{T}, \underset{B}{F}, \underset{G}{F}, \underset{S}{T}]$

$[F, \underset{\text{Breeze}}{\underbrace{T}}, F, F]$

Knowledge Base / Rules of the game

- If there is a pit in cell [X, Y],
  there will be Breeze in [X+1, Y]
  [X, Y+1]
  [X+1, Y+1]
  [X-1, Y-1]
  neighbouring cells.

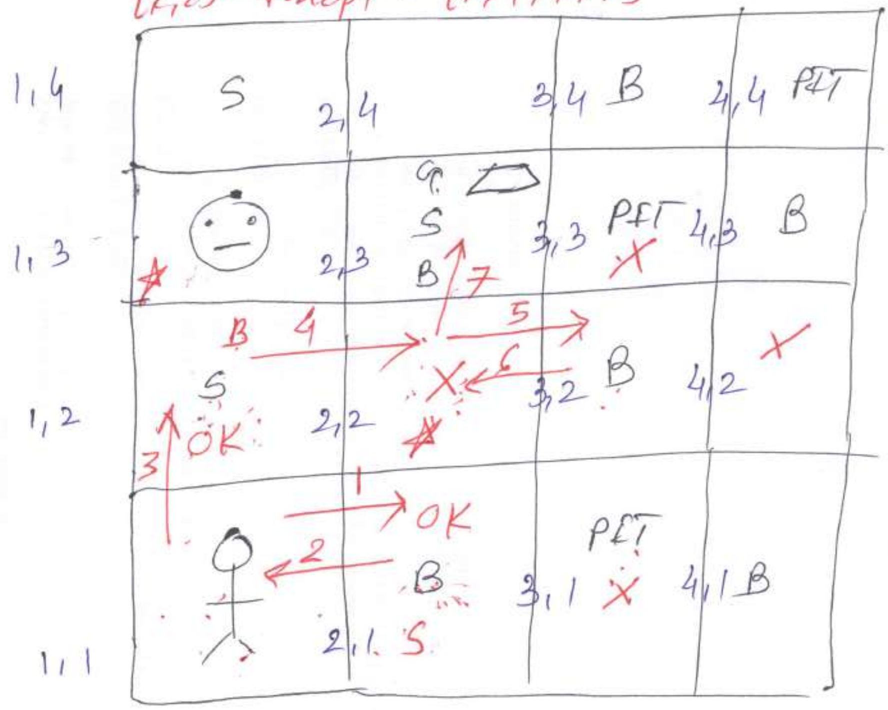
- If there is a wumpus in [X, Y], there will be stench in neighbouring cells.

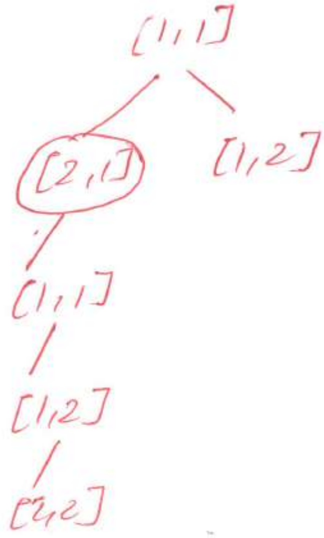
- If there is gold in [X, Y], there is glitter in [X, Y]

- If the wumpus dies, it will scream.

- There is only one arrow available.

[2,2] - Percept - [F, F, F, F]





[1,1] : Percept → [F, F, F, F]

stench, Breeze, Glitter, scream

[1,2] : Percept : [T, F, F, F]

Stench

[2,1] : Percept : [F, T, F, F] - Pit may be in [3,1] or [4,2]

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3. Logic

- To store Knowledge and to infer an action given a percept.
- Proposinal logic, Predicate Logic — Knowledge Representation Languages
- Syntax — rules of language — well formed
- Semantics — meaning of sentence

ex $x + y = 4$

x — no of male members
y — no of female members

To fully understand the semantics,
we have to find truth value of the sentence
in all possible worlds or models.

$0 \le x \le \cancel{4}^{2}$, $0 \le y \le \cancel{4}^{2}$

model no.	x	y	$x+y$
1 →	0	0	0
2 →	0	1	1
3 →	0	2	2
4 →	1	0	1
5 →	1	1	2
6 →	1	2	3
7 →	2	0	2
8 →	2	1	3
9 →	2	2	4

The sentence $x+y=4$
is true only in
model no. 9

$A \vee B$ – Boolean expression

Truth table → Semantics

Model no.	A	B	$A \vee B$
1 →	T	F	T
2 →	T	T	T
3 →	F	T	T
4 →	F	F	F

· Entailment: ($\models$)

$\alpha \models \beta$: From α, we are able to derive β (where α and β are two sentences)

or

β is logically inferred from α.

P: Today is Monday (Atomic)

¬P: ¬ (Today is Monday) = It is not true that Today is Monday.

Q: We had an AI lecture today.

R: We had programming lab today.

Q ∧ R: We had an AI lecture today and we had programming lab today.

∨ – OR

Q ∨ R: ——

S: You work honestly. If S is true then

T: You will earn more money. T is true.

If you work honestly, you will earn more money. ($S \Rightarrow T$)

You earn more money if and only if you will work honestly ($S \Leftrightarrow T$)

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$$S \Leftrightarrow T = (S \Rightarrow T) \wedge (T \Rightarrow S)$$

- U: It rains.

  V: Roads are wet.

