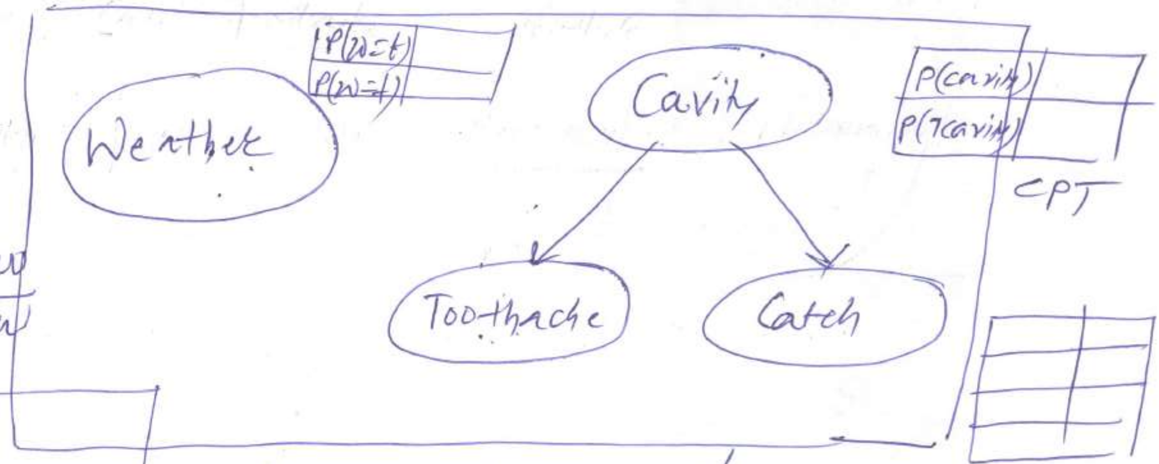
Bayesian N/W   DAG

- It is a graphical representation of relationship between Random Variables of given problem domain.



$$\frac{P(t \wedge c)}{P(c)} = \frac{5000/10000}{6000/10000}$$

P(t/c)	
P(t/¬c)	
P(¬t/c)	
P(¬t/¬c)	

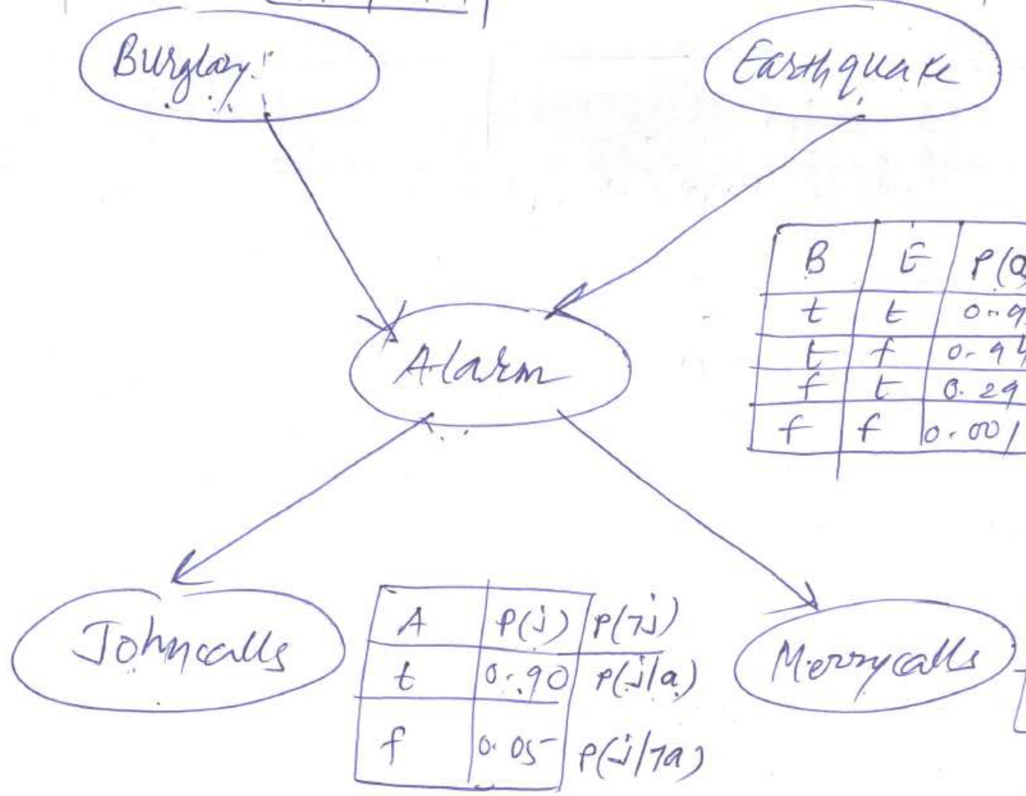
Example Bayesian N/w.

