Lukhdhirji Engineering College - Morbi

General Department

Tutorial - 1

Subject Name: Discrete Mathematics **Subject Code: 3140708**

Branch: IT **Sem: 4**

Solve the following examples.

(1) Define: Relation, reflexive, symmetric and transitive relation with examples.

(2) Define: Poset. Prove that (S_{45}, D) is poset, where D =divisibility relation.

(3) Show that $(P(X), \subseteq)$ $(X \neq \emptyset)$ is a poset.

(4) Define: Cover of an element in a poset. Let $(P, \leq)$ be a poset with $P = \{1, 2, 3, 4\}$ and $\leq$ is with usual meaning then find cover of each element of P.

(5) Draw the Hasse diagram of the following posets: (S_6, D), (S_{30}, D), (S_{45}, D), (S_{45}, D), (S_{100}, D), $(P(X), \subseteq)$ $(X = \{a, b, c\})$.

(6) Define: Least element, greatest element, minimal element, maximal element. Find least, greatest, minimal and maximal element(s) for (S_6, D), (S_{36}, D) $(P(X), \subseteq)$ $(X \neq \{a, b, c\})$.

(7) Define: GLB and LUB in poset. Find GLB and LUB of 6 and 10 in (S_{30}, D).

(8) Define: Lattice. Give two examples of lattice.

(9) Define: Lattice as an algebraic system. Show that $(S_{30}, *, \oplus)$ is a lattice as an algebraic system, where for every $a, b \in S_{30}$, $a * b =$ gcd of a and b and $a \oplus b =$ lcm of a and b.

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Tutorial - 2

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Solve the following examples.

(1) Define: Graph, loop, multiple edges, simple graph. Draw the following graphs: K_3, K_4, $K_{2,3}$, P_5, C_6, Petersen graph

(2) What is the smallest integer n such that the complete graph K_n has at least 500 edges ?

(3) Let G be a k regular graph, where k is an odd number. Prove that the number of edges in G is multiple of k.

(4) Prove that there are always an even number of vertices of odd degree in a graph.

(5) Prove that if u is an odd vertex in a graph G then there must be a path in G from u to a another odd vertex v of G.

(6) Let G be a simple graph. Show that if G is not connected then its complement $\bar{G}$ is connected.

(7) Define the following by drawing graphs (i) weak component (ii) unilateral component (iii) strong component.

(8) Let G be a graph each of whose nonempty connected components is a bipartite graph. Assuming that G has at least one nonempty component, show that G is bipartite graph.

(9) Determine whether the Petersen graph is bipartite.

(10) Let G be the graph with vertex set $\{1, 2, \ldots, 15\}$ in which i and j are adjacent if and only if their greatest common factor exceeds 1. Count the components of G and determine the maximum length of a path in G.

(11) Determine the value of m and n such that $K_{m,n}$ is Eulerian.

(12) Prove or disprove: Every Eulerian bipartite graph has an even number of edges.

(13) Let G be a simple graph having no isolated vertex and no induced subgraph with exactly two edges. Prove that G is a complete graph.

(14) Prove that G is a tree if and only if G is connected and every edge is a cut-edge.

(15) Prove that a graph is a tree if and only if it is loopless and has exactly one spanning tree.

(16) Let T be a tree in which every vertex has degree 1 or degree k. Determine the possible values of number of vertices.

(17) Prove that among trees with n vertices, the star has the most independent sets.

(18) Prove that an edge e of a connected graph G is a cut-edge if and only if e belongs to every spanning tree.

(19) Prove that the eccentricities of adjacent vertices differ by at most 1.

(20) Define: Adjacency and incidence matrix of a graph. Give adjacency and incidence matric of the following graphs: P_3, C_4, K_5, Peterson graph.