  $U \Rightarrow V$: If it rains (LHS), roads are wet (RHS).

  $U \Leftrightarrow V$: roads are wet (LHS) if and only if it rains (RHS).

- Truth tables:

A	B	$A \wedge B$	$A \vee B$	$A \Rightarrow B$ ✓	$A \Leftrightarrow B$	$B \Rightarrow A$ ✓
T	T			T	T	T
T	F			F	F	T
F	T			T	F	F
F	F			T	T	T

$$A \Leftrightarrow B = (A \Rightarrow B) \wedge (B \Rightarrow A)$$

$$A \Rightarrow B = \neg A \vee B = \neg A \wedge \neg B$$

De Morgan Rules: $\neg(A \wedge B) = \neg A \vee \neg B$

$\neg(A \vee B) = \neg A \wedge \neg B$

$$A \Rightarrow B = \neg A \vee B$$

$$\neg(\neg A \vee B) = A \wedge \neg B$$

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$a * b = b * a$

Equivalence / logical equivalence ($\equiv$)

- Two sentences α and β

  $\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$.

- Ex: $\alpha \wedge \beta \equiv \beta \wedge \alpha$ is true because $\alpha \wedge \beta \models \beta \wedge \alpha$ and $\beta \wedge \alpha \models \alpha \wedge \beta$

- Truth tables are same.

$\alpha \vee \beta \equiv \beta \vee \alpha$ (commutativity)

$\neg(\alpha \vee \beta) \equiv \neg\alpha \wedge \neg\beta$

$\neg(\neg\alpha) \equiv \alpha$

$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$ (associativity)

$\alpha \Rightarrow \beta \equiv \neg\beta \Rightarrow \neg\alpha$

$\alpha \Rightarrow \beta \equiv \neg\alpha \vee \beta$

✓ $\alpha \Leftrightarrow \beta \equiv (\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$ (Bicond. Elimination)

$\Leftrightarrow$ Biconditional

$\alpha \wedge (\beta \vee \gamma) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$ (distributivity)

- **Validity:**
  - A sentence is valid if it is true in all the models.

ex: P: The sun rises in the east.

- tautology - valid sentence
- contradiction - a sentence which is false in all the models.

- **Satisfiability:**

  If a sentence S is true in model m, it is said that m satisfies S, or m is a model of S.

5. Reasoning Patterns in Propositional Logic:

- Goal: To derive new facts or actions using KB and percepts.
- Process of deriving new facts is called Reasoning.

Rules useful for Reasoning:

Modus Ponen: $\dfrac{\alpha \Rightarrow \beta, \ \alpha}{\beta}$ → facts given/assumed to be true
→ what we derive

And-Elimination: $\dfrac{\alpha \wedge \beta}{\alpha}$ or $\dfrac{\alpha \wedge \beta}{\beta}$

$P_{i,j}$ - there is a pit in cell $[i,j]$

$B_{i,j}$ - there is Breeze in $[i,j]$.

$W_{i,j}$ - there is a Wumpus in $[i,j]$ } let us ignore

$S_{i,j}$ - there is stench in $[i,j]$ } for now.

KB : Rules of game

R_1 : $\neg P_{1,1}$ (there is no pit in $[1,1]$)

R_2 : $B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1}$

(there is Breeze in $[1,1]$ if and only if there is a pit in $P_{1,2}$ or there is a pit in $P_{2,1}$)

R_3 : $B_{2,1} \Leftrightarrow P_{1,1} \vee P_{2,2} \vee P_{3,1}$

$\cancel{X}$ $B_{i,j} \Leftrightarrow P_{i+1,j} \vee P_{i,j+1} \vee P_{i+1,j+1} \vee P_{i,j-1}$

$$B_{3,1} \Leftrightarrow P_{4,1} \vee P_{3,3} \vee P_{2,1}$$

$$\neg W_{1,1}$$

$$W_{1,2} \Leftrightarrow \cancel{W_{1,3} \vee W_{2,2} \vee W_{1,1}}$$

$$S_{1,3} \vee S_{2,2} \vee \cancel{S_{1,1}}$$

$$\neg S_{1,1}$$

$$W_{2,1} \Leftrightarrow S_{3,1} \vee S_{2,2} \vee \cancel{S_{1,1}}$$

Percepts:

P_1: $\neg B_{1,1}$ P_4: $B_{2,1}$

P_2: $\neg S_{1,1}$ P_5: $\neg S_{2,1}$

P_3: $\neg G_{1,1}$ P_6: $\neg G_{2,1}$

Q: Given KB (R_1, R_2, R_3) and Percepts (P_1, P_2, P_3, P_4, P_5, P_6), can we derive $P_{1,2}$ (can we prove that $P_{1,2}$ is true?)

$P_{1,2}$: there is a pit in cell [1,2].

A: R_6 $\quad B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1}$ (given in KB) (Rule R2)

$\equiv (B_{1,1} \Rightarrow P_{1,2} \vee P_{2,1}) \wedge (P_{1,2} \vee P_{2,1} \Rightarrow B_{1,1})$ (Biconditional Elimination)

$R_7 \equiv P_{1,2} \vee P_{2,1} \Rightarrow B_{1,1}$ (And-Elimination)

$R_8 \equiv \neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1})$ $\quad (\because \alpha \Rightarrow \beta \equiv \neg\beta \Rightarrow \neg\alpha)$

$$\frac{\neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1}), \quad \neg B_{1,1}}{R_9 \quad \neg(P_{1,2} \vee P_{2,1})}$$ (Modus Ponens)

At this point, $\neg(P_{1,2} \vee P_{2,1})$ is true

$$\neg(P_{1,2} \vee P_{2,1}) \equiv \underline{\overset{T}{\neg P_{1,2}} \wedge \overset{T}{\neg P_{2,1}}} \quad \text{(De Morgan Rule)}$$

R10 Neither $P_{1,2}$ is true nor $P_{2,1}$ is true.