Level

0

1

2



P(b)	0.001
P(¬b)	0.999

P(e)	0.002
P(¬e)	0.998

P(¬a)

B	E	P(a)	
t	t	0.95	P(a/b, e)
t	f	0.94	P(a/b, ¬e)
f	t	0.29	
f	f	0.001	

A	P(j)	P(¬j)
t	0.90	P(j/a)
f	0.05	P(j/¬a)

A	P(m)
t	.70
f	.01

* Semantics of Bayesian Networks:

$$P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(x_i / \text{parents}(X_i))$$

$$P(\overset{x_1}{j} \wedge \overset{x_2}{m} \wedge \overset{x_3}{a} \wedge \overset{x_4}{\neg b} \wedge \overset{x_5}{\neg e}) = \prod_{i=1}^{5} P(x_i / \text{Parents}(X_1))$$

$P(\neg b, j, m, \neg e, a)$

$= P(j/a) \times P(m/a) \times P(a/\neg b, \neg e) \times P(\neg b) \times P(\neg e)$

$= 0.9 \times 0.7 \times 0.001 \times 0.999 \times 0.998$

$= 0.00062.$

* Inference in Bayesian Networks :

$$P(B|j,m) = \langle \underbrace{P(b|j,m)}_{1}^{0.295},\ \underbrace{P(\neg b|j,m)}_{2}^{1-0.295} \rangle$$

$$(1)\quad P(b|j,m) = \frac{P(b,j,m)}{P(j,m)} = \frac{0.00059}{0.002} = 0.295$$

$$* \; P(b,j,m) = P(b,j,m,e,a) + \underbrace{\cancel{P(b,j,m,\neg e,a)}}^{\cancel{0.00062}} + P(b,j,m,e,\neg a) + P(b,j,m,\neg e,\neg a)$$

$$P(b,j,m,e,a) = P(j|a) \times P(m|a) \times P(a|b,e) \times P(b) \times P(e) = 0.000001$$

$$P(b,j,m,e,\neg a) = P(j|\neg a) \times P(m|\neg a) \times P(\neg a|b,e) \times P(b) \times P(e) = 0.00000002$$

$$P(b,j,m,\neg e,\neg a) = P(j|\neg a) \times P(m|\neg a) \times P(\neg a|b,e) \times P(b) \times P(\neg e) = 0.0000$$

$$P(b,j,m,\neg e,a) =$$

$$P(b,j,m) = \quad = 0.00059$$

$$P(j,m) = P(j,m,a,b,e) + P(j,m,a,b,\neg e) +$$
$$P(j,m,a,\neg b,e) + P(j,m,a,\neg b,\neg e) +$$
$$P(j,m,\neg a,b,e) + P(j,m,\neg a,b,\neg e) +$$
$$P(j,m,\neg a,\neg b,\neg e) + P(j,m,\neg a,\neg b,e)$$