Hence, we conclude that $P_{1,2}$ is not true.

There is no pit in $P_{1,2}$.

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Model checking:

KB:

R_1 : $\neg P_{1,1}$

R_2 : $B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1}$

R_3 : $B_{2,1} \Leftrightarrow P_{1,1} \vee \underline{P_{2,2}} \vee P_{3,1}$

R_4 : $\neg B_{1,1}$

R_5 : $B_{2,1}$

$KB = R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5$

α : $P_{2,2}$ = There is a pit in cell [2,2]

Q: Given KB, can we check whether α is true or not.

OR

Q: Can we prove that $KB \models \alpha$ is true?

$KB \models \alpha$ is true if α is true in every model in which KB is true.

To show that $KB \models \alpha$ is true, first find truth value of KB in all the possible models. Then check whether α is true in all the models in which KB is true.

Consider

$$KB = R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5$$

$\alpha \wedge \beta$

	α	β	$\alpha \wedge \beta$
1	T	T	T
2	T	F	F
3	F	T	F
4	F	F	F

	R_1	R_2	R_3	R_4	R_5	$KB = R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5$	α $(P_{2,2})$
m1 :	T	T	T	T	T	T	T
m2 :	T	T	T	T	F	F	T
		⋮				⋮	⋮

No. of models $= 2^5 = 64$

atomic sentences

	$P_{1,1}$	$B_{1,1}$	$P_{1,2}$	$P_{2,1}$	$B_{2,1}$	$P_{3,1}$	$P_{2,2}$	R_1	R_2	R_3	R_4	R_5	KB	α $P_{2,2}$
m_1:	T	T	F	T	T	T	T							
m_2:														
⋮														

Correct no. of models $= 2^7 = 128$

Time complexity is exponential $- \; 2^n$ (n = no. of atomic sentences)

Not an efficient approach.

Resolution

$$\left(\beta = \neg(\neg\beta)\right)$$

ex: $$\frac{\alpha \vee \beta, \quad \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

fact 1: $\alpha \vee \beta$ is true $\rightarrow$ Either α is true

OR

β is true

fact 2: $\neg\beta \vee \gamma$ is true $\rightarrow$ $\neg\beta$ is true $=$ β is false

OR

γ is true

$$\frac{l_1 \vee l_2 \vee l_3 \vee \cdots \vee l_k, \quad m_1 \vee m_2 \vee m_3 \vee \cdots \vee m_n}{l_1 \vee l_2 \vee \cdots \vee l_{i-1} \vee l_{i+1} \cdots \vee l_k \vee m_1 \vee m_2 \vee \cdots \vee m_{j-1} \vee m_{j+1} \cdots m_n}$$

(l_i and m_j are contradictions) or ($l_i = \neg m_j$)

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$KB \models \alpha$ < Model Checking / Inference Algo. (Modus Ponen, And-Elimination, Equivalences, Resolution)

Example for Resolution Rule:

- In Wumpus world, player starts at (1,1). Then goes to (2,1). Then comes back to (1,1). Finally moves to (1,2).

$R_{11}: \neg B_{1,2}$ (Percept)

$R_{12}: B_{1,2} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{1,3})$

We want to check whether there is a pit in (2,2) or not. In other words, check if $P_{2,2}$ is T or F.

$B_{1,2} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{1,3})$

$\equiv [B_{1,2} \Rightarrow (P_{1,1} \vee P_{2,2} \vee P_{1,3})] \wedge [(P_{1,1} \vee P_{2,2} \vee P_{1,3}) \Rightarrow B_{1,2}]$

(Bicond. Elimination)

$\equiv (P_{1,1} \vee P_{2,2} \vee P_{1,3}) \Rightarrow B_{1,2}$ (And Elimination : $\frac{\alpha \wedge \beta}{\beta}$)

$\equiv \neg B_{1,2} \Rightarrow \neg (P_{1,1} \vee P_{2,2} \vee P_{1,3})$ ($\alpha \Rightarrow \beta \equiv \neg\beta \Rightarrow \neg\alpha$)

$$\frac{\neg B_{1,2} \Rightarrow \neg (P_{1,1} \vee P_{2,2} \vee P_{1,3}),\ \neg B_{1,2}}{\neg (P_{1,1} \vee P_{2,2} \vee P_{1,3})}$$ (Modus Ponen)

$\neg (P_{1,1} \vee P_{2,2} \vee P_{1,3})$ is true.

$\neg (P_{1,1} \vee P_{2,2} \vee P_{1,3}) \equiv \neg P_{1,1} \wedge \neg P_{2,2} \wedge \neg P_{1,3}$ (DeMorgan Rule)

R_{13} : $\neg P_{2,2}$

R_{14} : $\neg P_{1,3}$

R_3 : $B_{21} \Leftrightarrow P_{11} \vee P_{22} \vee P_{31}$

$\equiv (B_{21} \Rightarrow P_{11} \vee P_{22} \vee P_{31}) \wedge (P_{11} \vee P_{22} \vee P_{31} \Rightarrow B_{21})$

$\equiv B_{21} \Rightarrow P_{11} \vee P_{22} \vee P_{31}$

$$\frac{B_{21} \Rightarrow P_{11} \vee P_{22} \vee P_{31},\ \overset{R5}{B_{21}}}{\underline{\underline{P_{11} \vee P_{22} \vee P_{31}}}} \quad \text{(Modus Ponen)}$$

$\{ R_1 : \neg P_{11}$

R_{15} : $P_{11} \vee P_{22} \vee P_{31}$

Apply Resolution between R_{15} and R_1

$$\frac{\cancel{P_{11}} \vee P_{22} \vee P_{31},\ \neg \cancel{P_{11}}}{R_{16} : \longrightarrow P_{22} \vee P_{31}}$$

Apply Resolution between R_{16} and R_{13},

$$\frac{\cancel{P_{22}} \vee P_{31},\ \neg \cancel{P_{22}}}{P_{31}}$$

$R_{17} : P_{31}$

* CNF (Conjunctive Normal Form)

- Conjunction of Disjunctions. (Product of Sums)
  $(\wedge)$ $(\vee)$

Ex: $(l_{11} \vee l_{12} \vee l_{13}) \wedge (m_{11} \vee m_{12} \vee m_{13}) \wedge$

$(n_{11} \vee n_{12} \vee n_{13})$

Above sentence is in CNF.

- Example for converting an expression/sentence into CNF.

$$B_{11} \Leftrightarrow (P_{12} \vee P_{21}).$$

Convert above sentence into CNF.

$$\equiv [B_{11} \Rightarrow (P_{12} \vee P_{21})] \wedge [(P_{12} \vee P_{21}) \Rightarrow B_{11}]$$

$$\equiv [\neg B_{11} \vee (P_{12} \vee P_{21})] \wedge [\neg(P_{12} \vee P_{21}) \vee B_{11}]$$

$$(\because \alpha \Rightarrow \beta \equiv \alpha \wedge \neg\beta \equiv \neg\alpha \vee \beta)$$

$$\equiv [\neg B_{11} \vee P_{12} \vee P_{21}] \wedge [(\neg P_{12} \wedge \neg P_{21}) \vee B_{11}]$$

$$\equiv (\neg B_{11} \vee P_{12} \vee P_{21}) \wedge [B_{11} \vee (\neg P_{12} \wedge \neg P_{21})]$$

$$\equiv (\neg B_{11} \vee P_{12} \vee P_{21}) \wedge [(B_{11} \vee \neg P_{12}) \wedge (B_{11} \vee \neg P_{21})]$$

$$\equiv \boxed{(\neg B_{11} \vee P_{12} \vee P_{21}) \wedge (B_{11} \vee \neg P_{12}) \wedge (B_{11} \vee \neg P_{21})}$$ In CNF.

- **Resolution Algo :**

  - Purpose is we want to check whether $KB \models \alpha$ is true or not.

- Requirement : All the clauses of KB ~~should~~ must be in CNF. So also α.

- O/p → True, if $KB \models \alpha$ is true.
  → False, otherwise

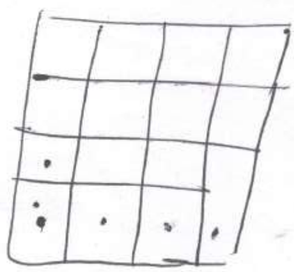
AI 7/3/22.

First Order Logic

* Properties of Representation languages

- Expressiveness - Can express facts in a concise manner.

e.g. If there is a breeze in a cell, a pit may be present in neighbouring cells.



16 Sentences

$B[1,1] \Leftrightarrow P[2,1] \vee P[1,2]$

$B[2,1]$

$B[3,1]$

$B[4,1]$

$\forall x,$ ______

$\exists y,$ ______

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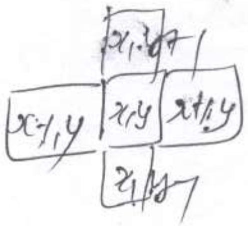
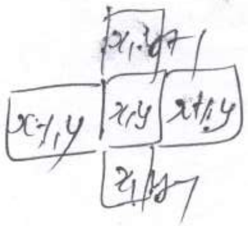
$$\forall x, y \quad B[x, y] \Leftrightarrow P[x+1, y] \vee P[x, y+1] \vee P[x, y-1] \vee P[x-1, y]$$



- <u>Context free</u>
  - English → context dependent
- ~~Compositeness~~ <u>Compositionality</u> : Meaning → Semantics of a sentence is understood if we understand parts of a sentence

  <u>$a \vee b$</u> – is true if a is true or b is true or both – truth table

$$\forall x, y \quad B[x, y] \Leftrightarrow P[x+1, y] \vee P[x, y+1] \vee P[x, y-1] \vee P[x-1, y]$$



- Context free
  - English → context dependent

- ~~Compositeness~~ Compositionality : Meaning of a sentence is understood if we understand parts of a sentence

Meaning → Semantics

$a \vee b$ – is true if a is true or b is true or both – truth table

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$\frac{a}{\cdot}$ $\frac{b}{\cdot}$ $\underline{a \times b}$

* Syntax and Semantics of First Order Logic:

- The world is made of objects and relationship between the objects.

John, Richard ⇒ constant objects

Term: Object → constant, variable, function

Ex: John is brother of Richard.