$$= 0.000001 + 0.00059 + \underline{0.00036} + \underline{0.00062}$$

$$+ 0.00000002 + 0 + \underline{0.000000} + \underline{\qquad}$$

$$= 0.002$$

$$P(a|b) = \frac{P(a,b)}{P(b)} \quad \therefore P(a,b) = P(a|b) \cdot P(b)$$

$$P(x_1, x_2, \dots, x_n) = P(x_n | x_1, x_2, \dots, x_{n-1}) \times P(x_1, x_2, \dots, x_{n-1})$$

$$= P(x_n | x_1, \dots, x_{n-1}) \times P(x_{n-1} | x_1, x_2, \dots, x_{n-2}) \times P(x_1, x_2, \dots, x_{n-2})$$

$$= P(x_n | x_1, \dots x_{n-1}) \times P(x_{n-1} | x_1, x_2 \dots x_{n-2}) \times P(x_{n-2} | x_1 \dots x_{n-3}) \times \dots P(x_2|x_1) \times P(x_1)$$

$$= P(x_n | \text{Parents}(x_n)) \times P(x_{n-1} | \text{Parents}(x_{n-1})) \times \dots$$

$$= \prod_{i=1}^{n} P(x_i | \text{Parents}(x_i))$$

$$P(B|j,m) = \alpha \cancel{P(B|j,m)} P(B,j,m)$$

$$\alpha = \frac{1}{P(j,m)}$$

$$P(B|j,m) = \alpha P(B,j,m) = \alpha \langle P(b|j,m), P(\neg b|j,m) \rangle$$

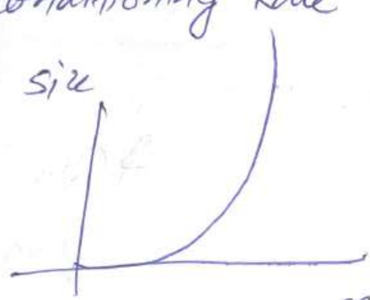
$$P(b|j,m) = \alpha \sum_e \sum_a P(b) P(e) P(a|b,e) P(j|a) P(m|a)$$

$$P(\neg b|j,m) = \alpha \sum_e \sum_a P(\neg b) P(e) P(a|\neg b,e), P(j|a) P(m|a)$$

$$\boxed{P(a|b) = \frac{P(a,b)}{P(b)} \Rightarrow P(a,b) = P(a|b).P(b)}$$

$$P(A|B) = P(A,B)/P(B) \Rightarrow P(A,B) = P(A|B).P(B)$$

$$\boxed{P(Y) = \sum_{Z} P(Y/Z)\,P(Z)}$$ → conditioning Rule



cavity ∧ tooth ∧ cadue tch   0 0 0

cavity ∧ tooth ∧ ¬catch   0 0 1

0 1 0

Pg

· There are n variables, table has 2^n values. Exponential

$P(Cavity, toothache) = \langle P(cavity, toothache), P(\neg cavity, toothache) \rangle$

$= \langle 0.6, 0.4 \rangle$

Cavity < true Cavity = true or cavity
false Cavity = false or $\neg$cavity

Marginlization:

$$\boxed{P(Y) = \sum_z P(Y, z)}$$

of

$P(Y) = \langle P(Y = true), P(Y = false) \rangle$

$P(Y = true) = P(Y = true, z = true) + P(Y = true, z = false)$

$P(Y = false) = P(Y = false, z = true) + P(Y = false, z = false)$

* Conditional Independence

$$P(a|b) = P(a) \text{ if } a \text{ is independent of } b.$$

Cavity, Catch, Toothache Weather

$$P(\text{toothache} \mid \text{cavity}, \text{catch}, \underline{\text{weather} = \text{true}}) = P(\text{toothache} \mid \text{cavity}, \text{catch})$$

$$P(\text{toothache}) = P(\text{toothache}, \text{cavity}, \text{catch}) + P(\text{toothache}, \text{cavity}, \neg\text{catch}) + P(\text{toothache}, \neg\text{cavity}, \text{catch}) + P(\text{toothache}, \neg\text{cavity}, \neg\text{catch})$$

$$= 0.108 + 0.012 + 0.016 + 0.064$$

$$P(\neg\text{cavity} \mid \text{toothache}) = \frac{P(\neg\text{cavity}, \text{toothache})}{P(\text{toothache})}$$

$$P(\neg\text{cavity}, \text{toothache}) = P(\neg\text{cavity}, \text{toothache}, \text{catch}) + P(\neg\text{cavity}, \text{toothache}, \neg\text{catch})$$

* Inference Using Full Joint Distribution.