$\underbrace{\text{IsBrotherOf}}_{\text{relationship}}(\underbrace{\text{John}}_{\text{obj1}}, \underbrace{\text{Richard}}_{\text{obj2}})$

brothers (J, R)

Ex: Raju is son of Mahesh

son (Raju, Mahesh)

Variables : $x, y, z,$

brothers (x, y)

Function : It is an alternate way of representing constants.

Left leg of John	crown of the King
→ LLJ	Crown (John)
→ $\underbrace{\text{LeftLeg}}_{\text{function}}$ (John)	

Quantifiers : $\forall$ - for all - Universal $\quad \vee, \wedge, \neg, \Rightarrow, \Leftrightarrow$

$\exists$ - there exists - Existential

Complex sentence :

$\quad\quad T \quad\quad\quad\quad\quad F$

King (Richard) $\vee$ King (John)

$\neg$ King (John)

P

Q

R

$S \vee T$

$P(x) \vee Q(z)$

$P(x, y) \vee Q(l, m)$

Universal Quantifier ($\forall$)

All IT students are from Morbi

ItStudent(x) – x is an IT student.
FromMorbi(x) – x is from Morbi.

a	b	$a \Rightarrow b$
T	T	T
T	F	F
F	T	T
F	F	T

x = Rajesh

$$\forall x, \underbrace{\overset{F}{\underset{F\,T}{\text{student}(x)}} \Rightarrow \underset{F}{\text{FromMorbi}(x)}}_{P(x)}$$

$\forall x$, $P(x)$ is true if $P(x)$ is true for all possible values of x.

$x \Rightarrow$ student name

$\forall x$, ~~student~~ ItStudent(x) $\wedge$ FromMorbi(x) X

F

* Existential Quantifier ($\exists$)

$\exists x, P(x)$: $P(x)$ is true for at least one value of x.

There are some IT students who are from Morbi

$\exists x$, $\underline{\text{ItStudent}(x) \wedge \text{FromMorbi}(x)}$

$\forall x,$ ~~student~~ ItStudent(x) $\wedge$ FromMorbi(x) X

F

* Existential Quantifier ($\exists$)

$\exists x, P(x)$: P(x) is true for at least one value of x.

There are some IT students who are from Morbi

$\exists x,$ ItStudent(x) $\wedge$ FromMorbi(x)

Nested Quantifiers

* $\underbrace{\forall x \ \forall y}$ $\underline{Brother(x, y) \Rightarrow Sibling(x, y)}$

For all x and all y, if x is brother of y then ~~y~~ x is sibling of y.

OR

- All brothers are siblings.

$\forall x \ \forall y \longrightarrow \forall x, y$

* $\forall x \, \exists y \; helps(x, y)$

For all x, there exist a y such that
x helps y.

x = we take from IT students

$x \in S_{IT}$

y = we take from MECH students

$y \in S_{MECH}$

Everyone helps some one

* $\exists y \; \forall x \; helps(x, y)$

There exists at least one y such that for all x, x helps y.

There is someone who is helped by everyone

* $\forall x \; [Crown(x) \lor (\exists x) \; [Brother(Richard, x)]]$

$\forall x \; [Crown(x) \lor (\exists y \; Brother(Richard, y))]$

```
int x;
void main()
{

}
int sum()
{
int x;
x=5;
}
```

Connection between $\forall$ and $\exists$

$\forall x \; \neg likes(x, Pizza)$

For all x, it is not true that x likes Pizza

$\neg \exists x, \; likes(x, Pizza)$

There does not exist an x s.t. x likes Pizza

$$\boxed{\forall x, \neg P(x) = \neg \exists x, P(x)}$$

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$$\boxed{\begin{aligned}\forall x,\ P(x) &= P(A) \wedge P(B) \wedge P(C)\\ \exists x,\ P(x) &= P(A) \vee P(B) \vee P(C)\end{aligned}}$$

$$\neg \forall x\ P(x) = \neg [P(A) \wedge P(B) \wedge P(C)]$$

$$= \neg P(A) \vee \neg P(B) \vee \neg P(C)$$

$$\boxed{\neg \forall x\ \neg P(x) = \neg \exists x\ P(x)} \qquad \exists x\ \neg P(x)$$

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$$\forall x, \ \text{likes}(x, \text{Pizza})$$

$$\equiv$$

$$\neg \exists x, \ \neg \text{likes}(x, \text{Pizza})$$

There does not exist even at least one x such that x does not like Pizza

$$\boxed{\forall x, \ P(x) \equiv \neg \exists x \ \neg P(x)}$$

$$\neg \forall x, \ P(x) \equiv \neg\exists x \ \neg P(x)$$

$$\exists x \ P(x) = \neg \forall x \ \neg P(x)$$

$$\forall x \ \neg P(x) = \neg P(A) \wedge \neg P(B) \wedge \neg P(C)$$
$$= \neg (P(A) \vee P(B) \vee P(C))$$

$$\boxed{\forall x, \neg P(x) = \neg \exists x, P(x)}$$

* Equality

$x == y$

Father (John) = Henry

$$\exists x, y \ [\text{Brother}(x, \text{Richard}) \wedge \text{Brother}(y, \text{Richard}) \wedge x \neq y]$$

Richard has at least two brothers.

Exercises

1) Some students took French in spring 2001

2) Every student who took French passes it.

3) only one student took Greek in spring 2001

4) The best score in Greek is higher than the best score in French.

5) Every person who buys a policy is smart.

F/W and B/W chaining in FOL

- It is a crime for an American to sell weapons to a hostile nation.

$\forall x, y, z$ American$(x) \wedge$ Weapon$(y) \wedge$ Sells$(x, y, z) \wedge$ Hostile$(z) \Rightarrow$ Criminal(x)

- Nono has some missiles

$\exists x$ [Missile$(x) \wedge$ Owns(Nono, x)] (Not a definite clause)

Missile(m_1) ✓
Owns(Nono, m_1) ✓

- Nono is an enemy of America

Enemy (Nono, America)

- The missiles owned by Noro were sold by Col. West.

$\forall x \quad Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$

- Col. West is an American.

American(West)

- All missiles are weapons.

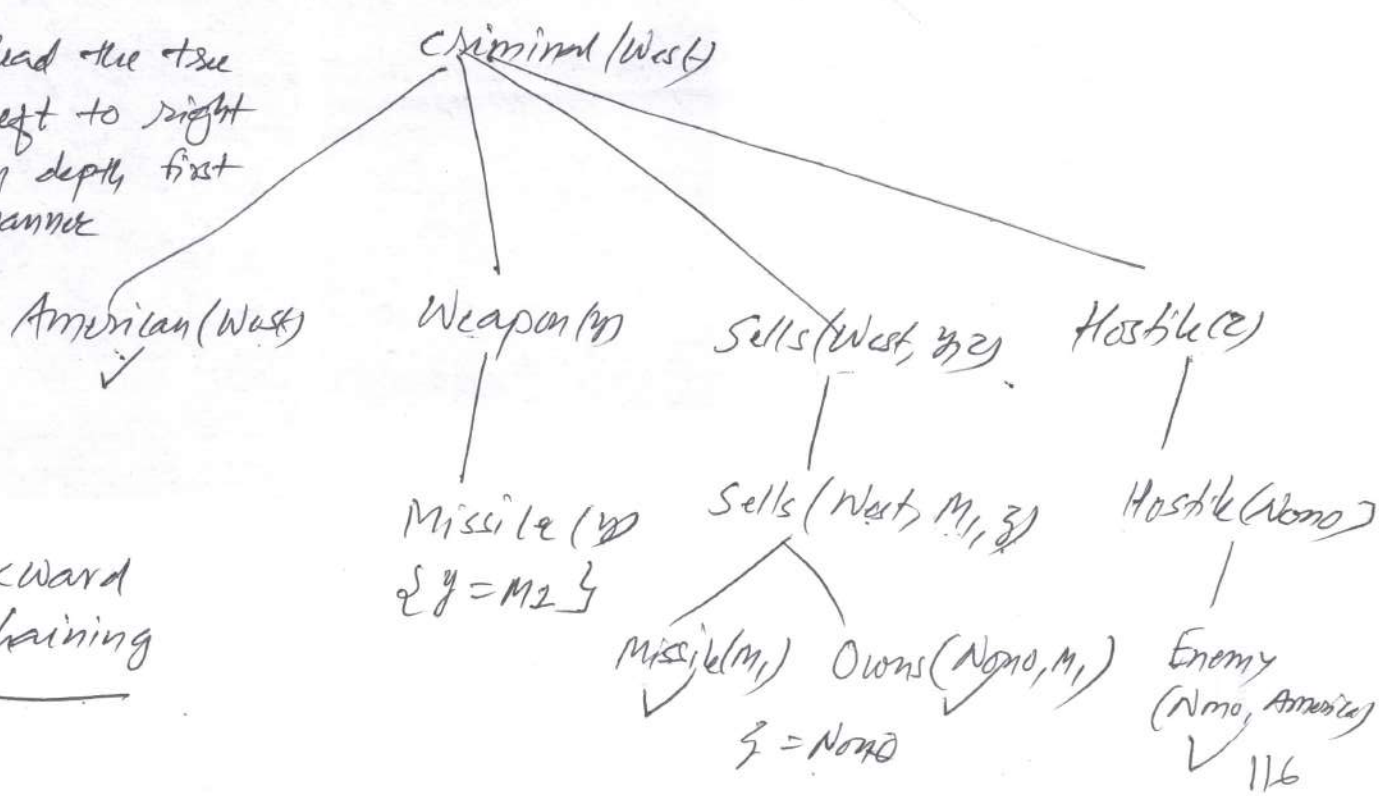
$Missile(x) \Rightarrow Weapon(x)$.

- ~~All enemies are hostiles.~~ / An enemy of America is considered "hostile"