	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.088
¬cavity	0.016	0.064	0.144	0.576

$$P(cavity \mid toothache) = ?$$

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)}$$

$$= \frac{P(cavity \wedge toothache)}{P(toothache)}$$

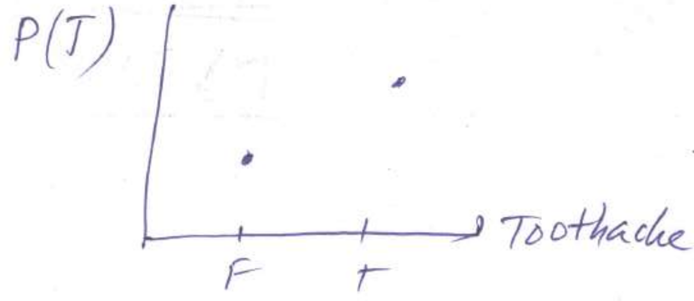
$\wedge = ,$

$$P(cavity, toothache) = P(cavity, toothache, catch) + P(cavity, toothache, \neg catch)$$

$$= 0.108 + 0.012$$

Joint Probability Distribution

Toothache $\langle$ T, F   Cavity $\langle$ T, F



	cavity	$\neg$cavity
toothache	X	Y
$\neg$toothache	Z	W

Joint Prob. Distri table

$P(toothache, cavity) = \underline{X}$ $P(\neg toothache, cavity) = \underline{Z}$

$P(toothache, \neg cavity) = \underline{Y}$ $P(\neg toothache, \neg cavity) = \underline{W}$

Product Rule:

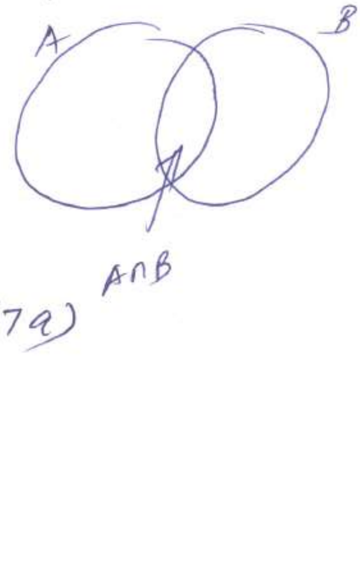
$$P(a \land b) = P(a|b)\, P(b)$$

* Axioms of Probability

1) $0 \le P(a) \le 1$

2) $P(true) = 1, \quad P(false) = 0$

3) $P(a \lor b) = P(a) + P(b) - P(a \land b)$



$A \cup B = A + B - (A \cap B)$

4) $P(a) = 1 - P(\neg a)$

$$P(a \vee \neg a) = P(a) + P(\neg a) - P(a \wedge \neg a)$$

$$\therefore P(true) = P(a) + P(\neg a) - 0$$

$$\therefore 1 = P(a) + P(\neg a)$$

$$\therefore \boxed{P(a) = 1 - P(\neg a)}$$

Conditional Prob.

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

$a|b$ = a given that b is true.

Experiment : Throwing a dice

O/p : $\{1, 2, 3, 4, 5, 6\}$

$P(a \wedge b) = P(a, b)$

a : outcome is 4

b : outcome is an even no.

$\underset{\text{Prior}}{P(a)} = \frac{1}{6}$ $\qquad$ $\underset{\text{Posterior}}{P(a|b)} = \frac{1}{3}$

$a \wedge b = \{4\}$

$P(a \wedge b) = 1/6$

$b = \{2, 4, 6\}$

$P(b) = 3/6$

$P(a|b) = \frac{1/6}{3/6} = 1/3$

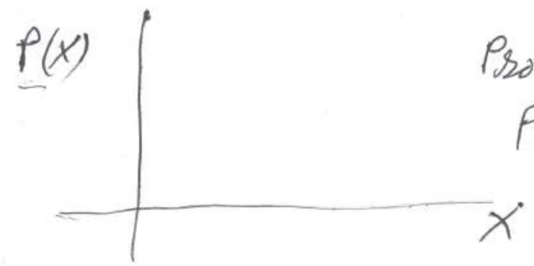
* X = water level of a river

RV

Discrete
• boolean

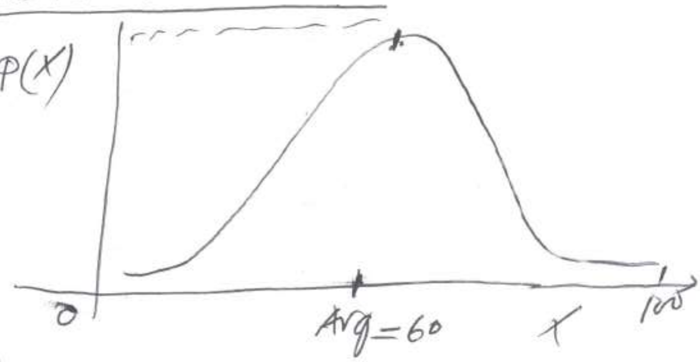
Continuous

$X \in [0, 500m]$



Prob. Density Function

* X = marks of a student



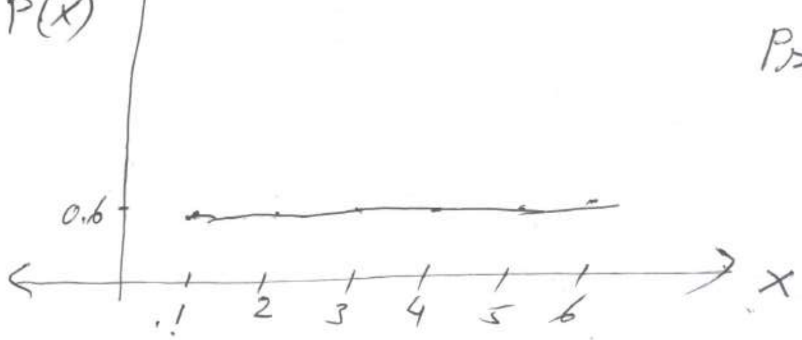
- Experiment : throwing a dice
  Use RV X to store the outcomes.

  X will take any value from 1 to 6.

$P(X=1) = 0.6$ $\quad P(X=4) = 0.6$

$P(X=2) = 0.6$ $\quad P(X=5) = 0.6$

$P(X=3) = 0.6$ $\quad P(X=6) = 0.6$



Prob. ~~Density~~ Distribution Function of X.

Basic Probability Notation

```
main()
{
 r=rand();
 if(r≤0.7)
   pt("hi");
 else
   pt("hello");
}
```

$P(a)$ – Prob. of event a.

$0 \le P(a) \le 1$

$a \to P(a) = 0.7$

~~a~~ Experiment – Toss a coin

a – outcome is head $P(a) = 0.5$

RV – Random Variable (Stochastics)

- Used to store ~~of~~ outcome of an experiment.

```
count = 100
while (~~count~~ i ≤ count)
{
 r = rand()
 if (r ≤ 0.5)
   heads++
 else
   tails++;
}
```

Reasoning With Uncertanity

$KB \not\models \alpha$?

$B[2,2] \Leftrightarrow P[3,2] \vee P[1,2] \vee P[2,3] \vee P[2,1]$

$\forall p$ Toothache$(p) \Rightarrow$ Cavity(p)

$\forall p$ fever$(p) \Rightarrow$ Covid19$(p) \vee$ Malariya$(p) \vee \ldots \vee$

Probabilistic Reasoning