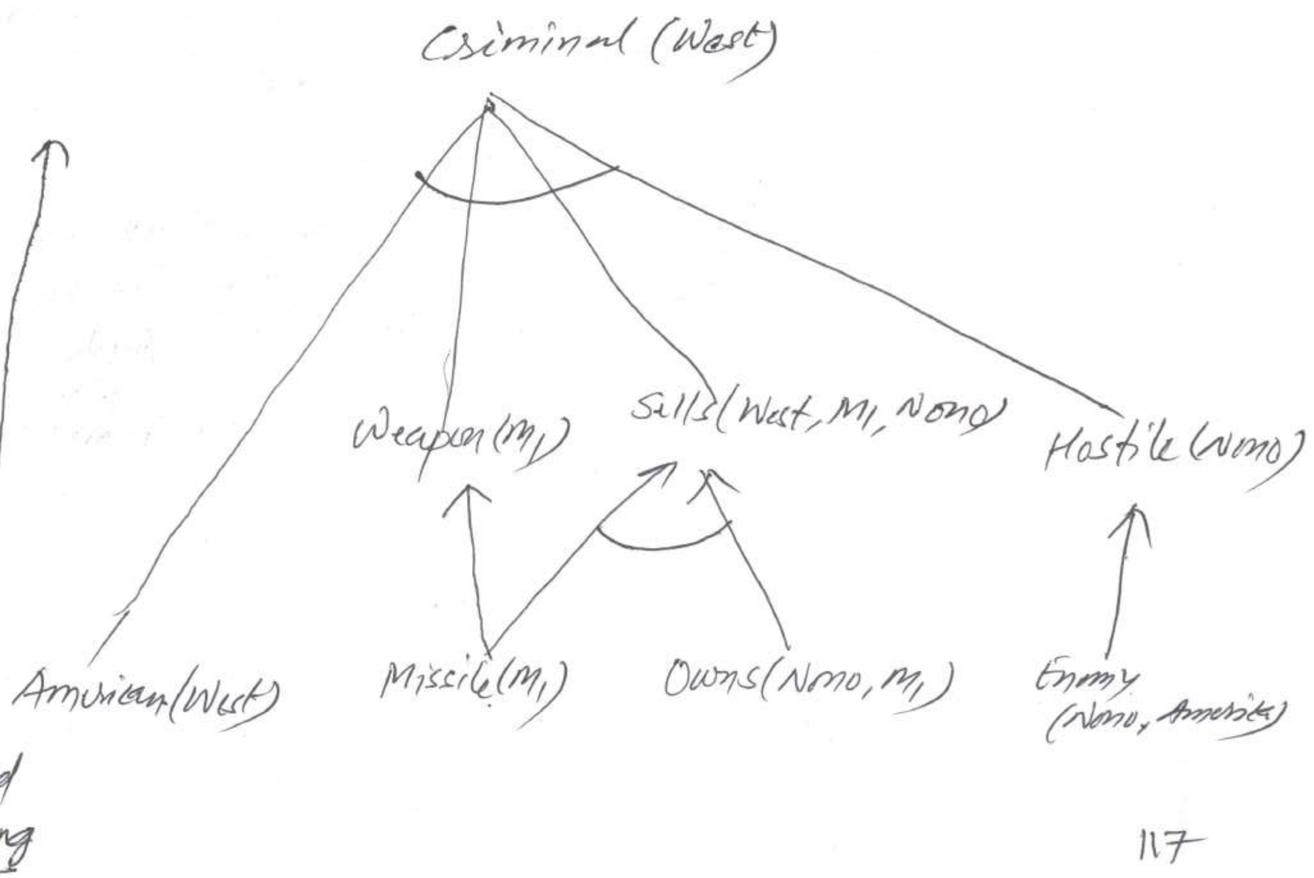
$Enemy(x, America) \Rightarrow hostile(x)$.

American(West) $\land$ Weapon(y) $\land$ Sells(West, y, z) $\land$
Hostile(z) $\Rightarrow$ Criminal(West)

Read the tree left to right in depth first manner



Backward chaining



Read the tree bottom to top

Forward chaining

Resolution in FOL

- clauses in CNF - Conjunctive Normal Form.
- $KB \models \alpha$ is proved by showing $KB \wedge \neg\alpha$ is unsatisfiable.

$$KB \wedge \neg\alpha = F$$
$$(KB \wedge \neg\alpha)' = T$$
$$KB' \vee \alpha = T$$
$$\therefore KB \Rightarrow \alpha \text{ or } KB \models \alpha$$

$$\alpha \Rightarrow \beta$$
$$= \neg\alpha \vee \beta$$

In propo. logic,

$$\frac{p \vee q, \ \neg q \vee r}{p \vee r}$$

In FOL,

$$\frac{l_1 \vee l_2, \ m_1 \vee m_2}{\cancel{SUBSTI} \ SUBST(\theta, l_1 \vee m_2)} \quad \text{if} \quad \underline{UNIFY(l_2, m_1) = \theta}.$$

Ex:

$$\frac{\overset{l_1}{Animal(F(x))} \vee \overset{l_2}{Loves(G(x), x)}, \quad \overset{m_1}{\neg Loves(u, v)} \vee \underset{m_2}{\neg Kills(u, v)}}{Animal(F(x)) \vee \neg Kills(G(x), x)}$$

$\theta = \{(u, G(x)), (v, x)\}$

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✓ $c_1 = \neg American(x) \vee \neg Weapon(y) \vee \neg Sells(x, y, z) \vee$
$\neg Hostile(z) \vee Criminal(x)$

$c_2 = \neg Missile(x) \vee \neg Owns(Nono, x) \vee Sells(West, x, Nono)$

$c_3 = \neg Enemy(x, America) \vee Hostile(x)$

$c_4 = \neg Missile(x) \vee Weapon(x)$

$c_5 : Owns(Nono, m_1)$

✓ $c_6 : Missile(m_1)$

✓ $c_7 : American(West)$

$c_8 : Enemy(Nono, America)$

✓ $\neg\alpha : \neg Criminal(West)$

$KB : c_1 \wedge c_2 \wedge c_3 \wedge c_4 \wedge c_5 \wedge c_6 \wedge c_7 \wedge c_8$

$\alpha : Criminal(West)$

$KB \models \alpha$ is true?

$a \wedge b \wedge$

Everyone who loves all animals is loved by someone.

$$\forall x\ [\underbrace{\forall y\ Animal(y) \Rightarrow Loves(x,y)}] \Rightarrow [\exists y\ Loves(y,x)]$$

$$\exists z\ Loves(z,x)$$

$a \Rightarrow b : \neg a \vee b$

$$\forall x\ [\underbrace{\forall y\ \neg Animal(y) \vee Loves(x,y)}] \Rightarrow [\exists y\ Loves(y,x)]$$

$$\forall x\ [\neg(\forall y\ \neg Animal(y) \vee Loves(x,y))] \vee \exists y\ Loves(y,x)$$

$$\forall x\ [\underbrace{\neg \forall y\ (\neg Animal(y) \vee Loves(x,y))}] \vee \exists y\ Loves(y,x)$$

$$\neg \forall x\ P = \exists x\ \neg P .$$

$$\neg \exists x\ P = \forall x\ \neg P$$

$$\left\{ \begin{array}{l} \neg(A \vee B) \\ = \\ \neg A \wedge \neg B \end{array} \right.$$

$$\forall x\ [\exists y\ \underbrace{\neg(\neg Animal(y) \vee Loves(x,y))}] \vee \exists y\ Loves(y,x)$$

$= \forall x \ [\exists y \ \neg\neg Animal(y) \wedge \neg Loves(x, y)] \vee \exists y \ Loves(y, x)$

$= \forall x \ [\underbrace{\exists y \ Animal(y) \wedge \neg Loves(x, y)}] \vee \exists y \ Loves(y, x)$

$= \forall x \ [\exists y \ Animal(y) \wedge \neg Loves(x, y)] \vee \exists z \ Loves(z, x)$

$\neq$

$\times \ \forall x \ [Animal(Dog) \wedge \neg Loves(x, Dog)] \vee \exists z \ Loves(z, x)$

Skolemization:

$\Rightarrow \forall x \ [Animal(F(x)) \wedge \neg Loves(x, F(x))] \vee \exists z \ Loves(z, x)$

Skolem Function $\rightarrow$ $F(x)$: An animal ~~~~ not loved by x.

$G(x)$: A person who loves x

~~A~~ ~~(1, A)~~ RESOLV(4, ¬d)

$$\frac{\text{(1)}\quad \neg American(\overset{West}{x}) \vee \neg Weapon(y) \vee \neg Sells(\overset{West}{x}, y, z) \vee \neg Hostile(z) \vee Criminal(\underset{West}{x}) \;,\; \neg Criminal(West)}{}$$

(A1) ← ¬American(West) ∨ ¬Weapon(y) ∨ ¬Sells(West, y, z) ∨ ¬Hostile(z)

(2) RESOLV(A1, C7)

$$\frac{p \overset{\vee}{\cancel{\wedge}} q,\ \neg q \vee r}{p \vee r}$$

$$\frac{\neg American(West) \vee \neg Weapon(y) \vee \neg Sells(West, y, z) \vee \neg Hostile(z) \;,\; American(West)}{}$$

(A2) ← ¬Weapon(y) ∨ ¬Sells(West, y, z) ∨ ¬Hostile(z)

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③ RESOLV (A_2, C_4)

$$\cancel{\neg Weapon(y)} \vee \neg Sells(West, y, z) \vee \neg Hostile(z),$$
$$\neg Missile(\underset{y}{x}) \vee \cancel{Weapon(\underset{y}{x})}$$

A_3 ← $\neg Sells(West, y, z) \vee \neg Hostile(z) \vee \neg Missile(y)$

④ RESOLV (A_3, C_6)

$$\neg Sells(West, \overset{M_1}{y}, z) \vee \neg Hostile(z) \vee \cancel{\neg Missile(\overset{M_1}{y})},\ \cancel{Missile(M_1)}$$

$$\neg Sells(West, M_1, z) \vee \neg Hostile(z)$$

$= \underset{X}{\cancel{\forall x}}\ [Animal(F(x)) \wedge \neg Loves(x, F(x))] \vee Loves(G(x), x)$

$= [Animal(F(x)) \vee Loves(G(x), x)]\ \underline{\wedge}$

$[\neg Loves(x, F(x)) \vee Loves(G(x), x)]$

KB $\qquad \alpha$

$clauses = \{ \{C_1, C_2, C_3\}, \neg\alpha \}$

$C_1 C_2 \rightarrow A1$

$C_1 C_3 \rightarrow A2$

$C_2 C_3 \rightarrow A_3 \quad \underline{F}$

$C_1 \alpha \rightarrow \quad new = \{A_1, A_2, A_3\}$

$C_2 \alpha \rightarrow$

$C_3 \alpha \rightarrow$

$\underline{KB \models \alpha\ ?}$

$P \wedge \neg Q$

$\cancel{P \Rightarrow Q}$

L

m

$new \subseteq clauses \Rightarrow KB \not\models \alpha$

$$\frac{Enemy(\cancel{Nono}, \cancel{America}) \quad , \quad \neg Enemy(\cancel{Nono}, America)}{False}$$

$\boxed{KB \wedge \neg\alpha = False}$

$\therefore KB \models \alpha$

$\boxed{KB \wedge \neg\alpha = T}$

$\therefore KB \not\models \alpha$

Lect_19

Model checking :

KB :

$R_1 : \neg P_{1,1}$

$R_2 : B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1}$

$R_3 : B_{2,1} \Leftrightarrow P_{1,1} \vee \underline{P_{2,2}} \vee P_{3,1}$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

$KB = R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5$

$\alpha : P_{2,2}$ = There is a pit in cell [2,2]

Q: Given KB, can we check whether α is true or not.